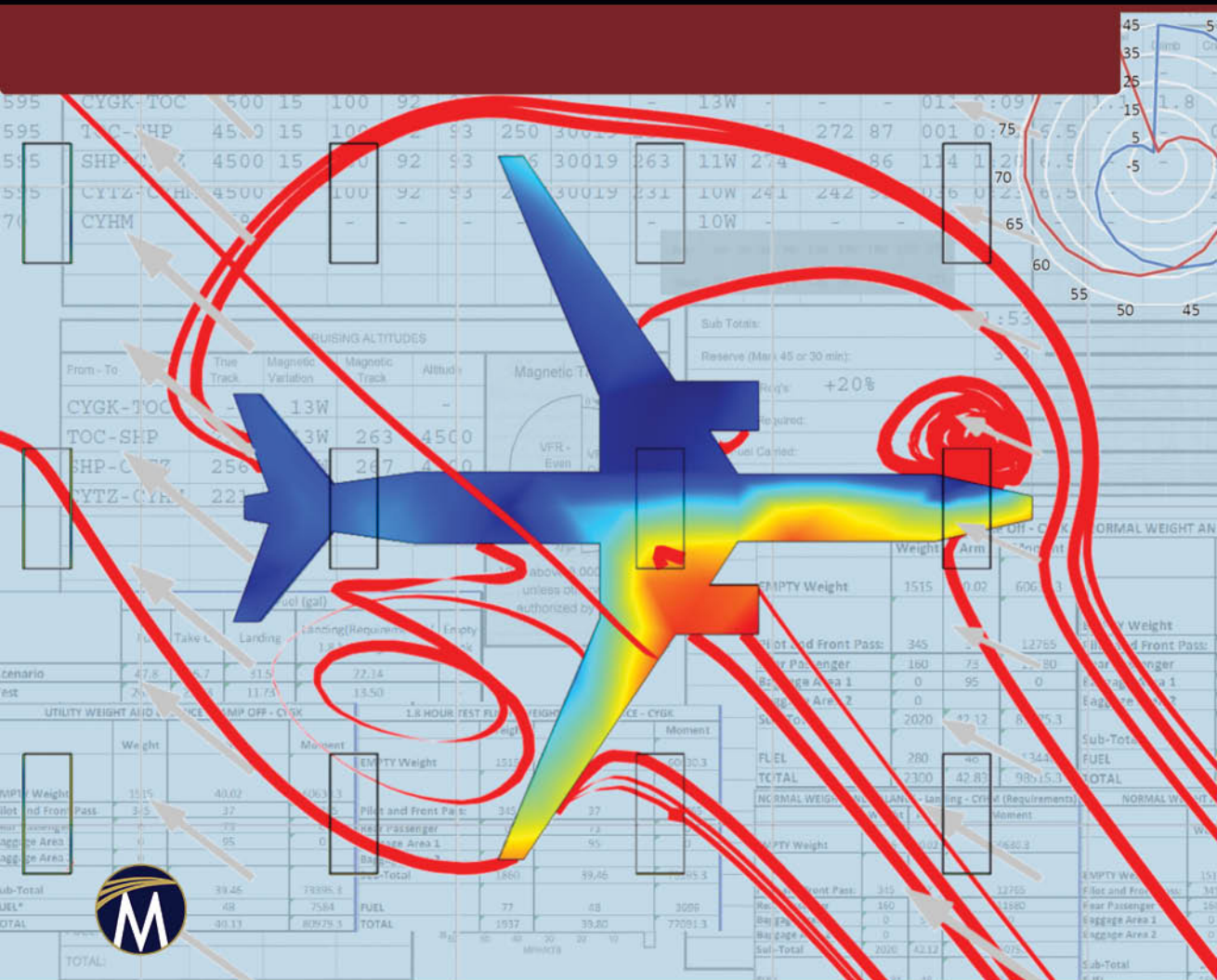


FLIGHT SCIENCE

Mathematics • Techniques • Sensibility



L. S. MAYBOUDI

FLIGHT SCIENCE

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FLIGHT SCIENCE

MATHEMATICS • TECHNIQUES • SENSIBILITY

Layla S. Mayboudi, PhD



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Boston, Massachusetts
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Publisher: David Pallai
MERCURY LEARNING AND INFORMATION
22841 Quicksilver Drive
Dulles, VA 20166
info@merclearning.com
www.merclearning.com
800-232-0223

L. S. Mayboudi. *FLIGHT SCIENCE: Mathematics, Techniques, Sensibility.*
ISBN: 978-1-68392-215-5

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Library of Congress Control Number: 2018964989

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It is high time. . . . To my parents, Captain Mayboudi, my father, the top student and captain in the Boeing 747 and first officer in the Airbus 300, and at home, the second-in-command to my mother, Mrs. Mayboudi.

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PREFACE

Only by the power of knowledge can one pass through the regions of the Sky and the Earth [1].

Am I Runway Ready – Lights, Camera, Action

Learning to fly is a decision of a lifetime. This work is intended for aviation enthusiasts in particular and life enthusiasts in general, combining technical flight-related topics with a personal journey. Both a reader new to the field working toward a recreational, private, or commercial license as well as a reader who is already flying a single-piston or multi-engine aircraft will benefit from the content, and so will the readers who do not know how to make that first step, the dreamers who are equipped with the wide-angle vision of becoming a pilot, taking this challenge as a hobby or a profession, whatever their age.

It is a common misconception to think that success is a “random act of luck” because some people make it look so easy, so accessible, and so effortless. No, it is not. The journey is long and eventful, and a wide range of events, expected and unexpected, happen along the way. When it comes to flying, the smallest changes, from the environment to personal rituals such as diet, exercise, or studying habits, can make a difference. A lot of things can contribute to success—small things such as how snug your belt is, how often you drink water from a bottle, having bananas, taking organic ginger supplements before each flight, the flying club you adopt as your new family, the instructor(s) you pick, and your relationship with the aircraft and how respectfully you treat it the minute you set your eyes on that beauty.

What can you expect along this journey? The moments of tears and joy when flying solo or duo; the first solo experience; the short and long cross-country flights; the first time on top of the clouds; the haze spread by the wind turbines in the distance; the walkaround on a quiet morning with birds singing, the Sun shining, and dew covering the ground; the challenges of controlling the aircraft when flying low during a crosswind landing—where you scramble to hold on to the throttle but settle for the instrument panel instead, since the throttle is already taken—fighting with mother nature; diving into the study resources, taking online webinars, watching YouTube videos, reviewing official information resources such as the Aeronautical Information Manual (AIM), and surfing the web for more information; they are all part of that original decision and the journey. They are all part of the experience, like musical notes in a symphony.

This book includes technical information which should help you understand the related sciences and technology that go into flying. It also talks about the do’s and don’ts, and what to watch for when starting out as a novice along the journey to become a professional pilot. This journey is full of events, a lot of enjoyment,

and also moments that make the bleeding heart speechless. The purpose is to help and encourage the reader to persist in their dreams in a safe fashion. Thus, move in the direction that your heart desires, to the tune to which it sings and dances. Move toward the good ethical tingles in the bottom of your belly; your love and passion at moments of doubt and defeat; the smile and the warmth that fills your heart when you fly an old tail dragger for the first time, and the excitement of landing it—looking forward to soft-landing on a grass field; the dream to take off and land a floater from the neighborhood lake bordered by maple trees; the full *à la carte* of emotions, colored with shades of saffron, spread by mother nature with utmost perfection when fighting for control of the airplane, when flaring to a safe destination; the reactions of your seven chakras (mainly, heart, solar, sacral, and root ones) to everything around you, knowing that you are grounded, deeply rooted, even if you are on your own, all by yourself, soaking your feet in one of the Great Lakes. Flying is not loving the smooth and hating the bumpy sky; as it turns out, indifference is the opposite of love and not hate. It is all about professionalism, work ethic, understanding, deep respect, and self-control. Are you runway-ready? Lights, Camera, Action.

Road Map

A *road map* is an approach or stratagem intended to accomplish a specific goal. This section provides a road map for this work. It presents a clear picture of where the journey starts, is headed, and ends—which is really the start of the next journey. The recommended stepping stones as well as the milestones are provided. This book introduces certain aspects of science and mathematics, including mechanics and dynamics, in the context of topics related to aviation, from the wheels-up to the wheels-down, and the time in between—in that order. The author recommends following this sequence, without skipping sections; furthermore, the author encourages you to attempt the problems to make the experience more rewarding. There are a number of case studies presented at the end of each chapter under the title “Examples” that are to be studied carefully to improve the reader’s comprehension of the chapter’s subject.

Chapter 1 introduces the connection between aeronautics and thermo-fluid sciences, which comprise thermodynamics, heat transfer, and fluid mechanics. The focus is on the historical significance of these sciences and their contribution to aeronautics. Additionally, the history of progress in atmospheric thermodynamics is highlighted. An introduction is given to thermodynamic diagrams to characterize weather in the short term and climate in the long term. The concept of energy balance as it applies to a parcel of air (or any other medium) is provided.

Chapter 2 focuses on meteorology related to flying to that dream destination, such as the factors affecting the decision of whether to fly, given the flight rules,

either visual or instrument. Examples of weather minima are provided, and the reader is encouraged to check for regulations when flying in different regions.

Chapter 3 discusses climate and atmospheric thermodynamics, the layers and their characteristics, variation of temperature with the altitude—also known as the lapse rate, temperature as an interdependent property along with related terms such as dry-bulb, dew-point, virtual, potential, and equivalent temperatures and the influence it has on the speed of sound. Pressure, another interdependent property, is discussed afterwards along with related terms such as stagnation and dynamic pressure, and pressure altitude. Airspeeds, such as indicated, calibrated, equivalent, and true, are defined, accompanied by the relationships that present them for both subsonic and supersonic speeds. Other thermodynamic properties, including density, density altitude, humidity, enthalpy, and their effects on the atmospheric stability, conclude this section.

Chapter 4 is about thermodynamics of air systems. It starts with an introduction of thermodynamics of air in general, progressing to the characteristics of moist air, and introduces the models that characterize weather in the short term and climate in the long term. The most accurate methods are selected and presented, including well-known weather diagrams such as the Stüve diagram, Emagram, Tephigram, Skew-T Log-P diagram, and Theta-Z diagram, as well as the Psychrometric Chart as a more general diagram to obtain properties of the moist air. Weather data, mainly consisting of dry-bulb and dew-point temperatures, as well as wind information such as magnitude and direction as a function of the altitude, are plotted on these diagrams; other properties such as lifting, convective condensation, and equilibrium levels, as well as equilibrium temperature, can also be calculated. Stability criteria, such as convective available potential energy and convective inhibition, can also be estimated using charted weather data. Weather reports including aviation routine weather reports, pilot reports, and terminal aerodrome forecasts are presented. A section on wind concludes this chapter.

Chapter 5 focuses on the dynamics of air and applicable forces. The importance of the shape of airfoils is discussed. Historical examples are provided to present airfoils that made a large impact on the aviation industry. The influence of angle of attack on lift and drag forces is discussed. The difference between pitch angle and angle of attack is clarified. The forces applied to descending and ascending aircraft, such as weight, drag, lift, and thrust, are discussed in the last portion of this section. A real-world example as part of a case study is presented in this chapter: the first electric aircraft ever, made in Perth, Australia (Electro.Aero), which is environmentally friendly with an approximate continuous flight time reaching 1.5 hrs and with growing potential.

Chapter 6 discusses the piston engine and the combustion process. Some aircraft systems, such as electrical, fuel, power plant, and vacuum subsystems, are reviewed. The mechanical implications along with thermo-fluids are the center

of attention—conservation of energy and mass in particular. Thermodynamic relationships for the flow inside the Venturi tube are derived. The combustion process is investigated in some detail, with the octane and heptane fuels considered individually and also in combination. The chemical formulae, mass-balance relations, and methods to calculate the heat of combustion are presented for the complete combustion process as well as for one with excess air in the products, resulting in the production of extra oxygen, nitrogen, and carbon oxides.

Chapter 7 presents an overview of unmanned flying objects, including drones. Their purpose and a brief history of their development is provided along with the advancements they have undergone up to their current state. These objects are either unpowered or equipped with their own propulsion systems. They can also have different configurations. For example, they may be fixed-wing and require some distance for takeoff and landing, or rotor style, like copters, requiring little space to take off and land. Each of them has its own characteristics, including maneuverability and operational requirements. The applications of various unmanned flying objects are identified: they may be used in combat missions or for taking photographs of natural beauties or disasters. The mechanical forces applicable to the drones are discussed as the next step, followed by the introduction of the proportional, integral, and derivative control concepts.

Chapter 8 discusses performance charts. This part is for aviation enthusiasts but also for math enthusiasts who want to apply their mathematical tools as a “hammer” to “nail down” all kinds of problems; the “hammer” used here is *regression analysis*. There were times when making complex calculations would have taken hours or even days. Nowadays, with the progress of computing tools and the introduction and advancement of high-performance computing machines and virtual laboratories to facilitate computing capabilities among educational institutions, the difficulties of the past (perhaps only twenty years ago) are long forgotten. The author recalls the master’s supervisor’s comment on the need to queue in order for a Fortran card to be processed (punched) when completing one’s doctoral thesis; it would have taken hours for the computational results to be produced, and if revisions were required, another set of cards would need to be created and put in a queue for a new set of calculations, and so on. Nowadays, computational lives seem so hassle-free, don’t they? Therefore, the engineer-mathematician devotee should explore some of these methods and tools to gain a better understanding of the current state of affairs of the mathematical world and the capabilities it can offer to enhance the performance of any organization.

In the aviation industry, performance chart data are one potential area for application of regression analysis methods, which can easily be developed into a long-time relationship with *regular customers*. The reader may be familiar with the concept of the *customer* emphasized in *performance review charts* where, in a matrix management environment, those in a position of authority relating to certain tasks can ask a colleague to complete engineering tasks in a timely

fashion—also known as internal and external customers—which is applicable to one’s next-door colleague or a colleague working for a parent company somewhere overseas. On performance charts, the influence of predictor data such as pressure altitude, temperature, and revolution per minute on response data such as true airspeed or brake horsepower can be investigated. Additionally, the takeoff and landing data may be analyzed given the pressure and temperature altitude, and the results may be employed in making urban planning decisions when designing and building aerodromes or runways, or when selecting aircraft to operate at individual runways. It seems relevant that only after thousands of hours of individual, dedicated work along with millions of hours of collective work, a detailed data analysis can be performed with results applicable to recognition of the product, its efficiency, and the direction in which the design and production of the future platform is headed.

Chapter 9 is about navigation. This topic is one of the most important elements of flying; perhaps it is the concluding one, as all the previously learned skills are to join together and make sense of the entire experience. The significance of this field is evident in how it has been included as part of a liberal arts education. Navigation is an *art*, and that must be for the lives, countries, and continents it has saved during history from the time of Magellan, when land and sea exploration and navigation were of the utmost importance for trade and power expansion; to the time of Galileo, when space exploration and navigation became an irresistible thought for curious minds, the *outliers* who were introduced as “fugitives” of their time and were ridiculed, shunned, interrogated, and executed. If it were not for those individuals, the Earth would have still been flat and the center of the universe, with the Sun as one of its moons.

The concepts of mapping and land surveys are discussed as well, explaining how King William the Conqueror introduced the idea of bookkeeping land locations along with their areas and owners, which was further developed by Napoleon Bonaparte in France and spread to the rest of Europe. Concepts such as latitude, longitude, great circle, rhumb line, variation, zenith, and azimuth are also discussed. Coordinate systems are discussed, as well as vector algebra to provide tools required to connect the dots on the map, to find one’s way. Charts are then discussed, including the Mercator Conformal Projection Chart with its most useful variations (Transverse and Oblique), the Lambert Conic Conformal Projection Chart, and the Polar Stereographic Projection Chart. Pilotage and dead-reckoning are introduced as methods to navigate through unknown lands. Related subjects including time zones, distances between locations on the Earth, identifying geographical references for locations using grid systems, and the use of an almanac to calculate the Sun’s position when doing celestial navigations are presented as well.

Chapter 10 discusses the flight plan, the modern heir to navigation. Terminologies such as visual and instrument flight rules are presented along

with tools such as direction measurement equipment systems. The concept of airworthiness is discussed in detail (what it takes for an aircraft to fly) as well as weight-and-balance (how to ensure the forces on the aircraft are in equilibrium). Technical methods of communication and line of sight are discussed. Wind as a vital element in making flight plans is presented, along with how to use it in calculations to determine true and magnetic headings. It is explained how the crosswind and headwind components affect flight time and ground speed. Cross-country flight planning is covered, including diversions. Practices that assist with making cross-country flights more pleasant and rewarding experiences are shared with the reader. The collection of case studies presented in this chapter may be used as a guide when the reader attempts to make a detailed flight plan in a flight simulation or in real-life situations.

Chapter 11 is dedicated to *special scenarios*; these are the cases in which ground speed calculations, given desired and constant headings, are formulated. Point of safe return as well as critical point are discussed along with derived equations and applicable conditions. Methodologies to calculate ground speed as an approximate or exact technique are provided. Diagrams are shown presenting the relationships between ground speed, crab angle, wind magnitude and direction, and distances (point of safe return and critical point). Precision approaches are briefly discussed, and relationships used for non-precision approaches, such as rate of descent, descent angle, and gradient of descent, are derived. The mechanics and glide distance in an engine failure scenario are presented. Gyro errors are discussed along with their possible sources. Cabin pressurization as a technological advancement and also its effect on human physiology are discussed. Related formulae are presented to determine the maximum altitude at which an aircraft is capable of flying—given the difference between the interior cabin pressure and that of the exterior.

Chapter 12 presents a case study based on a recently carried out, self-made conversion of a plane from a piston engine to a turboprop (the Wilga *DRACO*) in the United States that was first presented at the 2018 Experimental Aircraft Association (EAA) Air Venue in Oshkosh. This aircraft is a powerful monster with a takeoff and landing performance almost rivaling that of a helicopter, with backcountry flying abilities that can only be envied. Mechanical forces (i.e., lift and drag) and simulated lift and drag coefficients are analyzed along with a simulated flight plan that considers wind magnitude and direction along with magnetic variation. The minimum true airspeed required to keep the aircraft aloft is calculated for different angles of attack.

Chapter 13 provides some insight on preparation for a flight, including upper air work and groundwork on regular days and the day of the check ride, the test day. It talks about good practices that prepare the student to take steps as efficiently as possible, to make the experience pleasant, more rewarding, and to promote good airmanship.

Chapter 14 talks about good practices and lean six sigma implementation applicable to engineering and non-engineering fields in which the product lifecycle (Conceive, Design, Implement, and Operate) is used. It talks about preferred approaches to flight training and learning, as well as whether it is even appropriate to talk about a *preferred* approach. Crew resource management after interactions among flight crew members have gone awry has been given plenty of attention. Thousands of hours have been dedicated to analyzing what happened before, during, and after each incident, and only a few of them point fingers at the responsible individual(s); these examples are meant as learning experiences and to make things better in the future.

Chapter 15 contains concluding remarks, the last remarks on practicing good judgment when organizing flight-related activities, including learning. It provides some examples of recent developments in space exploration and what is to come next. This work coincides with NASA celebrating its sixtieth anniversary; the fiftieth anniversary of the Apollo program; the launch of Parker—the Sun’s first close-orbiting satellite and the fastest among its peers—by NASA; the launch of Falcon Heavy, a space launch vehicle designed and manufactured by SpaceX, to Mars’s orbit; the launch of ICESat-2—a laser-armed satellite—into space to monitor the seasonal change of ice thickness with an accuracy of a fifth of an inch; a lunar masonry study by ESA to explore how one could use lunar dust for making habitable structures on the Moon; and a solar-powered version of the Zephyr drone, also called a high-altitude-pseudo-satellite, by Airbus, setting a flight record of approximately 26 days, and flying at an altitude of 70,000 ft (21,300 m). This is only to name a few examples.

The flight experience may be chosen as a profession or hobby; nevertheless, how it is perceived does not necessarily reflect the sense or level of commitment that the adaptor brings to the experience or feels deep down. It does depend on the will and level of competency to take the experience to higher altitudes and make it meaningful for the self or others.

If you are a student working toward your private or commercial pilot license, you can expect to be supported by your flight school. If such support does not exist or the instructor tells you that “you will have to pay me to give you instructions,” you should seriously consider switching the instructor or flight school before wasting any more money.

Finally, all the uncredited illustrations in this work were generated by the author using Computer Aided Design and other software. Modern liberal arts of history, languages, literature, mathematics, philosophy, and science along with their medieval predecessors (i.e., arithmetic, astronomy, geometry, grammar, logic, music, and rhetoric) are closely intertwined throughout this work as they are also in the world around us.

ACKNOWLEDGMENTS

Although I recall the individuals and instances that have had a positive influence in my life, from my family, teachers, professors, and publisher to the janitor of the aerospace company I once worked for, I cannot fully express the depth of my gratitude toward these individuals whose sacrifices and kind, polite, welcoming, and understanding attitudes have been deeply touching. They guided me during this long voyage by improving upon my perceptions and making me envision new possibilities. They did all that with respect and love, which I remember as this work is being completed.

I am grateful to my father and his colleagues—pilots, engineers, doctors, and innovators—who, despite technical challenges, protected the independence and sovereignty of their land and its people, both in times of war and peace.

I also wish to recognize the good pilots and astronauts (both real and virtual) who keep cool when heated; whose virtue, honor, temperance, perseverance, patience, prudence, and proven character endure with persistence to accomplish the purpose to an eternal perspective; and who make it possible for the rest to dream, live a fulfilling life, feel safe, love, and respect themselves and others.

The world of scientific publication entails challenges but also rewards, especially at the time of completion. This work, from its conception to its publication, underwent a long journey, and numerous challenges; thus, it has been entwined with a personal voyage.

My thoughts on the aforesaid individuals are as follows:

*There are some who raise you up to be a better person than
you have aspired to be . . .*

There are some who put fire in your heart to create and to succeed . . .

*There are some who bring hope and love of life back to you
when you are tired and hopeless . . .*

There are some who make positive impacts in your life . . .

*There are some who nurture you and give you wings to fly to the
wild blue yonder and to continue flying . . .*

*There are some who encourage your confidence, dignity, and
humanity so you rise after you fall . . .*

There are some who walk on the earth and spread love and respect . . .

There are some who keep giving with minimal or no expectations . . .

*There are some who believe that life is about dreams and
teach others how to make them come true . . .*

There are some who bring you a towel to dry your tears and sweat . . .

*There are some who stand by you in adversity when you need it most . . .
And there are others . . .*

CONNECTION BETWEEN AERONAUTICS AND THERMO-FLUID SCIENCES

Looking up *flight* in a dictionary, you will find several meanings: (a) traveling through the air like a bird or an airplane, (b) a flock of birds in the sky, (c) a number of military planes flying in formation, (d) an escape, (e) a set of steps between two levels, (f) an extravagant or implausible idea or the process to reach one, and (g) a tail of an arrow [2].

Each of the previous definitions in some way relates to the idea of flying. All cases entail an action or object that ideally reaches for something distant. The remoteness of the concept or thing, and the mission to grasp it, has been the subject of speculation by humans for many years. Humans have expressed these thoughts throughout the ages as they enviously watched the creatures capable of flight [3].

1.1 Thermo-Fluid Sciences

Thermodynamics, fluid mechanics, and heat transfer, known as thermo-fluid sciences, are the three sciences that contribute to atmospheric thermodynamics and fields of study associated with climate variation, buoyancy of flying objects in the air, and heat and work created to make the operation of buoyant systems and subsystems possible. Examples of the latter are the combustion process for piston and jet engines and force balance in flight dynamics. The historical significance of these sciences and their contribution to aeronautical sciences, especially in flight-related subjects, are the focus of this work in general and this section specifically. The following paragraphs provide a brief historical overview of these sciences.

Thermodynamics is related to temperature and change of internal energy and their association with other forms of energy such as heat or work. Four laws of thermodynamics have been identified as the main governors of the physical phenomena occurring in nature: (a) zeroth law—two systems each in equilibrium with a third system are in equilibrium with each other, (b) first law—internal energy of systems does not change, (c) second law—heat flows from higher temperature to lower temperature, and (d) third law—entropy of a system approaches zero as its temperature approaches absolute zero. A system is defined as a group of particles whose interactions and properties are defined by the equations of state. The molecules are not necessarily homogeneous, and therefore their properties may be defined by means of thermodynamic laws for reacting and non-reacting mixtures that consider each property and its significance [4,5].

Fluid mechanics describes the mechanics of fluids (e.g., gases, liquids, and plasmas), including the forces applied to them or exerted by them. The latter is commonly known as the third law of Newtonian mechanics or action-reaction law. It states that for every action, there is a reaction that is equal in its magnitude but opposite in its direction. Depending on the spatial and temporal variation of fluid molecules, fluid statics or fluid dynamics disciplines apply. The former involves studying fluids at rest, and in the majority of cases in an equilibrium state, while the latter involves the study of fluids in motion. For example, the oleo system of the Cessna aircraft takes advantage of a hydraulic-pneumatic system and, combined with a dynamic system, acts as a shock absorber, softening the impact of the nose wheel on the runway when landing. Fluid statics, also known as hydrostatics, investigates the behavior of a still fluid when it is in equilibrium, such as change of atmospheric pressure with altitude. If you wonder about the flatness of liquids within any container, hydrostatics provides the explanation. Fluid statics is the foundation for use and transfer of liquids in engineering applications. Fluid dynamics is related to fluid flow, focusing on the active forces applied to a stationary or moving buoyant object submerged in either moving or stationary flow. The nature of this complex interaction is investigated in computational fluid dynamics [6,7,8,9,10,11].

Heat transfer is the science that examines the effect of temperature change in physical phenomena. It involves spatial and temporal change of internal or external temperatures, whether it is a closed system, in which only energy in the form of heat and work crosses boundaries; or an open system, where mass in addition to energy (heat and work) can cross boundaries. A heat transfer process may also include cases of temperature

remaining constant for the entire period (isothermal); for example, when a phase change occurs. Heat transfer involves two main modes: conduction and radiation, depending on the media in which the energy is transferred. For conduction to take place, either in its pure or subsidiary forms (such as convection, where the gaseous molecules come in contact with solid surfaces), matter (as atoms or molecules) needs to be present. When fluid in the form of gas comes in contact with a solid surface, the heat transfer mechanism takes a hybrid approach. It combines the conduction mode in solid and liquid. Additionally, the momentum of the bulk of the fluid contributes to transferring some of the heat (advection). In the radiation mode of heat transfer, electromagnetic waves transmit energy. This explains why the Soyuz (Союз) spacecraft is covered with a thermal blanket—the blanket protects it from overheating when exposed to the Sun’s radiation. Isohels (lines of constant solar radiation of the same intensity as sunshine) may be employed to identify the areas of space where the spacecraft is exposed to maximum solar radiation. This also explains why we are interested in thermal modeling of satellites when they are propelled into Mars’s orbit. Excessive heat caused by the radiated energy may damage the electronics and main body parts, but it also may be harvested to generate power for the onboard equipment [12,13].

Each of the previously discussed sciences embraces natural physics and utilizes analytical laws, those derived from mathematical relations, as well as empirical laws, those obtained from experimental observations, to correlate these two physical modeling approaches. In most of these studies, the purpose is to calculate temporal and spatial fluid properties such as temperature, pressure, density, and velocity. Depending on the type of fluid, either liquid or gas, more specialized disciplines of hydrodynamics and aerodynamics were developed. The laws affecting the latter are relevant to this work. Aerodynamics investigates the flow patterns, momenta, and forces over an aircraft, or any flying object in general, and predicts the evolution of weather [14,15].

1.2 A Brief History of Temperature Measurements

Thermo-fluid sciences share an interesting historical background that needs to be appreciated, as these sciences allowed humans to understand the forces of nature that they were to reckon with when venturing into the sky. So, how did humans get to the present state of knowledge in these fields? Let us take a brief historical journey.

The development in these fields can be said to have started when the need for measuring temperature was first recognized. The idea of heat dates back to the prehistoric era, mainly associated with fire as part of the primitive categorization of elements into four—air, fire, water, and earth. It has been referenced in a Greek text called *Kore Kosmou*, attributed to Hermes Trismegistus. First efforts in this field are ascribed to Heron of Alexandria, a mathematician and engineer, from the Roman province of Egypt (about 10 AD to 70 AD) who noticed air contraction and expansion within a tube partially filled with air whose end was submerged inside a tub of water. In 1593, Galileo Galilei, an Italian polymath, employed this concept and invented the thermoscope, which worked based on changes in sensible heat. The air within the tube with a balloon-shaped, air-filled region was heated, and due to the changes in the temperature, the level of water changed as well, causing water to experience perpetual motion.

One may argue that Galileo set the stage for an atomistic conception of heat transfer. The term *temperature* was still unknown at that time. In 1624, Jean Leurechon, a French mathematician, first used the term *thermometer* for a similar device that was in fact a barometer as well. Galileo's student, Joseph Solomon Delmedigo, an Italian physician, mathematician, and music theorist, reportedly was the originator of the sealed liquid-in-glass thermometer. However, Ferdinando II de' Medici, an Italian, primarily a musician, in 1654 produced the latter instrument, which was independent of air pressure and only relied on expansion of the liquid. A number of thermometers were subsequently experimented with using different liquids and tubes. The first standardized one that was proposed based on melting and boiling points of water was in 1694 by Carlo Renaldini, a French philosopher and experimenter. The one proposed by Sir Isaac Newton, a British natural philosopher, in 1642, was based on the 12-degree scale between the ice melting point and body temperature. It was only in 1714 that Daniel Gabriel Fahrenheit, a Dutch-German-Polish scientist, introduced the concept of the modern thermometer with an accurate 96-degree scale that he developed based on mercury's high coefficient of thermal expansion between the equilibrium temperatures of ice-water-ammonia chloride, ice-water, and human body. In 1742, Anders Celsius, a Swedish scientist, proposed a 100-degree scale between the freezing and boiling points of water. In 1848, William Thomson, a Scottish mathematician and engineer, later known as Lord Kelvin, introduced the idea of absolute zero, a negative reciprocal of the expansion coefficient of gas (0.00366) at the water freezing point per degree Celsius. One Kelvin is also defined as the fraction (1/273.16) of the triple point of water (0.01 °C) which bears the Celsius scale.

Francis Bacon, a British scientist, philosopher, and statesman, distinguished between heat and temperature in 1620 [16]; however, Joseph Black, a Scottish physician and chemist, quantitatively distinguished between the two. He applied the conservation of mass—earlier suggested by Newton and adopted by Antoine-Laurent Lavoisier, a French chemist, for all chemical reactions—and developed calorimetry and latent heat. Specific heat that accompanies melting or vaporization was introduced by Joseph Black (1763), who also articulated the theory of latent heat. Black's work was later published by John Robinson, a Scottish scientist, in 1803 [13].

1.3 Atmospheric Thermodynamics Through the Ages

In 1824, French physicist Sadi Carnot proposed the equivalency of heat and mechanical work, the principle which became known as the Carnot cycle [17,18,19,20]. Based on his observation, every thermodynamic system in whatever state, when moving from the initial state through intermediate states and eventually getting back to the initial state, is said to have completed a thermodynamic cycle. This system may exchange energy in the form of heat and work with its surroundings. The conversion of these two main forms of energy to each other is the basis for the Carnot cycle. The Carnot heat engine is an application for a perfect Carnot cycle that has been achieved on a microscopic level. Carnot also measured the equivalent work done by a gas expanded under isothermal conditions. In 1851, a Scottish mathematician William Thompson re-emphasized this equivalency and that heat is not a substance and presented *Dynamical Theory of Heat*, which developed into the science of thermodynamics. Terms such as *specific heat*, *calorimetry*, and *combustion*, introduced in the nineteenth century, were rooted in this theory.

Benoît Paul Émile Clapeyron, a French engineer and physicist, was another founder of modern thermodynamics through his most well-known work, *Memoir on the Motive Power of Heat*, published in 1834, based on Sadi Carnot's work on theory of heat and work and employing caloric theory. He introduced the pressure-volume diagram, known today as a Clapeyron's graph. Clapeyron's other influential work is on introducing optimal valve position for a piston in an internal combustion engine, hypothesizing which valves are to be opened and closed during a piston-engine operation [21]. The second law of thermodynamics, related to reversible processes and rooted in Carnot's investigation of heat—is also among Clapeyron's contributions. Characterization of perfect gases—similar to ideal gases but in which the Van der Waals forces are neglected—is another contribution made by this scientist [22].

Rudolf Julius Emanuel Clausius, a German physicist and mathematician, is another important contributor to our knowledge of thermodynamics [23]. He redefined the Carnot cycle's theory of heat on a truer and sounder basis and is well-known for his 1850 paper titled "On the Moving Force of Heat," in which the second law of thermodynamics is presented. He also introduced the concept of entropy in 1865 and the virial theorem of heat in 1870. His work was further expanded by Clapeyron, resulting in the formula known as the Clausius-Clapeyron relationship related to phase transition. Clausius is the figure associated with the Stefan task, also known as the Stefan problem, in which the boundary column changes with time during a phase transition process [24].

The work of these three main contributors (Carnot, Clapeyron, and Clausius) on the development of thermal sciences led to the development of key theoretical and empirical models for atmospheric thermodynamics that started in the 1860s with the introduction of dry and moist adiabatic concepts. From the middle of the eighteenth to the end of the twentieth century, scientists introduced properties and concepts that are now used in the discipline of atmospheric physics. The highlights of these developments are: (a) the definition of the dew-point temperature in 1751 (Charles Le Roy—a French physician and encyclopedist [25]); (b) measuring pressure and temperature using hydrogen balloons (Jacques Charles—a French inventor, mathematician, and balloonist in 1782 [26] and Joseph Louis Gay-Lussac—a French chemist and physicist in 1804 [27]); (c) the concept of vapor partial pressure in 1802 (John Dalton—a British physicist and meteorologist [28]); (d) the variation of temperature with altitude in 1805 (Pierre Simon Laplace—a French scholar [29]); (e) convection energy for cyclones in 1841 (James Pollard Espy—an American meteorologist [30]); (f) the concept of potential temperature in 1889 (Hermann von Helmholtz—a German physicist and physician who used adiabatic lapse rate and pseudo-adiabat [31] and John William von Bezold—a German physicist and meteorologist [32]); (g) an areological sonde to correlate pressure-temperature-humidity in 1893 (Richard Assmann—a German meteorologist and physician [33]); (h) the concept of equivalent temperature in 1894 (John Wilhelm von Bezold—a German physicist and meteorologist); (i) the introduction of the tephigram in 1926 (Sir Napier Shaw—a British meteorologist [34]); (j) describing the influence of condensational growth of ice crystals in the presence of water drops in the formation of precipitation from supercooled fluid in 1933 (Tor Bergeron—a Swedish meteorologist [35]); (k) the first cloud-seeding experiment in 1946 (Vincent J. Schaeffer—an

American chemist and meteorologist [36] and Irving Langmuir—an American chemist and physicist [37]); and (l) the conceptualization of tropical cyclones as Carnot heat engines in 1986 (Kerry Emanuel—an American meteorologist [38]).

Heinrich Rudolf Hertz, a German physicist who proved the existence of electromagnetic wave theory by James Clerk Maxwell and whose name is associated with Hertz—the cycle per second unit—created the first thermodynamic diagram known as the emagram in 1884 [39]. This diagram is mainly used in European countries. Willis Haviland Carrier, an American engineer well known for modern air conditioning, introduced in 1904 a psychrometric chart, now used extensively by ASHRAE. This chart is a graphical representation of the main thermodynamic properties of the moist air—such as relative humidity, dry-bulb temperature, and dew-point temperature at a specific altitude—governed by equations of state. Napier Shaw, a British meteorologist, invented the tephigram in 1915; it is used mainly in commonwealth countries. The coordinates of isotherms and isobars have been skewed and have slight curves. In 1927, Georg Stüve, a German meteorologist, developed what we now call a Stüve diagram as a means for plotting dry-bulb temperature and dew-point temperature data using radiosonde measurements [40]. In 1947, Nicolai Herlofson, a Norwegian meteorologist, proposed a modification to emagram coordinates that depicts horizontal-straight isobars and displays a wide angle between isotherms and dry adiabats [41]. This diagram, similar to the tephigram, is known as a skew-T log-P diagram and is still used in weather forecast applications and especially by glider pilots to determine thermals and the associated cloud base.

Johann Friedrich Wilhelm von Bezold—a German physicist and meteorologist known for discovering the Bezold effect—identified a pseudo-adiabatic process describing parcels of air during expansion, contraction, and precipitation in 1888 when he also published his substantial work *On Thermodynamics of the Atmosphere* [32]. Samuel Pierpont Langley, an American astronomer, physicist, inventor of the bolometer, and aviation pioneer, published a widely read paper entitled “On the Moon Observations” in 1890 [42]. Frank Washington Very, an American astronomer, applied a bolometer to measure the temperature of the surfaces of the Moon and other planets [34]. In 1891, Very published his paper entitled “Distribution of the Moon’s Heat,” which also included measurements taken during a lunar eclipse. He published *Radiant Properties of the Earth from the Standpoint of Atmospheric Thermodynamics* in 1919. Craters on Mars and the Moon are named in Very’s honor. Alfred Lothar Wegener, a German polar

researcher, geophysicist, and meteorologist who published *Thermodynamics of Atmosphere* in 1911, is the pioneer in the application of thermal sciences to atmospheric studies [43]. Wegener, mainly known for his 1912 theory of continental drift, also contributed to meteorology and polar research. By the late 1970s various textbooks on the subject began to appear. Today, atmospheric thermodynamics is an integral part of weather forecasting.

1.4 Energy Balance

Energy balance is to be complied with for a small and identifiable portion (element) of the material of any shape that satisfies continuity in order for the conservation of energy to be valid. Energy can enter and exit the continuum; however, the boundaries remain constant, and so it forms a closed system. The continuum is identified by its size, mass, and thermo-physical properties. Thermo-physical properties may be temporal (transient—change with time), spatial (non-homogeneous—change with location or non-isotropic—change with direction within the geometry), temperature-dependent, or constant. Energy is defined in different forms inside this environment. It is either in the form of heat conduction entering the continuum, Heat Generation (HG) inside the continuum, changes of internal energy, or heat conduction leaving the continuum. The heat leaving the continuum by conduction is the same as the heat entering the continuum by conduction plus the spatial variations due to the length of travel, and it is time-independent (steady-state). On the other hand, the internal energy is time-dependent (transient); it focuses on the variation of internal energy, which is presented in the form of thermal capacity of the continuum. Conservation of energy requires that the total energy inputted ($Energy_{in}$) to the system is the same as the total energy outputted ($Energy_{out}$) from the system—considering the heat generation within the system ($Energy_{generated}$)—along with the change of the system's internal energy ($Energy_{storage}$). Figure 1 shows the schematic of the general form of energy balance for a continuum—equation (1).

$$Energy_{in} + Energy_{generated} = Energy_{out} + Energy_{storage} \quad (1)$$

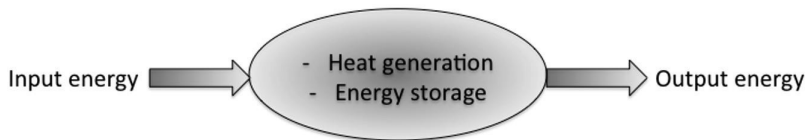


FIGURE 1 Energy balance diagram for a continuum (e.g., a parcel of air).

METEOROLOGY

So, you chose your destination, reviewed the map, selected the route, identified alternate aerodromes, noted the radio frequencies, and completed the flight plan. You picked from the list of the Fixed-Base Operator (FBO) services the restaurant known for their scrumptious pumpkin pie, your lunch box is packed with cold meat sandwiches, and there is hot tea in your thermos—you are ready to take off. But no matter how well prepared you are, there is still a chance for turbulent conditions (during the warm season) or a wicked snow storm (during the cold season). Even though as you accumulate flight hours you may become more tolerant of turbulence (that you rate by the degree of strain you feel against your seat belt), you should not dare to take that trip just to show how capable you are to tackle the forces of nature—you may end up not winning that contest.

So, the first item on the checklist of every pilot before taking off is to verify that the weather is conducive to the situation and the flight is a *go*. Part of your due diligence is to collect all the available weather information before making the *go-no-go* decision. Sometimes, the wisest plan is to postpone that flight after learning that the conditions at the destination may not allow a safe landing. It is expected for the weather not to vary considerably within 24 hrs of the prediction, so a responsible pilot is to check the conditions of the departure and destination aerodromes before commencing the flight. If the weather unexpectedly deteriorates during the flight, the pilot is to either perform a diversion or head back to the

departure aerodrome or the closest safe aerodrome en route. A number of factors such as experience or comfort level of the pilot are also to be considered when making such decisions. The most experienced pilots will not fly until the weather minima are met—surely doing so is safe and responsible conduct to aspire to.

Depending on the flight rules you are following, either Visual Flight Rules (VFR) or Instrument Flight Rules (IFR), there are minimum acceptable conditions to be complied with. The conditions for the VFR flights are also known as Visual Meteorological Conditions (VMC). The following are examples of weather minima in the United States for a fixed-wing aircraft. Note that the FAA 111-rule states there must be 1,000 ft vertical clearance above and the same below the clouds, and 1 statute-mile horizontal clearance from the clouds. Similarly, the FAA 152-rule states that there must be 1,000 ft vertical clearance above the clouds, 500 ft vertical clearance below the clouds, and 2,000 ft horizontal clearance from the clouds.

For Class E airspace and at altitudes up to 10,000 ft Mean Sea Level (MSL), 5 statute-mile visibility and the 111-rule apply. For Classes C, D, and E at altitudes below 10,000 ft MSL, 3 statute-mile visibility and the 152-rule apply. For Class B at altitudes below 10,000 ft MSL, 3 statute-mile visibility and clear of clouds conditions apply. For Class G airspaces for altitudes above 1,200 ft Above the Ground Level (AGL), 3 and 1 statute-mile visibilities during the night and day and the 152-rule apply. For Class G airspaces for altitudes below 1,200 ft AGL, 3 statute-mile visibility during the night and the 152-rule apply. For Class G airspace for altitudes below 1,200 ft AGL, 1 statute-mile visibility during the day and clear of clouds conditions apply.

The aircraft may fly at 500 ft AGL over water and unpopulated areas, and 1,000 ft above the highest obstacle over populated areas, with a horizontal clearance of 2,000 ft (600 ft in the United Kingdom). Other requirements such as flying at the minimum altitude of 2,000 ft above the ground over caribou-populated areas or some other animal habitats are also to be observed [44]. It is recommended to check the weather minima for each specific flight region, as the local regulations may vary. For example, Canadian regulations call for a vertical clearance of 500 ft from the clouds in controlled airspace and at altitudes of 1,000 ft AGL in uncontrolled airspace [44]. Weather minima may be also different for rotary aircraft (e.g., 3 and 1 statute-mile visibilities during the night and day and clear of clouds below 1,000 ft AGL in uncontrolled Canadian airspace).

Weather minima are to be checked over different airspaces and jurisdictions—especially when a weather advisory is not available, provided discretionally, or if tower workload does not permit such services that are also known as Traffic Service (TS) in the United Kingdom and the International Virtual Aviation Organization (IVAO) or Flight Information Center (FIC) in other countries. If a tower is to provide weather advisory services, the aircraft needs to be equipped with two-way radio communication and a special transponder (Mode C). VFR night flights are to be conducted under the minimum safe altitudes and may require additional ratings. If for whatever reason the takeoff and landing using the VFR conditions are to be conducted in local conditions that are below the VFR weather minima, the First-In-Command (FIC) is responsible to ask for Special VFR (SVFR) clearance from the tower with ground visibility that must be still over 1 statute mile. No VFR flights are permitted in Class A airspace (FL180-FL600) and Class B VFR is only allowed if there is clear sky (no clouds). The cruising altitude for VFR flights starts at 3,000 ft AGL up to, but not including, FL180 (18,000 ft) altitude.

The acceptable weather conditions for IFR flights, also known as Instrument Meteorological Conditions (IMC), are more relaxed, with the minimum visibility indicated in a table at the bottom of the Instrument Approach Procedure Charts (IAPC). In some IFR-restricted airspaces (Classes B, C, D, and E), weather advisory, traffic separation, and flight conflict resolution services are either provided or may be provided upon request, or workload permitting, to the VFR flights. An exception is mandatory service provided under Controlled VFR (CVFR) conditions in some regions (e.g., Class B airspace in Canada and some European countries) where Air Traffic Control (ATC) provides the bearing and altitude for the VFR pilots without the need to hold an IFR rating. In Class A airspace, an IFR rating is required along with a two-way radio and a Mode C transponder.

There are a number of ways to obtain weather data. The most common ones are the Meteorological Aviation Routine Weather Report (METAR), Terminal Aerodrome Forecast (TAF), and Pilot Report (PIREP). If you have already opened a flight plan with the FIC, you can count on them as a knowledgeable source, rain or shine. Air Traffic Control (ATC) may provide information, depending on the aerodrome type and their workload at the time the request is made. Feel free to discuss the weather in detail with them and ask whatever questions you may have. This is possible in the air with ATC, aerodrome category and workload permitting, as previously

mentioned. ATC, specifically, may not be able to provide you with detailed instructions as to what technical maneuvers to perform or which runway to take, because you are the responsible decision maker; however, they would most probably inform you of the runway in use or air traffic transient conditions. Sometimes, a combination of all these resources is what you need in order to make an informed decision.

Before talking to the FIC or ATC, assuming that their jurisdiction, airspace, and workload permit such services—aside from their temper that may be weather-dependent—it is a good idea to check the web resources that are both accurate and available for free. Most of the flight plan programs and smart device applications also have a built-in link which graphically displays the weather data as a map overlay. METAR, TAF, Significant Meteorological Report (SIGMET), Airmen's Meteorological Information (AIRMET), Tropical Cyclone Advisory (TCA), and Volcanic Ash Advisory (VAA) are examples of the reports that follow standards set by the International Civil Aviation Organization (ICAO) and the World Meteorological Organization (WMO).

CLIMATE AND ATMOSPHERIC THERMODYNAMICS

3.1 Thermodynamics of Air

Air is a mixture of gases (by volume, 78 percent nitrogen, 21 percent oxygen, and 1 percent argon, carbon dioxide, water vapor, and other gases), each showing behavior approximating that of an ideal gas. Thus, air may be treated as a mixture of ideal gases, with applicable thermodynamic equations of state to describe the relationships between heat, enthalpy, entropy, temperature, pressure, density, relative humidity, and other properties [45]. The three main properties of the ideal gases—pressure, temperature, and density—are interrelated, meaning change in one likely results in change in the others. The majority of these short-term transitions occur during an adiabatic process in which no extremal heat or energy is added to or taken away from the system. Recall the principle of conservation of energy discussed earlier—Figure 1. This is particularly noticeable when parcels of air move in vertical columns due to the changes in their density, which is directly related to the variation of temperature decrease due to the increase in altitude (i.e., pressure). Thermal radiation from the Sun is responsible for this temperature variation, and therefore the term *thermodynamics* (i.e., aerodynamics due to variations in temperature—the relationship between heat and mechanical energy) manifests itself in the form of motion induced by heat. A fourth factor, known as relative humidity, an indication of water content in the atmosphere, is a direct collaborator with other main properties (particularly temperature) contributing to fronts—bundles of air mass that agglomerate and are

protected by boundaries that can affect a wide area as they form and vanish. This moisture is the result of the hydrologic cycle and is fed by the Earth's water resources that cover two-thirds of our planet. Water evaporates and the vapor moves and transforms through the atmospheric layers and gets back to the Earth in the form of liquid or solid precipitation. The resistance of the atmosphere to the vertical convective movement of air parcels affects atmospheric stability—greater resistance to this push-and-pull is associated with greater stability. Depending on the type of investigation, either short- or long-term, the atmospheric processes may be treated as both reversible and irreversible. This section focuses on these properties and their interaction, also known as weather and climate.

Weather describes atmospheric conditions due to the convection of air masses, cloud development, instability of layers, humidity, formation of fronts, as well as severe conditions such as storms and hurricanes. Temperature is an inherent property of a molecule and is the driving force for the movement of air parcels as well as heat transfer in atmospheric layers. Frontal movement occurs when parcels of air with different temperature and humidity levels collide. Depending on which parcel moves up or down, atmospheric stability is defined. This movement is due to the convective air parcels that are mobilized by thermal radiation from the Sun. Heat is transferred from the air parcel with the higher temperature to the one with the lower. This exchange of energy or force endures until the two air parcels reach an equilibrium state, meaning their temperatures equalize and forces become balanced. Unlike the steady-state problems in which the status is fully stabilized, transient conditions are dominant in an atmosphere whose complexities result in different air fronts and short-term weather variations.

During any of these processes, energy and heat are inter-transformed, and as a result weather manifests. The main thermo-physical properties previously described, namely density, temperature, relative humidity, pressure, and heat capacity, form the foundation of all heat transfer and force-balance problems in the atmosphere. Temperature—combined with relative humidity—is the driving force in many heat transfer problems, either directly or indirectly, by converting other types of energy to heat. Any type of conservation, either in the form of mass, energy, or momentum, employs one or more of these properties. The classical laws of thermodynamics discussed earlier are applicable to atmospheric thermodynamics as well.

Theoretically, the parcel of air with a higher density moves down while the one with a lower density, being lighter, rises to the top. Factors such as air pressure and Coriolis forces affect this dynamic. There are a number of assumptions made to simplify the modeling of this process: (a) during the

air parcel exchange of position, no heat is accepted by or rejected from the system (i.e., an adiabatic process); and (b) entropy remains the same (i.e., an isentropic process).

3.2 Layers and Components of Atmosphere

The layers of atmosphere extend from the Earth's surface up to 1,000 km above the MSL. However, 99 percent of the atmosphere's mass is contained below 30 km above the MSL. The atmospheric content of gases, including water vapor, and pollutants, such as smog, remains the same at altitudes up to 80 km above the MSL. The atmosphere consists of seven layers, in sequence, starting at the surface: troposphere, tropopause, stratosphere, stratopause, mesosphere, mesopause, and thermosphere. Figure 2 shows an approximate depiction of temperature versus the altitude for atmospheric layers.

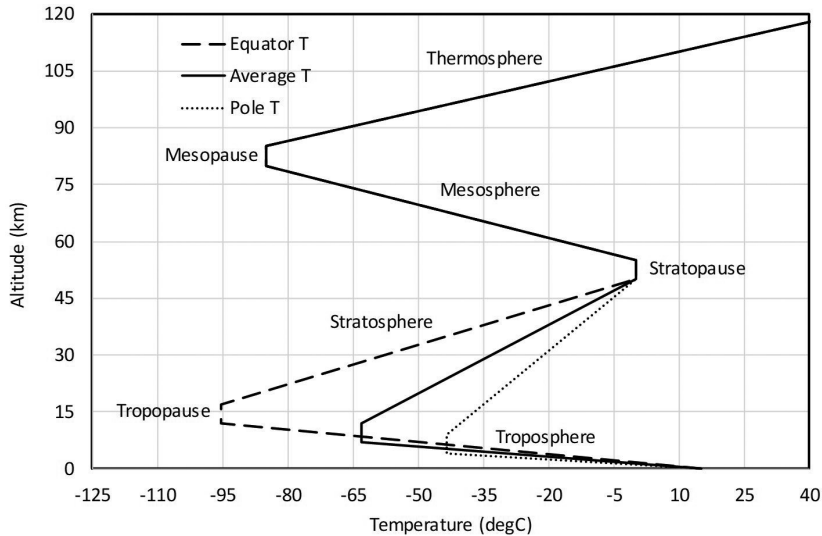


FIGURE 2 Altitude versus the temperature for atmospheric layers.

The three layers of the troposphere, stratosphere, and mesosphere are called the homosphere, where gases are mixed by turbulence. The upper boundary is called the turbopause; it extends to 100 km from sea level and includes part of the thermosphere, also known as the edge of space. The two layers of the thermosphere and exosphere make the heterosphere, where gases are mixed based on their weight, thus changing the chemical composition with altitude. Therefore, the heavier gases, such as nitrogen and oxygen, remain at lower levels while the lighter ones, such as hydrogen, float above [46].

The troposphere is the lowest layer of the atmosphere, and the temperature within this layer decreases with increasing altitude by about 6.5 °C for every km (1.98 °C per 1,000 ft); the height of the troposphere itself changes from approximately 9 km over the Poles to 17 km over the Equator. The lowest part of the troposphere, also known as the planetary boundary layer, is where the flow over the surface is affected by the air flow and temperature in the form of wind and thermal radiation, resulting in turbulence. The layer increases approximately 100 m for every 1 °C increase of surface temperature and varies between 50 m at the Arctic to an average of 4.5 km over deserts and under 1 km over the ocean, with an average thickness that is higher by 7 km over the Equator versus the Poles [47,48]. The turbulence in return redistributes the heat and flow to the layers within the boundary layer. The upper boundary of the troposphere is called the tropopause. The temperature within this layer remains constant with increasing altitude. Over 75 percent of atmospheric air content can be found in the troposphere.

The stratosphere is the layer above the tropopause and extends up to 50 km from sea level. The temperature within this layer increases with increasing altitude due to higher absorption of Ultraviolet (UV) radiation; the temperature reaches about zero degrees Celsius at the layer's upper boundary. The upper layer of the stratosphere, where the temperature remains constant with increasing altitude, is called the stratopause.

The mesosphere is the layer above the stratopause and extends up to 80 km from sea level. The temperature decreases with altitude, with temperatures reaching a low of -95 °C. The upper part of this layer is called the mesopause, with the temperature staying constant with altitude.

The thermosphere is the layer above the mesopause; there the temperature increases with increasing altitude due to the absorption of energetic UV rays and X-rays from the Sun. Solar radiation separates the electrons from the atoms, and therefore positive ions are created whose concentration and temperature vary daily and seasonally. Therefore, the thermosphere's extent varies from approximately 500 km to 1,000 km from sea level, and temperatures may rise up to 2,000 °C, since it is highly affected by solar activities—it is usually 200 °C warmer during the day than at night, and it increases by 500 °C when the Sun is more active. Reflection and absorption of radio waves happen in this layer—this is how you manage to listen to shortwave radio broadcasts around the world. Canada (CFCX) and the United States (WWV) reportedly had the world's first shortwave radio stations [49,50]. The main components of the air in the

thermosphere are ozone, water vapor, carbon dioxide, and interplanetary gases. The International Space Station (ISS) orbits the Earth in this layer at an altitude between 350 km (220 mi) and 420 km (260 mi). The layer on top of the thermosphere is called the exobase [51,52,53,54,55].

The exosphere is the layer above the exobase; it extends up to 10,000 km from sea level and mainly consists of rarely colliding rare oxygen and hydrogen atoms. When conformed to gravity, they follow a ballistic path, and when they enjoy the freedom of escaping into space, they choose their own trajectory. The Aurora Borealis and Aurora Australis occasionally occur at the interface of the exosphere and thermosphere. Most of the satellites orbiting the Earth operate in the exosphere. Isochasms are lines of constant aurora occurrence or mean frequency. Every time electrons and protons attempt to free themselves into vast space, the Earth relays to the atmospheric layers, “I got you!” and holds them with its magnetic power. The magnetic kingdom known as the magnetosphere extends from 3,000 km to 16,000 km from the MSL to the Van Allen radiation belts. It causes the charged particles to swirl and spiral down, following magnetic field lines [56].

Ozone molecules consist of three oxygen atoms; the ozone’s concentration varies with altitude and geographical location due to absorption of UV radiation. The ozone tends to descend and accumulate in the lower levels of the stratosphere, about 15 km to 25 km above the MSL. Some of it may also find its way through the lower levels of the atmosphere in the proximity of the Earth due to the electrostatic discharge. Pollutants such as aerosols and other floating chemicals have hurt this layer, causing issues for living creatures. This particularly affects the polar areas, and this phenomenon has been named the Arctic and Antarctic ozone hole. Due to past and ongoing efforts to control ozone-harming chemicals, this layer has recovered to some extent.

Water vapor is mainly found in the lower troposphere, up to an altitude of approximately 6 km from sea level—the thickness of this layer changes with latitude, decreasing over the tropics and increasing over the Poles. Even relatively small variations in atmospheric water content can be the cause of significant weather events. Theoretically, with increasing altitude, the water content in the atmosphere decreases; however, there are scenarios in which the situation reverses. The water vapor entering the atmosphere transforms through different elevations and falls under gravity in the form of drizzle, rain, sleet, snow, and hail. Other forms of precipitation such as dew, mist, fog, and frost may be seen on the ground surface during the cold hours of the early morning.

Carbon dioxide is released into the atmosphere by living creatures through the process of exhalation (human and animals), deterioration of organic compounds, and volcanic activities. Most of this gas is absorbed by vegetation through the process of inhalation. It is believed that 99 percent of the Earth's carbon dioxide is dissolved in water, mostly that of the oceans; however, this solubility depends on the water temperature, and therefore the gas density varies throughout. The highest carbon dioxide content is measured over dry land.

If you have ever considered why atmospheric gases are contained to the Earth versus floating away into the vastness of space, gravity is to be thanked for that. Atoms possess a kinetic energy which is directly related to their temperature. Particle physics states that particles on the microscopic scale have higher energy and velocities compared to the heavier ones on the macroscopic scale, and therefore longer mean free paths—described by the Knudsen number, which will be discussed later. The effect of gravity is smaller on these particles, so they can travel farther and therefore may be seen in the upper layers of the atmosphere, such as the exosphere and interplanetary space. Atmospheric gases remain in the layers since they cannot resist the gravitational forces.

3.3 Temperature

Temperature is a physical quantity and is the property of an atom; it is the result of the conversion of kinetic energy to heat due to the atom's vibration, rotation, and translation. Temperature can be measured by a thermometer and is scaled to units of measurement known as Kelvin, or degrees of Celsius, Fahrenheit, or Rankine. In thermodynamic relations, temperature—an intensive variable—is expressed as a derivative of one extensive variable with respect to another extensive variable (e.g., partial derivative of internal energy with respect to entropy). An intensive variable is independent of the physical characteristics of the system such as size or shape. A piece of gold retains its mechanical and thermo-physical properties independent of its size. An extensive property of a system can be identified by adding up the extensive properties of its individual sub-units. The total mass or volume of a gold bar is the agglomeration of infinitesimally small nuggets of gold. This is not to be confused with homogeneity of the variable, which is related to the inconsistency of the spatial distribution of its atoms. Intensive properties may be either homogeneous or non-homogeneous.

The concept of temperature is meaningful only when the particles in a system are at thermal equilibrium, meaning that heat in the form of kinetic energy transferred from one particle to the other remains constant. The temperature difference between the systems is the driving force showing the direction of the heat transfer, which occurs from the system at a higher temperature to that at a lower one, with maximum microstate combination (i.e., microscopic arrangement of a system with certain probability during a thermal process) happening at their lower state, also known as entropy (S). Using the concept of the Carnot heat engine, the relationship between heat, work, and temperature is presented, where the difference between heat from a higher temperature source and that of a lower temperature is converted to work. The efficiency is then the work by the heat input to the system, which is directly related to the temperature of the two sources. Thermodynamic definition of the temperature (T) is then defined as the change of entropy (dS) with respect to the change of internal energy (dE). When entropy is a function of its internal energy, thermodynamic temperature is given with equation (2).

$$\frac{1}{T} = \frac{dS}{dE} \quad (2)$$

On the atomic level, thermodynamic temperature is directly related to an elementary particle's kinetic energy ($K_E = 0.5 mv^2$), meaning that particles of equal mass (m) moving with the same speed (v) possess the same kinetic energy and therefore temperature. Since kinetic energy due to the translation motion is dominant for gases (compared to rotational and vibrational energies) and there are three degrees of freedom associated with translational motions—since particles are able to move in the x , y , and z directions—one may deduce equation (3) to calculate particle temperature distribution (T)—where v_{rms} is the root mean square velocity of the atoms and k_B is Boltzmann's constant 1.380649×10^{-23} J/K. Note that the molecules have varied spacing, and therefore number of molecules having the same velocity and occupying the same regions are different.

$$T = \frac{mv_{rms}^2}{3k_B} \quad (3)$$

3.3.1 Dry-Bulb Temperature

Dry-bulb temperature (T) is the temperature of the dry component of air and may be measured by a regular thermometer when the measuring device is not exposed to moisture or radiation.

3.3.2 Wet-Bulb Temperature

Wet-bulb temperature (T_w) is the temperature read by a regular thermometer that is covered by a blanket of wet cloth, meaning the temperature of the moist air completes an adiabatic process at constant pressure and reaches a saturation condition. The humid cloth is dried by a rapid flow of dry air. It is the same as the dry-bulb temperature after the wet cloth has dried up. The mixing ratio (w), which is the ratio of the water vapor mass to the mass of the dry air in any air volume, varies as the wet cloth is drying out.

3.3.3 Dew-Point Temperature

Dew-point temperature (T_d) is the temperature at which a parcel of moist air reaches saturation during an adiabatic cooling process while the air pressure remains constant. During this process no vapor is added or removed from the air parcel. At the end of the process, the water vapor in the air parcel is ready to condense, that is, change to the liquid state. Specific humidity—the ratio of the water vapor mass to that of the moist air parcel that is approximately the same as the mixing ratio (the ratio of the mass of the moist air to that of the dry air)—remains constant during this adiabatic process up to the saturation state but decreases gradually afterward. Cooling the air parcel at this stage results in water vapor condensing into liquid water in the form of precipitation, fog, or dew. Cooling the air parcel further, below its freezing point, results in frost. Although it is possible to measure the dew-point temperature, it is not considered an independent property; it is mainly associated with other properties, such as humidity, in psychrometric charts and saturation curves in thermodynamic diagrams. Spread is the difference between the dry-bulb and dew-point temperatures. This quantity is used to predict the atmospheric level at which the cloud base is formed. The smaller the spread, the closer the air is to the saturation state, and the lower the level of cloud formation is expected.

3.3.4 Virtual Temperature

Virtual temperature (T_v) may be defined as the temperature of a parcel of moist air that a dry parcel of air with the same pressure and density possesses, where e is the partial pressure of water vapor and P is the pressure of the air parcel (i.e., water vapor and dry air). Given that $e/p = w/(w + R_d/R_v)$, equation (4) is obtained, where the relationship between the virtual temperature and the mixing ratio (w) is presented in equation (5)—where $\epsilon = R_d/R_v = 5/8 = 0.622$, R_v is the specific gas constant for water vapor, and

R_d is the specific gas constant for dry air. Note that temperature is in the unit of Kelvin.

$$T_v = \frac{T}{1 - \frac{3}{8} \frac{e}{P}} \quad (4)$$

$$T_v = \frac{T}{1 - \frac{3}{5} w} \quad (5)$$

For the case of unsaturated air, in which the rates of phase change from liquid to vapor and vice versa vary, the saturated relations are corrected for unsaturated effects, and equation (5) is replaced by equation (6). Note that temperature is in degrees Celsius.

$$T_v = T(1 + 0.6 w) \quad (6)$$

3.3.5 Potential Temperature

Potential temperature (θ), introduced by Poisson, is a state variable and is associated with an air mass with temperature T and pressure P which is either expanded or contracted, as needed, during an adiabatic process to attain a reference pressure of 1,000 mbar (100 kPa). The expression for potential temperature is shown in equation (7) as a function of the dry-bulb temperature, pressure, and the heat capacity ratio ($\gamma = c_p/c_v$), which is equal to 1.40 for dry air and 1.34 for water vapor [57]. This temperature is used in some thermodynamic charts to predict weather.

$$\theta = T \left(\frac{1000}{P} \right)^\gamma \quad (7)$$

3.3.6 Isentropic-Saturation Temperature

Isentropic-saturation temperature (T_c) is the temperature an air parcel reaches when it is cooled down to the saturation state during an adiabatic process at constant entropy, meaning the process is reversible. During this process no vapor is added or removed from the air parcel. This temperature is less than that of the dew-point temperature, since the expansion of water vapor causes the volume to increase until the saturation state is reached.

3.3.7 Speed of Sound

A number of factors affect the speed of sound in the atmosphere, and temperature is the most important of them. On a warm winter or cold spring day, snow ski in mountainous areas the next time you travel to

Monte Tremblant [58], the tallest peak in the Laurentian Mountains with a summit as high as 875 m (2,871 ft) in Canada, or Dizin in Iran, while standing in the middle of Alborz Range, face-to-face with the majestic Damavand peak—the highest volcano in Asia, with its summit of 5,628 m (18,465 ft) smiling at you on the horizon [59]. However, mathematically speaking, the temperature difference between a cool spring day and a cold winter day does not have a big effect on the speed of sound. For example, for a spring day where the temperature is about 5 °C, compared to a winter day where the temperature is about –15 °C, the ratio of the speed of sound in the warmer to the colder environment is about 1.04.

The relationship between the speed of sound (c) and absolute temperature (T) is given in equation (8), where the ratio of the air specific heat at constant pressure to that of constant volume ($\gamma = c_p/c_v$) is the same as mentioned in the preceding subsection in relation to equation (7)—1.40 and 1.34 for dry air and water vapor, respectively. R is the associated specific gas constant, presented in equation (4) [60,61].

$$c = \sqrt{\gamma RT} \quad (8)$$

Hence, it is inferred that the speed of sound in an ideal gas is mainly related to its temperature and composition, including the content and molecular weight. Temperature increases with altitude in some layers of the stratosphere and thermosphere (Figure 2), and that results in increasing speed of sound with increasing altitude in those layers. For dry air ($R_d = 287.0 \text{ J/kgK}$, $T = 0 \text{ °C} = 273 \text{ K}$, $\gamma = 1.4$), speed of sound is 331.3 m/s (644 kt, 741 mph). Figure 3 and Figure 4 present the speed of sound versus the altitude for the atmospheric layers up to the lower and upper thermosphere, respectively. Note that the sound speed gradient follows a similar profile to that of the temperature throughout the layers. The long-range temperature profile for the upper atmosphere has been linearly modeled, and it shows that the sound speed gradient is 35 percent of that of the temperature. The speed of sound increases with the humidity, since the specific vapor constant is larger for the saturated moist air than the dry air ($R_v = 461.5 \text{ J/kgK}$). The effect of 100 percent humidity on the speed of sound is presented in Figure 3 and Figure 4, the conversion factor being 1.27.

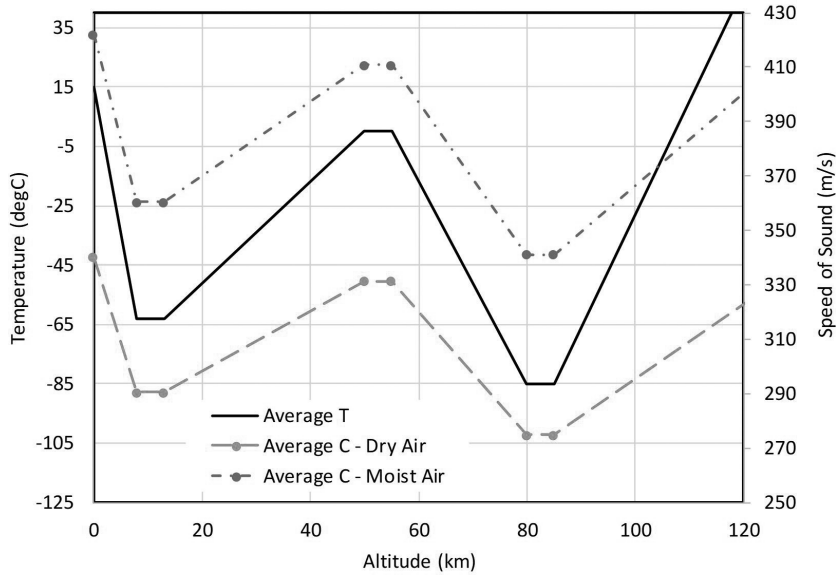


FIGURE 3 Temperature and speed of sound versus the altitude for atmospheric dry and moist layers up to the lower thermosphere.

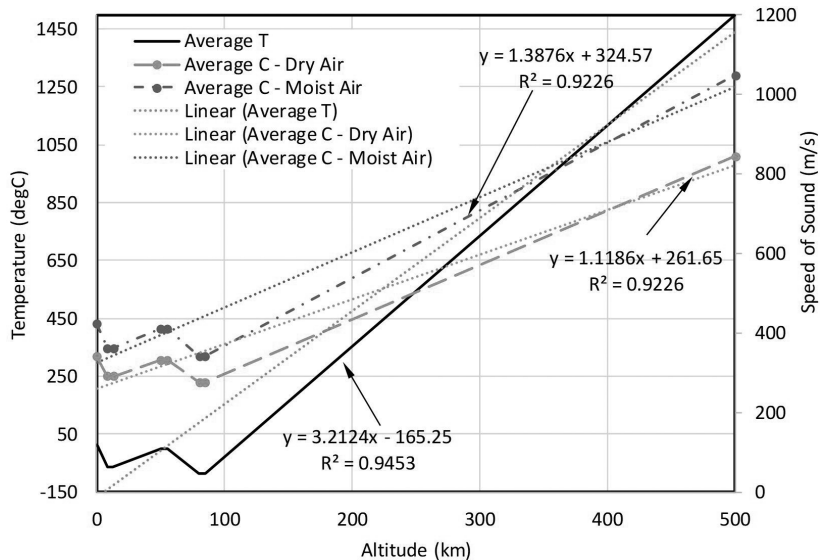


FIGURE 4 Temperature and speed of sound versus the altitude for atmospheric dry and moist layers up to the upper thermosphere.

Figure 5 and Figure 6 present the ratio of the speed of sound in the dry air to that in the moist air versus the altitude for the atmospheric layers up to the lower and upper thermosphere, respectively. The effect of 100 percent humidity on the speed of sound is presented in Figure 5 and Figure 6,

with a conversion factor of 1.27. Other factors such as composition of the atmosphere (e.g., carbon dioxide content) affect the speed of sound and therefore the ratio of the speed in dry air to that in the moist air. Changes of atmospheric pressure and sound frequency have a minor influence on the speed of sound.

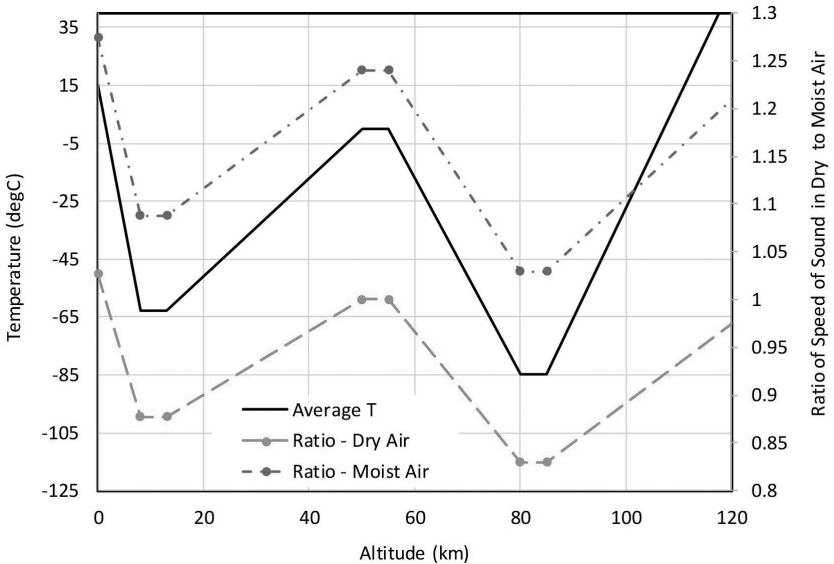


FIGURE 5 Temperature and ratio of the sound in the dry air to the moist air versus the altitude for atmospheric dry and moist layers up to the lower thermosphere.

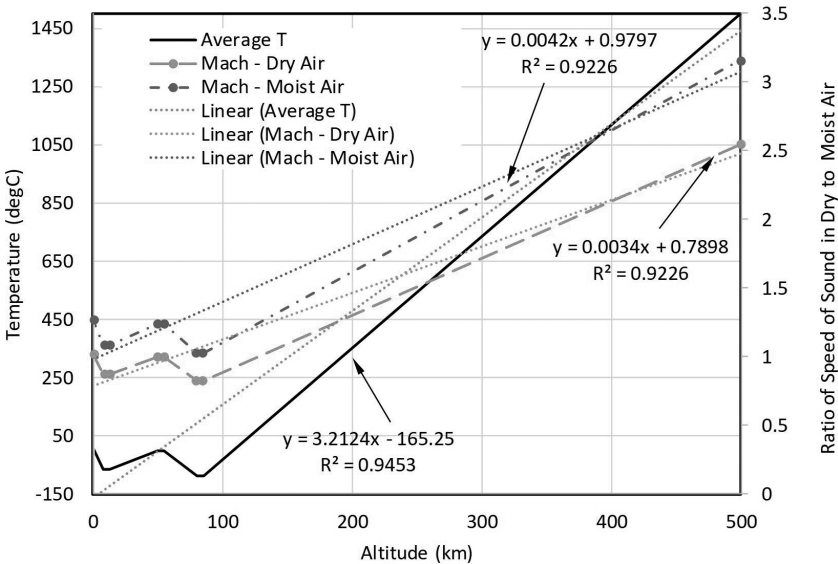


FIGURE 6 Temperature and ratio of the sound in the dry air to the moist air versus the altitude for atmospheric dry and moist layers up to the upper thermosphere.

3.4 Pressure

Pressure (P) is the perpendicular force applied to a surface per unit area. The force is spread over the surface. Pressure can be measured by a pressure gauge designed for a specific application, such as a manometer, which is used to measure gas pressures. It can be expressed in several different units, including Pascal (Newton per square meter, N/m^2), PSI (pound-force per square inch, lbf/in^2), PSF (pound-force per square foot, lbf/ft^2), atm (atmosphere—standard atmospheric pressure), Torr ($1/760$ of atm), and inHg (inches of mercury). Bar, a metric unit that is not part of the International System of Units (SI), is often used for atmospheric pressure measurements and equals 10^5 Pa.

Force, a vector quantity, is either mass-driven due to the gravitational field or is the result of the integration of pressure over an area to which it is applied with the force direction normal to the surface. Thus, you can calculate the buoyancy force by integrating the water pressure over the submerged body. Pressure in a gas is defined by equation (9), where N is the number of particles, V is the volume they occupy, m is the mass of an individual particle, ρ is gas density, and v_{rms} is the root mean square velocity of the particles.

$$P = \frac{Nmv_{rms}^2}{3V} = \frac{\rho v_{rms}^2}{3} \quad (9)$$

Dalton's law states that the total pressure (P) of a mixture of ideal gases may be represented by the summation of the pressures of the individual components—known as partial pressures. Thus, for air, one can define vapor partial pressure (e also known as P_e) and partial pressure of dry air (P_d).

3.4.1 Static Pressure

Pressure of a motionless fluid exerted on a boundary is the static pressure (P_s). The weight of a column of fluid over a point in this case is a good indicator of the pressure applied to the point. Gravitational forces due to the weight of the column of fluid over that point (F) spread over the projected plane area (A) at the center of which the point is located is the static pressure, equation (10), where H is the height of the column of gas over the point of measurement, ρ is the density, and g is the gravitational acceleration.

$$P_s = \frac{F}{A} = \rho g H \quad (10)$$

In an aircraft, static air pressure is measured by a static pressure system connected to a static port, and it is used in the pitot-static system connected to the altimeter, Vertical Speed Indicator (VSI), airspeed, and gyroscopic instruments—Figure 7. The static port is a small hole located on the main body of the fuselage [62].

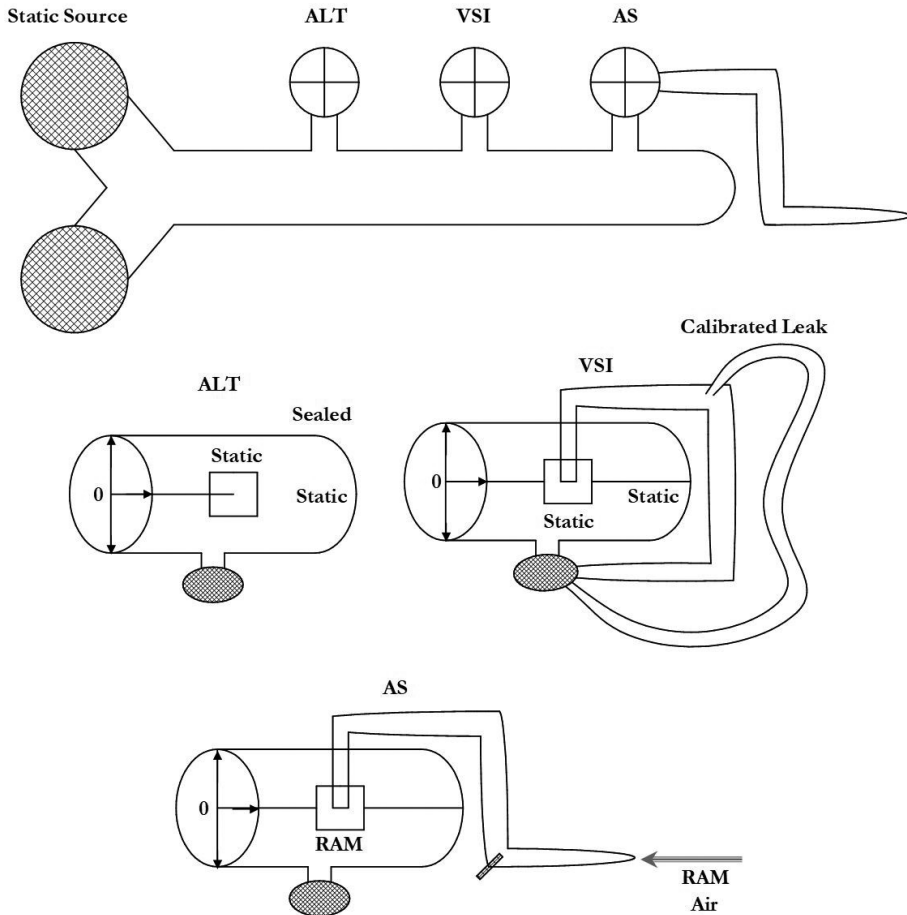


FIGURE 7 Pitot-static systems (drawings created using Solid Edge CAD tool).

3.4.2 Stagnation Pressure

The stagnation pressure, also known as the total pressure (P_t), is the pressure caused by a moving fluid. The pitot tube—connected to the exterior surface of an aircraft fuselage with its open end pointing toward the direction of the aircraft motion—measures the airspeed by measuring the dynamic pressure, the difference between the total pressure from the incoming ram air and the static pressure from the static port [63].

The total pressure (P_t) is defined based on Bernoulli's law presented by equation (11), where P_s is the static pressure, ρ is the density of the fluid in which the measurement is done (air in this case), and v is the airspeed. For an isentropic process (i.e., reversible and adiabatic), equation (12) is obtained. In this equation, T_t is the total temperature, T_s is the static temperature, M is Mach number, and γ is the ratio of the specific heat capacity at constant pressure (c_p) to the specific heat capacity at constant volume (c_v)— $\gamma = c_p/c_v$

$$P_t = P_s + \frac{\rho v^2}{2} \quad (11)$$

$$P_t = P_s \left(\frac{T_t}{T_s} \right)^{\frac{\gamma}{\gamma-1}} = P_s \left[1 + \frac{M^2(\gamma-1)}{2} \right]^{\frac{\gamma}{\gamma-1}} \quad (12)$$

3.4.3 Pressure Altitude

The average elevation of the surface of multiple bodies of water (e.g., oceans) from which the terrain elevation is to be measured is known as the Mean Sea Level (MSL)—it is a vertical reference datum. Terrain elevation where the dry surface is present is called the Ground Level (GL). Elevation measured from the GL is referred to as *Above Ground Level* (AGL). Flight Level (FL) is the vertical altitude expressed in hundreds of feet as indicated on the altimeter when it is set to the standard pressure (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi) [64,65]. Thus, 21,000 feet would be expressed as FL210.

QNE is the altitude indicated on the altimeter when one is located at the runway (or another location of interest) and the altimeter is set to the standard pressure [66]. Quasi Non-Hydrostatic (QNH) is related to the pressure altitude adjusted for the MSL [67]. When set on the altimeter, it should give the correct altitude above the MSL. Above the transition altitude (above 18,000 ft in Canada and the United States, 13,000 ft in Australia, and as low as 3,000 ft in some parts of Europe), the altimeter is set to the international standard atmosphere at MSL (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi).

Pressure altitude (PA) is defined as the altitude observed in the International Standard Atmosphere (ISA)—when the pressure at the MSL is equal to 29.92 inHg (101,325 Pa, 1 atm, 14.67 psi)—at which the pressure is equal to the pressure under the current atmospheric conditions. Thus, if the current atmospheric pressure is lower than the standard pressure, the pressure altitude will be higher than the actual one. Pressure altitude (PA) is calculated using equation (13) as the actual elevation (H) plus the

correction, which is proportional to the difference between the standard pressure (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi) and the actual current pressure at MSL (P_{inHg}). The latter pressure would be the one set on the aircraft's altimeter.

$$PA = H + 1000(29.92 - P_{\text{inHg}}) \quad (13)$$

The highest barometric pressure, as reported in *Guinness Book of Records* (1,083.8 mb- 32 inHg), was recorded at Agata, Siberia, Russia, at an altitude of 862 ft (262 m), on December 31, 1968. This pressure is equivalent to the standard pressure at an altitude of 2,080 ft (600 m) below the MSL. The highest barometric pressure reported in the United Kingdom was 1,054.7 mb (31.15 in), in Aberdeen on January 31, 1902 [68]. This pressure is equivalent to the standard pressure at an altitude of 1,230 ft (374.9 m) below the MSL. The lowest barometric pressure, as reported in *Guinness Book of Records* (870 mb- 25.69 inHg), was recorded on October 12, 1979 by the United States Air Weather Service at 483 km (300 mi) west of Guam in the Pacific Ocean in the eye of Super Typhoon Tip. Wind speeds of 165 kt (305 km/hr- 190 mph) were reported [69]. This pressure is equivalent to the standard pressure at an altitude of 4,230 ft (1,289.3 m) above the MSL. You may use said data in order to solve for the pressure altitude by adding the altitude at standard pressure to the elevation figure. For example, for case of Agata, Siberia, Russia, you will obtain a pressure altitude equivalent to a location that is 1,228 ft below the MSL.

The barometric pressure is how the height above the MSL is measured by an aircraft's altimeter, which is part of the pitot-static system—Figure 7. The sealed (airtight) case includes a vent to the static port. The altimeter is an aneroid-based barometer. The altimeter is a sensitive instrument (traditionally vacuum-based with some advanced electric ones with analog displays) that may over-read or under-read if there is any blockage of the nozzles. As the aircraft increases its altitude, the air pressure decreases. The exterior pressure must be balanced by the capsule's interior pressure; therefore, the capsule expands to equally decrease its own interior pressure, thus balancing the forces. With decreasing altitude the air pressure increases, causing the capsule to reduce in volume, therefore making the wafer contract. The expansion or contraction of the aneroid wafer causes the connected rocking shafts and associated gears to move, and therefore the end-pointer rotates, indicating numbers on the instrument's dial gauge. If the static port is blocked for whatever reason, such as icing, the case is isolated from the surroundings, and therefore with increasing altitude, the altimeter reading does not change—it remains at the last valid reading—Figure 7.

3.5 Speed of Aircraft

There are multiple airspeeds that are used as part of flight planning and as limit values for aircraft control and structural integrity. One can use the ICE-T mnemonic to help remember these airspeeds: (a) Indicated Airspeed (IAS), (b) Calibrated Airspeed (CAS), (c) Equivalent Airspeed (EAS), and (d) True Airspeed (TAS) [70,71].

Airspeed is measured by the pitot-static system (Figure 7). Ram pressure, also known as total pressure, is measured by a pitot tube, which is part of the pitot-static system connected to the airspeed indicator. The static pressure is measured by the static ports. The ram air feeds the pressure diaphragm embedded in a sealed (airtight) case filled with static pressure supplied from the static ports. The airspeed indicator shows the dynamic pressure—also known as the impact pressure (i.e., the difference between the total pressure and the static pressure)—and the instrument is calibrated to show this dynamic pressure as the IAS. TAS is the aircraft's speed relative to the air through which it is moving. Greater pressure difference means larger dynamic pressure and therefore higher airspeed, given similar air densities. The airspeed indicator is a sensitive instrument (traditionally vacuum-based or electric-based in glass-cockpit instruments) that may over-read or under-read if there is any blockage of the nozzles. As the aircraft increases its altitude, the dynamic air pressure decreases for similar TAS as air density decreases, causing the pressure diaphragm to contract as the pressure of gas inside the diaphragm decreases, as in equation (11). With decreasing altitude, the dynamic air pressure increases for a given TAS as the air density increases, causing the air pressure inside the capsule to increase and the capsule to expand, as in equation (11). The contraction or expansion of the pressure diaphragm causes the connected rocking shafts and associated gears to move and the end-pointer to rotate, indicating numbers on the instrument dial gauge. If the static port is blocked for whatever reason, such as icing, the case is isolated from the surroundings, and therefore with increasing altitude the airspeed indicator under-reads, and with decreasing altitude the airspeed indicator over-reads. If the pitot-system that measures the ram pressure is blocked, the airspeed acts like an altimeter—with increasing altitude it over-reads, and with decreasing altitude it under-reads. That is why one must use the pitot heater to melt the ice from the pitot tube; the heater consists of electric heating elements located inside the tube.

The rate of climb or descent is measured by the VSI, which is another component connected to the pitot-static system—Figure 7. The instrument's

case is vented to the static port through a calibrated leak, also known as the restricted diffuser. The pressure diaphragm is fed through the static port. The VSI is a sensitive instrument that may produce inaccurate readings if there is any blockage of the nozzles. As the aircraft increases its altitude, the air pressure decreases, causing the diaphragm to contract, since the rate of pressure decrease inside the diaphragm is faster than that of the calibrated leak. With decreasing altitude air pressure increases, causing the air pressure inside the diaphragm to increase and therefore expand the diaphragm, since the rate of pressure increase inside the diaphragm is larger than that of the calibrated leak. The contraction or expansion of the pressure diaphragm causes the connected rocking shafts and associated gears to move and the end-pointer to rotate, indicating numbers on the instrument dial gauge. If the static port is blocked by icing; for example, the case is isolated from its surroundings, and therefore with increasing altitude, the readings do not change—meaning it shows a zero-climb rate.

3.5.1 Mach Number

The Mach number is a dimensionless quantity which is a ratio of the TAS of an aircraft to that of the speed of sound at local conditions. The Machmeter in an aircraft uses the pressure differential versus the temperature to calculate the Mach number at high altitudes to prevent aircraft from exceeding the maneuvering speed—also known as the Mach number limit. Factors such as air temperature, humidity, and composition (e.g., carbon dioxide content) affect the speed of sound in the air (Figure 5 and Figure 6).

Equation (14)—a derivative of equation (46), $h = c_p T + 0.5v^2$ —shows Mach number (M) for an ideal gas for a compressible flow and based on Bernoulli's relation, considering the heat capacity ratio ($\gamma = c_p/c_v$) and the ratio of the stagnation (P_t) to static pressure (P_s).

$$M = \sqrt{\frac{2}{\gamma - 1} \left[\left(\frac{P_t}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]} \quad (14)$$

For a supersonic flow, the Mach number is obtained based on the Rayleigh relation—equation (15).

$$M = \frac{3\gamma}{\gamma - 1} \sqrt{\left(\frac{P_t}{P_s} + 1 \right) \left(1 - \frac{\gamma - 1}{2\gamma} \frac{1}{M^2} \right)^{\frac{1}{\gamma - 1}}} \quad (15)$$

3.5.2 Indicated Airspeed

The airspeed indicator shows the Indicated Airspeed (IAS), which is based on the difference between the static pressure (P_s) from the static ports and the stagnation pressure, also known as the total pressure (P_t), from the pitot system. The previous difference is the dynamic pressure that is converted to the airspeed and communicated to the dial gauge by means of shafts and gears moving a pointer.

Bernoulli's law presented by equation (11) then may be restated to present the IAS for a non-compressible flow as a function of the previous pressure difference and air density (ρ)—equation (16). For a compressible flow, equation (17) is used to calculate the IAS, where γ is the ratio of the heat capacities at constant pressure to that of constant volume. Equation (18) is applicable to a compressible and subsonic flow, in which the effect of Mach number (M) is considered using equation (14). Equation (19) is another form of equation (18) in which IAS is presented in terms of freestream speed of sound in air (c_s) at freestream air temperature (T_s). The Mach number from equation (15) can be used in equation (17) to derive a relation for a supersonic flow. The relationship between Calibrated Airspeed (CAS) and IAS is specific to each aircraft model and is presented by a table included in the Pilot Operating Handbook (POH) showing IAS for the associated CAS.

$$IAS = \sqrt{\frac{2(P_t - P_s)}{\rho}} \quad (16)$$

$$IAS = \sqrt{\frac{2\gamma}{\gamma-1} \frac{P_s}{\rho} \left[\left(\frac{P_t}{P_s} + 1 \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} \quad (17)$$

$$IAS = \sqrt{\frac{2P_s}{\rho} \frac{\gamma}{(\gamma-1)} \left[\left[\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{(\gamma-1)}} + 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right]} \quad (18)$$

$$IAS = c_s \sqrt{\frac{2}{(\gamma-1)} \left[\left[\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{(\gamma-1)}} + 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right]} \quad (19)$$

3.5.3 Calibrated Airspeed

Calibrated Airspeed (CAS) is the Indicated Airspeed (IAS) corrected for instrument and position errors. The corrections are functions of the IAS and account for both position and instrument errors due to the variation in location of the static ports and due to the interference between the aircraft systems and its structures. CAS is used to calculate TAS from IAS (or vice versa depending on the method of calculation) when navigating. CAS is also used as the main reference to ensure the dynamic pressure acting on the aircraft is within the allowable limits. CAS is used to calculate the ground speed for navigational purposes; however, the use of modern GPS and INS navigation systems has reduced the need for flight computers such as the E6B for ground speed calculations. If the weight of an aircraft remains the same, the CAS values at which the aircraft takes off or lands remain unchanged. These CAS values remain unchanged for different elevations, while the TAS or ground speed may vary; therefore, CAS is an important criterion for performing maneuvers such as rotation, climb, stall, cruise, or landing. CAS is the same as Equivalent Airspeed (EAS) when flying slower than Mach 1 and at standard atmosphere and sea level.

Measuring static freestream pressure is not always possible. Therefore, CAS has been introduced that is calculated by equation (20)—a derivative of equation (25)—where static freestream pressure (P_s) has been substituted with standard pressure at sea level, P_0 (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi). Equation (20) may be expressed in terms of the Mach number, using equation (14), to present equation (21). For a supersonic flow where the Mach number is over one—using the Rayleigh approximation method—CAS may be estimated using equation (22), which may be solved using Gauss-Seidel or any other iterative techniques, where c_0 is the freestream speed of sound in air at standard temperature at sea level, $c_0 = \sqrt{\gamma RT_0}$ (15 °C, 288 K, 59 °F). Equation (23) may be used to express calibrated airspeed (CAS) as a function of the equivalent airspeed (EAS), Mach (M), and pressure ratio ($\delta = P/P_0$).

$$CAS = c_0 \sqrt{\frac{2}{\gamma - 1} \left[\left(\frac{P_t - P_s}{P_0} + 1 \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]} \quad (20)$$

$$CAS = c_0 \sqrt{\frac{2}{\gamma - 1} \left[\frac{P_s}{P_0} \left[\left(\frac{\gamma - 1}{2} M^2 + 1 \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right] + 1 \right]^{\frac{\gamma - 1}{\gamma}} - 1} \quad (21)$$

$$CAS = c_0 \left[\frac{\left(\frac{P_t}{P_o} \right) \left(\frac{2\gamma}{\gamma-1} \left(\frac{CAS}{c_0} \right)^2 - 1 \right)^{\frac{1}{\gamma-1}}}{6^{\frac{1}{\gamma-1}} 1.2^{\frac{\gamma}{\gamma-1}}} \right]^{\frac{\gamma-1}{2\gamma}} \quad (22)$$

$$CAS = EAS \left[1 + \frac{1}{8}(1-\delta)M^2 + \frac{3}{640}(1-10\delta+9\delta^2)M^4 \right] \quad (23)$$

3.5.4 Equivalent Airspeed

Equivalent Airspeed (EAS) is Calibrated Airspeed (CAS) corrected for compressibility and Mach number. With airspeed increasing with pressure altitude, airspeed shows higher values than the real value, and therefore the number is to be corrected. EAS is the same as True Airspeed (TAS), where the standard temperature at sea level (T_0) has been substituted for the freestream static temperature (T_s)—equation (24)—where c_0 is the freestream speed of sound in air at the standard temperature at sea level ($c_0 = \sqrt{\gamma R T_0} = 340 \text{ m/s} = 1,116 \text{ ft/s}$, $T_0 = 15^\circ\text{C} = 288 \text{ K} = 59^\circ\text{F}$). Equation (24) may be expanded to express EAS versus the Mach by using equation (14) to present equation (25), where P_s is the static pressure and P_0 is the pressure at standard atmosphere. At sea level and ISA conditions, EAS and CAS are the same. At higher altitudes, EAS will be less than CAS. However, at speeds under 200 kt CAS and altitudes below 10,000 ft, the difference between EAS and CAS may be neglected. The Mach number from equation (15) can be used in equation (24) to derive a relation for a supersonic flow.

$$EAS = c_0 \sqrt{\frac{2}{\gamma-1} \frac{P_s}{P_0} \left[\left(\frac{P_t}{P_s} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} \quad (24)$$

$$EAS = c_0 M \sqrt{\frac{P_s}{P_0}} \quad (25)$$

3.5.5 True Airspeed

True Airspeed (TAS) is the aircraft speed relative to the surrounding air. TAS is the Equivalent Airspeed (EAS) corrected for altitude and non-standard temperature. TAS is used to calculate the ground speed for navigational purposes; however, the use of modern GPS and INS navigation systems has reduced the need for flight computers such as the E6B for ground speed calculations. The Indicated Airspeed (IAS) is

related to TAS, but it shows lower values than TAS as air density decreases due to the increase of temperature or altitude. For values over 100 kt (185 km/hr, 115 mph) TAS, the Mach number becomes an important factor, since air compressibility is to be taken into consideration. For isentropic and compressible air, where density changes with temperature during an adiabatic reversible process based on equation (67)— $PV^\gamma = cte$, combining the energy balance relation presented by equation (46)— $h = c_p T + 0.5v^2$, and stagnation pressure—also known as total pressure—presented by equation (12), results in true airspeed (TAS) presented by equation (26)—where c_0 is freestream speed of sound in air at sea level ($c_0 = \sqrt{\gamma RT_0}$), T_0 is the air temperature at sea level (15 °C, 288 K, 59 °F), and R is specific gas constant. Equation (26) may be expanded to express TAS versus the Mach, using equation (14), to obtain equation (27). The Mach number from equation (15) can be used in equation (26) to derive a relation for a supersonic flow. For Mach numbers much smaller than one, the higher order terms may be ignored and equation (27) may be simplified. Equation (28) expresses the TAS as a function of the temperature ratio ($\theta = T_s/T_0$). Equation (28) can be restated in terms of equivalent airspeed (EAS), pressure ratio ($\delta = P_s/P_0$), which is the ratio of the static pressure to a standard pressure at sea level, and temperature ratio ($\theta = T_s/T_0$), which is the ratio of the static temperature to a standard temperature at sea level—equation (29).

$$TAS = c_0 \sqrt{\frac{2}{\gamma-1} \frac{T_s}{T_0} \left[\left(\frac{P_t}{P_s} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} \quad (26)$$

$$TAS = c_s \sqrt{\frac{2}{\gamma-1} \left[\left[\left(1 + \left(\frac{\gamma-1}{2} \right) M^2 \right)^{\frac{\gamma}{\gamma-1}} - 1 \right]^{\frac{\gamma-1}{\gamma}} - 1 \right]} \quad (27)$$

$$TAS = c_0 M \sqrt{\frac{T_s}{T_0}} \quad (28)$$

$$TAS = \sqrt{\frac{T_s}{T_0} \frac{P_0}{P_s}} EAS = \sqrt{\frac{\theta}{\delta}} (EAS) = \sqrt{\frac{\rho_0}{\rho}} EAS \quad (29)$$

3.5.6 Comparison of Different Airspeeds

Recall the mnemonic formed from these airspeeds: Indicated, Calibrated, Equivalent, and True (ICE T)? “I” corrected for pressure (P) gives “C,” which corrected for compressibility (C) gives “E,” which when corrected for density (D) gives “T.” So, ICE T is a Pretty Cool Drink!

It is possible to express the static temperature (T_s) as a function of the total temperature (T_t), as in equation (30). Total temperature is also known as the stagnation temperature and may be expressed in terms of Mach number and ratio of the specific heats at constant pressure to that of constant volume ($\gamma = c_p/c_v$); Equation (31) presents the pressure (δ) and temperature (θ) ratios. In practice, the total temperature is measured by inserting a probe in the vicinity of the aircraft where the relative speed of the flow surrounding the aircraft (the difference between the airflow and aircraft speed) becomes zero. In reality, however, this relative velocity may not become zero, and therefore a correction factor (less than 1) is included in the calculations (e').

$$\frac{T_t}{T_s} = 1 + e' \frac{\gamma - 1}{2} M^2 \quad (30)$$

$$\theta = T_s/T_0, \quad \delta = P_s/P_0 \quad (31)$$

3.6 Lapse Rate

The variation of temperature between atmospheric layers is a determining factor characterizing atmospheric stability. The gradient of air temperature versus the altitude is referred to as the lapse rate.

Hydrostatic law governs the change of pressure for an air parcel, with the altitude versus the gravitational force applied vertically toward the center of the Earth. Assume that the mass of the dry air parcel over a specific point at altitude (z) is m . For an adiabatic process, the change of pressure (dP) for the incremental change in altitude (dz) may be obtained using forms of energy presented either by equation (70)— $dQ = Mc_v dT + PdV$ or equation (71)— $dQ = Mc_p dT - VdP$. Equation (32) is derived as a result.

$$dP = -\rho g dz \quad (32)$$

By substituting the general equation of state for the ideal gas, equation (55)— $PV = n\bar{R}T$, in the previous relation, you will find that change of temperature with increase in altitude ($\Gamma_d = dT/dz = -g/C_p$) for dry air is approximately 10 °C for every km or 3 °C per 1,000 ft, where Γ_d is dry-adiabatic lapse rate.

Assuming that there is a container partially filled with water, the molecules in the vicinity of the liquid are disturbed and separated from the surface in the form of water vapor. Some also collide with the liquid surface and adhere to it. The rates of phase change from liquid to gas and from gas to liquid—also known as evaporation and condensation—eventually equalize, and as a result the multi-phase temperature equalizes to the saturation state whose pressure is the saturated vapor pressure (e_s)

and is directly dependent on the absolute temperature—the higher the pressure, the higher the temperature is required and vice versa. The rate of phase change remains high at the beginning of the process; however, it is reduced as more water is transformed to gas in the form of vapor. This is the reason why the saturated lapse rate (change of temperature with respect to altitude for the saturated air parcel) is smaller than the lapse rate of dry air. More energy in the form of heat is required to cool down a parcel of air that includes a water vapor component due to the presence of contaminant nuclei and water vapor; a dry parcel of air can cool faster given its purity and lack of contamination. As a parcel of air is elevated, assuming that it starts from the dry state, it cools at a lapse rate of 3 °C per 1,000 ft (10 °C per 1 km) until reaching the saturation state.

The elevation at which the saturation state is achieved depends on the air parcel's initial temperature. The parcel is then cooled at a saturated adiabatic rate (1.5 °C per 1,000 ft, 4.9 °C/km) in an adiabatic-reversible process (Γ_w). The moving air facilitates this cooling process, moving the air particles to higher elevations. The saturation continues until the formation of liquid from vapor is completed under a latent heat that is negatively related to temperature (rain stage). The cooling continues down to the freezing point (0 °C), where the hail stage, which is an isothermal process, commences; the rate of cooling during this stage remains constant, since the latent heat of freezing is independent of temperature. This stage continues until the phase change from liquid to solid is completed. The parcel of air is still ascending with a reduced lapse rate due to the release of the latent heat of evaporation during the rain stage. Even after the majority of the liquid has transformed into a solid in the form of ice, there are still a number of vapor particles that are deposited into a solid in the form of ice crystals—the snow stage. The ascent of the air parcel continues until all vapor transforms into ice and is done in a dry adiabatic rate that is larger than that of the rain stage. Since the opposite process follows the same cooling rate and temperature profile in a majority of cases, it is called a reversible adiabatic process (Figure 8).

In a semi-adiabatic process, the formation of the precipitation happens at the moment of completion of each phase, starting with rain and then developing into snow. Hail does not form during this process since there is no liquid left in the air parcel after the completion of the rain stage to form ice. This process is irreversible. After completion of the snow stage, where precipitation in the form of snow has left the air parcel, meaning that

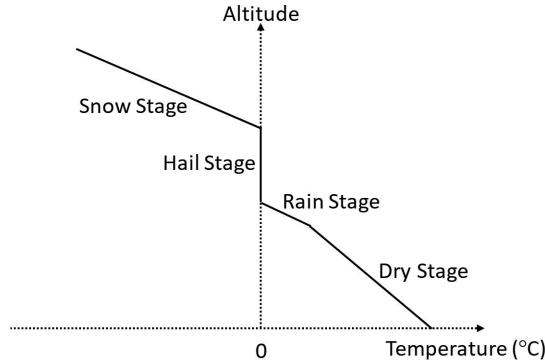


FIGURE 8 Cooling stages in the atmosphere versus the temperature.

there is no remaining moisture, the parcel of dry air therefore returns to its original state, which happens as a non-reversible adiabatic process and with a dry lapse rate. The temperature of the final state is higher than that of the initial one, and this final state temperature is known as the equivalent temperature.

Equation (33) presents the relationship between moist air properties and the change in the ratio of the water content to that of the dry air (dw_s) for an adiabatic or semi-adiabatic process, where L_v is the latent heat of vaporization. dw_s depends on the water content.

$$\frac{dT}{T} = \left(1 - \frac{1}{\gamma}\right) \frac{dP}{P} - \frac{L_v}{TC_p} dw_s \quad (33)$$

The temperature gradient between the atmospheric layers is often used to calculate the spread (i.e., the difference between the dry-bulb and dew-point temperatures) as well as the lapse rate as a function of the altitude. These two values are employed to calculate the saturation and freezing levels required to determine cloud base and icing. The levels (ft) are calculated by dividing the spread (°C) by the lapse rate for the given altitude (°C/ft). Note that the standard lapse rate set by ICAO is 1.98 °C per 1,000 ft (6.6 °C/km). Given the level of water vapor in the air parcel, there are three different types of lapse rate: dry, adiabatic, and saturated (moist) (Table 1).

TABLE 1 Dry, adiabatic, and saturated (moist) lapse rates.

Lapse Rate (°C/1,000 ft)			
Adiabatic Lapse Rate	Saturated/Moist	Standard/Environmental (ELR)	Dry
°C/1,000 ft	1.5	1.98	3.0
°C/km	4.9	6.5	9.8

3.7 Density

Density (ρ) is the ratio of the mass (m) of the parcel of air to its volume (V) and is related to the pressure and temperature—equation (34). Equation (55)— $PV = n\bar{R}T$ —can be rewritten to include molar mass (M), gas pressure (P), and absolute temperature (T), as well as specific gas constant (R)—equation (35). The equation shows that the density is directly related to the pressure and mass, and inversely related to the temperature. The second part of equation (35) is a function of mass (M_d, M_v) and partial pressure (P_d, P_v) of dry air as well as water vapor. The expression $1/T$ in the denominator of equation (36) is the thermal expansion coefficient for an ideal gas for isobaric and isochoric processes, presenting the changes of gas volume as a function of the changes in temperature (ΔT) [72]. Increasing the water content and temperature of the air mixture causes the air density to decrease.

$$\rho = \frac{m}{V} \quad (34)$$

$$\rho = \frac{P}{RT} = \frac{P_d M_d + P_v M_v}{RT} \quad (35)$$

$$\rho = \frac{\rho_0}{1 + \frac{\Delta T}{T}} \quad (36)$$

3.7.1 Specific Volume

Specific volume is the ratio of the volume of the mixture to its mass, which is the sum of the masses of dry air and water vapor content [73]. To obtain the specific volume for an ideal gas, you may simply calculate the inverse of density; however, for moist air, based on ASHRAE, this value is to be increased by percentage of the Humidity Ratio (HR).

3.7.2 Density Altitude

Density altitude is the pressure altitude adjusted for the temperature. In other words, it is a function of the density of the air at the altitude of interest above the MSL. Based on ideal gas laws, increase in temperature, decrease in pressure, and increase in humidity cause the density altitude to decrease. Density altitude is an important parameter when assessing a buoyant object's aerodynamic performance. The lift generated by the airfoils, the relation between the IAS and TAS, and the power generated by the engine all depend on the variation in air density and its composition [74]. Density altitude (DA) is shown by equation (37), where P is the static

pressure and P_{MSL} is ISA standard pressure at the MSL, P_0 (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi) are static atmospheric and MSL pressures, T is the outside air temperature (OAT) and T_{MSL} is the standard temperature at the MSL, T_0 (15 °C, 288 K, 59 °F), Γ is the temperature lapse rate, R is the specific gas constant, g is the gravity acceleration, and M is the molar mass. The subscript MSL means the parameter is associated with the mean sea level.

$$DA = \frac{T_{MSL}}{T} \left[1 - \left[\frac{P}{P_{MSL}} \right]^{\frac{\Gamma R}{gM - \Gamma R}} \right] \quad (37)$$

Equation (38) is the simplified form to calculate density altitude in which temperature (T_{ISA}) is the standard temperature at the given altitude (H) obtained from equation (39), where Γ_{MSL} is the standard-adiabatic lapse rate (1.98 °C/1,000 ft, 6.6 °C/km)— Γ_{ELR} .

$$DA = PA + 118.8(T - T_{ISA}) \quad (38)$$

$$T_{ISA} = T_{MSL} - \Gamma_{MSL}H \quad (39)$$

3.8 Humidity

Water vapor is a very important component of air. It is capable of transforming from gas to liquid (condensation) to solid (freezing), and from solid to liquid (melting) to gas (evaporation). It is responsible for the formation of clouds and precipitation. Water vapor is also capable of transforming directly into solid ice crystals (deposition) and from ice directly back to vapor (sublimation). The formation of clouds starts from particles called condensation nuclei. These small particles are present due to pollutants such as sulphur, salt, gas, smoke, and smog. These particles attract water droplets to form agglomerates that later expand to form clouds. You may employ isoplates (lines of constant acid precipitation) and isoflors (lines of constant areas of comparable biological diversity) to investigate the effect of human population and related pollutants on precipitation occurrence, duration, and intensity. Depending on the method of formation of these droplets, different temperature levels can be identified that are categorized into three main groups known as dew-point, dry-bulb, and wet-bulb temperatures.

There are two properties associated with humidity: relative humidity and humidity ratio, also known as moisture content, are associated with mole and mass fractions, respectively [75,76]. Relative humidity is the

ratio of the mole fraction of water vapor in moist air to the mole fraction of water in the saturated case at a constant temperature and is expressed as a percentage. It may also be expressed as the ratio of the vapor partial pressure to that of the equilibrium state at a constant temperature. At low temperatures, vapor partial pressure increases, resulting in less humidity in the air compared to that of high temperature cases. Fully saturated moist air is another way of expressing the relative humidity as 100 percent.

Humidity ratio, on the other hand, is the ratio of the mass of the water vapor to that of the dry air. You may come across the term *mixing ratio*, especially in thermodynamic diagrams, which is another term for *humidity ratio*. Using the dry-bulb and wet-bulb temperatures and a psychrometric chart, you can obtain the value of the humidity ratio for the relative humidity of 100 percent. Humidity is measured by a hygrometer.

Both excess and insufficient humidity can lead to undesirable effects for human and mechanical systems, especially in the aviation industry. For humans, lack of humidity may aggravate allergies and promote respiratory problems, dryness of nasal passages and bleeding as a result, and dry skin. High humidity may result in serious health issues, as the body may not be able to accomplish the heat dissipation required to moderate its temperature. The heat index and humidex are factors that combine the effects of temperature as well as humidity on the atmosphere cooling effect. A relative humidity of 50 to 60 percent may be recommended as a comfortable range for human beings.

When flying at high altitudes, temperature is low and therefore there is a low humidity content, which decreases even further with heating the cabin air. The cabin air relative humidity may drop to 10 percent or less, and this may lead to health risks for passengers and the crew. If your skin feels dry during the winter months, you may use a humidifier at your leisure; however, when flying thousands of feet above the MSL, the humidity that is generated by a supply of water to the system does not seem that practical for the extra weight it introduces, which may or may not be supported by the aircraft weight balance limitations. If, on the other hand, the captain decides to fly at a lower altitude or through the clouds, the increased level of humidity may result in water condensation on the skin of the buoyant object in contact with its surroundings or internal systems, which may be the cause of worry in close to sub-freezing temperatures for the systems—either internal or external—if not specifically designed for these operating conditions. Carburetor heat is a method to reduce the negative effects of cold humid air flowing into the piston engine of an aircraft. For this

reason, the dew-point temperature is provided with weather reports so that the spread can be evaluated, especially when making cross-country flight plans. Dry air is better at keeping an aircraft buoyant due to the lift forces it produces and higher flight efficiency compared to that of moist warm air. This is the reason for including weather conditions in the flight calculations related to the runway length required for takeoff and landing as well as ascent, descent, and cruise performance. Higher humidity combined with high temperatures requires longer takeoff and landing distances. Dry air in general is better able to hold moisture than the moist air.

A number of empirical relationships have been developed in which the saturation vapor pressure (e_s) may be obtained using absolute pressure along with the dry-bulb temperature (T). There are a number of assumptions associated with each formula: for example, the temperature must be between -80°C (112°F) and 50°C (122°F). Given their empirical nature, each formula poses its own complexity that is a function of the contamination level as well as the applicable temperature range. Equations (40) and (41) are among the most common empirical relations to estimate saturation vapor pressure in kPa versus the temperature in degrees Celsius [77,78,79]. Equation (42) presents Relative Humidity (RH) as the ratio of the partial vapor pressure (e) to saturation partial vapor pressure (e_s) at temperature T . These relations are also known as Arden Buck equations and are considered accurate for temperatures between -80°C (-112°F) and 50°C (122°F).

$$e_s(T) = 0.61121 \exp\left(\left(18.678 - \frac{T}{234.5}\right)\left(\frac{T}{257.14 + T}\right)\right) T > 0^\circ\text{C} \quad (40)$$

$$e_s(T) = 0.61115 \exp\left(\left(23.036 - \frac{T}{333.7}\right)\left(\frac{T}{279.82 + T}\right)\right) T < 0^\circ\text{C} \quad (41)$$

$$RH = \frac{e(T)}{e_s(T)} \quad (42)$$

3.8.1 Mixing Ratio

Mixing ratio is the ratio of the mass of the mixed components to the total mass—equation (43)—where w_1 is the weight fraction for component 1 with mass (m_1) and w_2 is the weight fraction for component 2 with mass (m_2), with (r_m) the ratio of the component masses ($r_m = m_2/m_1$). In weather applications where one deals with vapor and dry air, the mixing ratio (or humidity ratio) may be defined as the mass of the wet component (m_v) to that of the dry component ($m_d = m - m_v$)—equation (44)—where m is the total mass.

$$w = \frac{w_1 + w_2 r_m}{1 + r_m} \quad (43)$$

$$w = \frac{m_v}{m_d} \quad (44)$$

3.9 Enthalpy

Enthalpy (h) of a thermodynamic system is defined as the sum of its internal energy and the product of its pressure and volume, expressed as energy per unit mass [80]. In other words, it consists of the system's internal energy plus the amount of work required to displace its surroundings based on reaching the equilibrium state—volume and pressure. For an ideal gas, this value is a function of the specific gas constant as well as the gas pressure and temperature.

3.9.1 Static Enthalpy

Static enthalpy is the enthalpy of the fluid when it is motionless—equation (45).

$$h = c_p T \quad (45)$$

3.9.2 Stagnation Enthalpy

Stagnation enthalpy, also known as total enthalpy, is the enthalpy of a motionless fluid plus the added energy due to the movement—equation (46).

$$h = c_p T + \frac{v^2}{2} \quad (46)$$

3.10 Stability

Condensation occurs when the water vapor content within a moist parcel of air reaches its saturation capacity. Since the capacity to hold the water vapor depends on the temperature, this capacity may be increased by increasing the air temperature—for there is less moisture in a cooler air parcel. The condensation process is involved in the formation of clouds, fog, rain, hail, snow, and mist. This cooling process, either in the adiabatic or semi-adiabatic form, causes the density of the air to change—the density of dry air is greater than that of the moist air. Other processes that may help cooling and condensation may involve radiation, contact with the cooler parcel of air or freezing nucleus, and adiabatic cooling along with expansion of the parcel of air. Density of air, which is the determining factor for its buoyancy, is directly associated with air parcel stability. A parcel of air that

is warmer than its surroundings rises, and the rise continues (unstable) until the parcel's temperature equalizes with that of its surroundings. At this point it remains at the approached level (neutral) or becomes cooler so that it starts descending (stable).

In other words, to reverse a stable system to its original status, you only need to remove the driving force for causing the instability—the stressor. However, for an unstable system, lack of the driving force is not sufficient to make the system stable again. In a system with neutral stability, removing the driving force will leave the system at its current, and not initial, state. A technique to determine stability is to compare the actual lapse rate with that of the standard one. If the real-time lapse rate is smaller than the standard-adiabatic lapse rate, the air parcel is stable; if they are equal, the air parcel is neutral; for lapse rates larger than the standard-adiabatic lapse rate, the air parcel is unstable. Another method to identify the stability level is to calculate the gradient of potential temperature with respect to the altitude: positive values are associated with stable air while zero and negative values represent neutral and unstable conditions. Convective Available Potential Energy (CAPE)—the amount of energy an air parcel requires to be lifted vertically to a certain distance—is related to the positive buoyancy of the air parcel and is another indicator of atmospheric stability [81]. Figure 9 depicts the state of hydrostatic equilibrium (single-line arrows) as well as an accelerating cloud scenario (double-line arrows). Balance of forces is applicable to the arrows of similar line type (Figure 9).

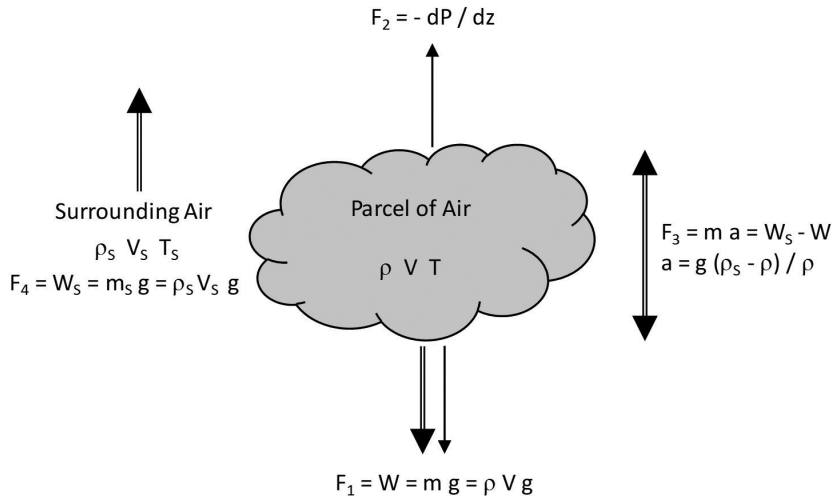


FIGURE 9 Air parcel in hydrostatic equilibrium and acceleration scenarios.

Applying the Archimedes' principle to the parcel of air shown in Figure 9 reveals that upward force imposed on the body from its surroundings and downward force imposed by gravity on the air parcel are to be equal for the case of a parcel in equilibrium with its environment. If the buoyancy force exerted on the air parcel equals its weight, hydrostatic equilibrium is in place and the air parcel does not move (single-line arrows). Equation (32), $dP = -\rho g dz$, is then transformed to equation (47). Average virtual temperature (T_v) may be represented by equation (48).

$$P = P_0 e^{\left(\frac{-g}{RT_v}(z - z_0)\right)} \quad (47)$$

$$\overline{T_v} = \frac{\int_{LnP_0}^{LnP} T_v d(LnP)}{(LnP - LnP_0)} \quad (48)$$

There are also cases in which the air parcel accelerates or decelerates—given the balance of said forces (double-line arrows). This scenario requires that the upward force imposed on the body by its surroundings and the downward force imposed by gravity on the air parcel are not equal—the difference is the acceleration force (F_3 in Figure 9). The balance of forces for an isobaric process results in equation (49), in which acceleration is given as a function of the air parcel temperature (T) and its surrounding temperature (T_∞). Equation (49) shows that if the temperature of the air parcel is lower than that of its surroundings, the acceleration is negative, meaning that the air parcel accelerates downward (stable). The opposite is the case when the temperature of the air parcel is higher than that of its surroundings, in which case the acceleration is positive, meaning that the air parcel accelerates upward (unstable).

$$a = \frac{(T - T_\infty)}{T_s} g \quad (49)$$

Substituting the hydrostatic relation presented by equation (32) into the semi-adiabatic process presented by equation (33) results in the saturated-adiabatic lapse rate (Γ_w) for the semi-adiabatic process presented by equation (50). ϵ is the ratio of the gas constant for the dry air to that of the water vapor ($\epsilon = R_d/R_v$).

$$\Gamma_s = \frac{dT}{dZ} = \Gamma \frac{\left(1 + \frac{Lw_s}{R_d T}\right)}{\left(1 + \frac{\epsilon L^2 w_s}{R_d C_p T^2}\right)} \quad (50)$$

Assuming that the air parcel in Figure 9 is homogeneous, meaning that its density and therefore volume do not vary spatially (i.e., horizontally or vertically), the condition that is possible in some hurricanes, you may calculate the lapse rate for the homogeneous atmosphere by combining the hydrostatic relation—equation (32)—and equation of state—equation (56)— $P = \rho RT$ —to obtain equation (51), in which the homogeneous gradient of temperature or lapse rate is a function of the mass of the air parcel and gas constant. By substituting the numerical values in equation (51), the homogeneous lapse rate of 10.42 °C per 1,000 ft is obtained (34.2 °C per km). If the lapse rate is higher than the homogeneous atmospheric lapse rate, the conditions are absolutely unstable, which is associated with the formation of tornadoes or hurricanes.

$$\frac{dT}{dZ} = -\frac{mg}{R} = -\frac{g}{R} \quad (51)$$

It is possible to calculate the altitude for the homogeneous atmosphere by combining the hydrostatic relation—equation (32)—and equation of state—equation (56)—and solving for the altitude. As a result, equation (52) is obtained, in which the homogeneous height is the function of the standard atmosphere absolute temperature, mass of the air parcel, and universal molar gas constant. By substituting the numerical values in equation (52), the homogeneous atmospheric height 8,267 m (27,119 ft) is obtained.

$$H = -\frac{\bar{R}T_0}{mg} = -\frac{RT_0}{g} \quad (52)$$

If the lapse rate of the air parcel is between the ones of the saturated and dry adiabatic, the air parcel moving upward first faces resistance due to a larger lapse rate and then less resistance due to a smaller lapse rate. This scenario is called conditional stability.

3.11 Examples

3.11.1 Case Study – Pressure Altitude

The weather report indicates the altimeter setting for Norman Rogers Airport (CYGK) in Kingston (Canada) is 30.09 inHg on the 11th of the month (METAR CYGK 111300Z ... A3009...). Assuming that the elevation of CYGK is 303 ft and the altimeter setting is 30.09 inHg, calculate:

(a) pressure altitude for CYGK, (b) pressure altitude for the aerodromes using the data in Table 2, and (c) pressure altitude for the altitudes presented in Table 3.

TABLE 2 Pressure altitudes for different aerodromes on the ground level.

Case	Aerodrome (AD)	AD Code	Elevation (ft)	Altimeter	P (inHg)	Pressure Altitude (ft)
1	Norman Rogers Airport	CYBK	303	A3009	30.09	133
2	Squamish	CYSE	171	A3014	30.14	-49
3	San Diego International	KSAN	16	A3017	30.17	-234
4	Oshawa Executive Airport	CYOO	460	A2962	29.62	760
5	Waterville Robert Lafleur	KWVL	332	A2935	29.35	902

TABLE 3 Pressure altitudes for different flight levels and aerodromes presented in Table 2.

Case	AD Code	Altimeter Data	P (inHg)	Altitude or Flight Level	Altitude (ft)	Pressure Altitude (ft)
1	CYBK	A3009	30.09	2,000	2,000	1,830
2	CYSE	A3014	30.14	4,000	4,000	3,780
3	KSAN	A3017	30.17	FL150	15,000	14,750
4	CYOO	A2992	29.92	FL180	18,000	18,000
5	KWVL	A2992	29.92	FL300	30,000	30,000

Pressure altitude is calculated using equation (13)— $PA = 303 + 1,000 \times (29.92 - 30.09) = 133$ ft. A similar approach is taken to calculate the pressure altitude for the aerodromes presented in Table 2.

3.11.2 Case Study – Lapse Rate

Assuming that the temperature pair in weather report values is provided as 25/18—where the higher value of 25 °C is the dry-bulb temperature and the lower one of 18 °C is the dew-point temperature, calculate:

(a) spread, (b) freezing spread, (c) cloud base levels assuming dry-, standard-, and moist-adiabatic lapse rates, and (d) freezing levels assuming the moist-adiabatic lapse rate.

The spread is 7 °C (i.e., 25 °C - 18 °C = 7 °C). For a saturated atmosphere, the cloud base is 4,667 ft (i.e., $7 [^{\circ}\text{C}] / (1.5 [^{\circ}\text{C}]/1,000 [\text{ft}])$)—Table 4.

TABLE 4 Cloud base and freezing level case studies.

Weather Data					AGL Cloud Base (ft)			Freezing Level (ft)	
METAR Format	Dry-Bulb Temperature (°C)	Dew-Point Temperature (°C)	Spread (°C)	Freezing Spread (°C)	Saturated	Adiabatic	Dry	Cloud Base	Ground Base
M5/M10	-5	-10	5	10	3,333	2525	1,666.7	6,666.7	10,000.0
0/M5	0	-5	5	5.0	3,333	2525	1,666.7	3,333.3	6,666.7
10/5	10	5	5	5.0	3,333	2525	1,666.7	3,333.3	6,666.7
25/18	25	18	7	18.0	4,667	3535	2,333.3	12,000.0	16,666.7
30/20	30	20	10	20.0	6,667	5051	3,333.3	13,333.3	20,000.0

3.11.3 Case Study – Density Altitude

The weather report indicates the altimeter setting for Norman Rogers Airport (CYGK) in Kingston (Canada) is 30.09 inHg on the 11th of the month, and the dry-bulb temperature is at 1 °C and the dew-point temperature is at -3 °C (METAR CYGK 111300Z ... 01/M03 A3009...). Assuming that the elevation of CYGK is 303 ft with an altimeter setting of 30.09 inHg, calculate:

(a) pressure altitude for CYGK, (b) pressure altitude for the aerodromes presented in Table 5, (c) standard temperatures for the elevations presented in Table 5 assuming the standard-adiabatic lapse rate (1.98 °C/1,000 ft), and (d) density altitude for the elevations presented in Table 5.

For Case 1, the pressure altitude is 1,830 ft [$PA = 2,000 + 1,000 \times (29.92 - 30.09) = 1,830$ ft]. The standard temperature (T_{ISA}) is 11.04 °C [$T_{ISA} = 15 - 1.98 \times (2,000/1,000) = 11.04$ °C] at the pressure altitude (PA), and the density altitude (DA) is 1,112.45 ft [$DA = 1,830 + 118.8 \times (5 - 11.04) = 1,112.45$ ft]. A similar approach is taken to calculate the aforementioned properties provided in Table 5.

TABLE 5 Pressure and density altitudes and standard temperatures for different flight levels and aerodromes presented in Table 2 and Table 3.

Case	Flight Level	AD Code	Altimeter	Altitude	P (inHg)	Pressure Altitude (ft)	Temperature (°C)	Standard Temperature (°C)	Density Altitude (ft)
1	2,000	CY GK	A3009	2,000	30.09	1,830	5	11.0	1,112.45
2	4,000	CY SE	A3014	4,000	30.14	3,780	-5	7.1	2,344.90
3	FL150	KSAN	A3017	15,000	30.17	14,750	-10	-14.7	15,308.36
4	FL180	CY OO	A2992	18,000	29.92	18,000	-15	-20.6	18,670.03
5	FL300	KWVL	A2992	30,000	29.92	30,000	-20	-44.4	32,898.72

3.11.4 Case Study – True Airspeed (TAS) versus the Calibrated Airspeed (CAS)

For the data presented in Table 6, using the standard pressure and temperature at sea level and elevation data, assuming the standard-adiabatic lapse rate of $1.98\text{ }^{\circ}\text{C}/1,000\text{ ft}$ and $CAS = 1.1 \times TAS / \sqrt{\rho}$, for the given Mach numbers, calculate:

(a) pressure, (b) density, (c) *TAS*, (d) *EAS*, (e) *CAS*, and (f) present a plot of *TAS*, *EAS*, and *CAS* versus the Mach number.

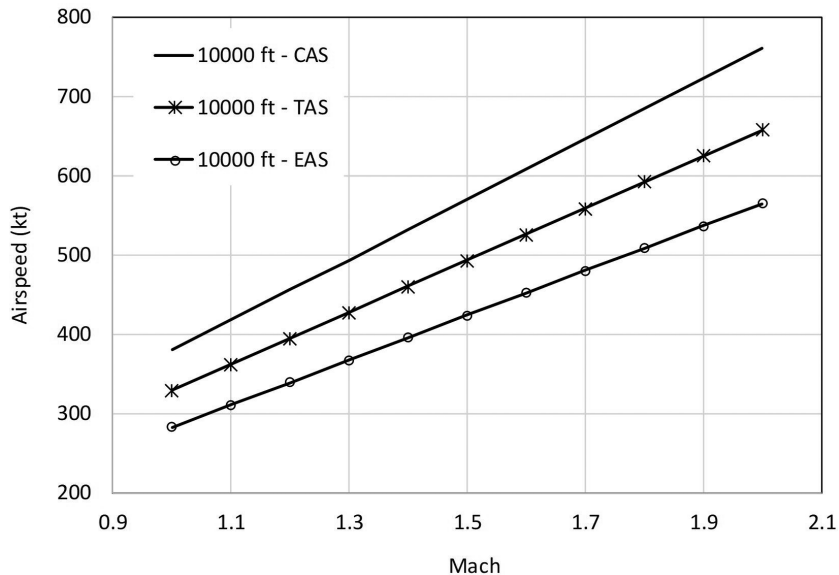
Using the standard pressure and temperature presented in Table 6, density ($\rho = P/RT$) is obtained. The pressure (in Pa) at a specified altitude is given by $P = 101,325(1 - 2.255,77 \cdot 10^{-5} H)^{5.255,88}$ —where H is the elevation in m. Temperature is obtained from $T = T_{SL} - 1.98(H - H_{SL})/1,000$. Given the standard-adiabatic lapse rate of $\Gamma_{ELR} = 1.98\text{ }^{\circ}\text{C}/1,000\text{ ft}$, *TAS* is obtained from $TAS = c_0 M \sqrt{T_s/T_0}$, *CAS* is obtained from $CAS = 1.1 TAS / \sqrt{\rho}$, and *EAS* is obtained from $EAS = \sqrt{\rho/\rho_0} TAS$ —where *TAS*, *CAS*, and *EAS* are in kt. Figure 10 presents the *TAS*, *CAS*, and *EAS* as a function of the Mach number—Table 7.

TABLE 6 Temperature, pressure, and density versus the elevation.

Elevation (ft)	Sea Level	10,000 ft	20,000 ft
ρ (kg/m ³)	1.22	0.90	0.65
P (kPa)	101.33	69.68	46.56
P (Pa)	101,325	69,682	46,563
T (°C)	15	-4.8	-24.6
Altitude (ft)	0	10,000	20,000

TABLE 7 True and equivalent airspeeds as functions of the calibrated airspeed and elevation.

Mach	TAS (kt)			EAS (kt)			CAS (kt)		
	Sea Level	10,000 ft	20,000 ft	Sea Level	10,000 ft	20,000 ft	Sea Level	10,000 ft	20,000 ft
1	340.3	328.4	316.0	340.3	282.2	230.7	338.2	379.8	430.3
1.1	374.3	361.2	347.7	374.3	310.4	253.8	372.0	417.8	473.4
1.2	408.4	394.1	379.3	408.4	338.6	276.8	405.9	455.8	516.4
1.3	442.4	426.9	410.9	442.4	366.9	299.9	439.7	493.8	559.5
1.4	476.4	459.8	442.5	476.4	395.1	323.0	473.5	531.7	602.5
1.5	510.4	492.6	474.1	510.4	423.3	346.0	507.3	569.7	645.5
1.6	544.5	525.4	505.7	544.5	451.5	369.1	541.1	607.7	688.6
1.7	578.5	558.3	537.3	578.5	479.7	392.2	575.0	645.7	731.6
1.8	612.5	591.1	568.9	612.5	508.0	415.2	608.8	683.7	774.6
1.9	646.6	624.0	600.5	646.6	536.2	438.3	642.6	721.6	817.7
2	680.6	656.8	632.1	680.6	564.4	461.4	676.4	759.6	860.7

**FIGURE 10** Calibrated, true and equivalent airspeeds as functions of the Mach number.

3.11.5 Case Study – Passenger Luggage

München Airport (EDDM) terminal in Munich (Germany) uses a circular vacuum tube lifter in order to facilitate handling passenger luggage up to 35 kg from the airplane to the luggage carrier and to the apron trolley for short

hauls. The main purpose is to improve ergonomics and efficiency—almost half of the pieces of baggage weigh between 15 and 20 kg. The vacuum tube is equipped with grippers. The operator uses a System Resource Controller (SRC) radio control to switch the vacuum on and off as required to improve productivity and energy costs. The vacuum gripper may also be connected to a mechanical hook system. The crane rail is made of aluminum with a 180-degree swivel angle, and the radius of the tube is 3.5 cm (Figure 11). Density as a function of the temperature for air and mercury is given in Figure 12. Using data presented in Table 8, calculate:

- (a) required vacuum pressure to lift 15 kg, 20 kg, and 30 kg loads,
- (b) maximum load that can be lifted assuming that atmospheric pressure is the driving force to generate the vacuum, and (c) maximum load that can be lifted assuming that varying ambient air pressure is the driving force to generate the vacuum.

Formulae are presented in Table 8. Note that pressure is defined as the force over the area. The cross section of the tube that has an area A is circular (with radius r). For the scenarios in which the atmospheric pressure is the driving force and it is dependent or independent of temperature, the associated force is employed for calculations.

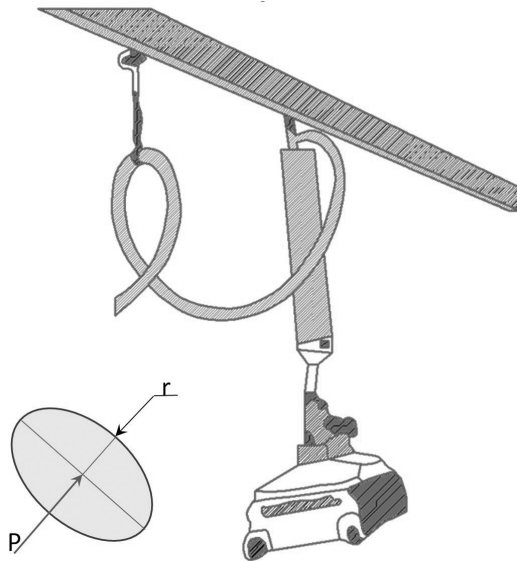


FIGURE 11 A vacuum tube lifter (drawings created using Solid Edge CAD tool).

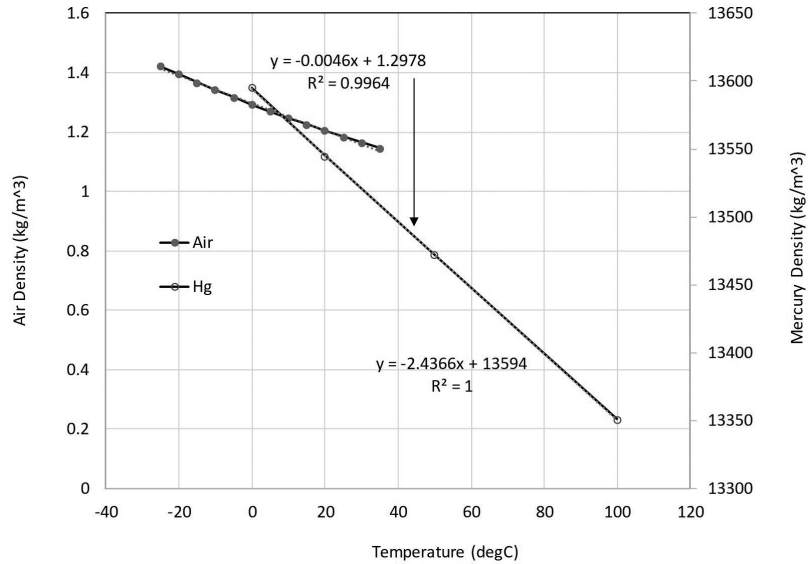


FIGURE 12 Mercury and air densities versus the temperature.

TABLE 8 Pressure versus the luggage mass calculations.

Gravitational acceleration (g)	9.8 m/s ²			
Air density at 15 °C (ρ_{air})	1.225 Kg/m ³			
Mercury density at 15 °C (ρ_{Hg})	13557.451 Kg/m ³			
Geometry Specification of the Tube				
Vacuum Tube Radius (r)	3.500	cm	0.03500	m
Vacuum Tube Area ($A = \pi r^2$)	38.485	cm ²	0.00385	m ²
Pressure Required to Lift the Loads				
Mass (M) (kg)	30	20	15	10
Weight ($F = W = Mg$)	294 N	196	147	98
Pressure ($P = F / A$) (Pa)	76,390	50, 909.09	38,181.82	25,454.55
Pressure ($P = F / A$) (kPa)	76.390	50.910	38.182	25.455
Pressure ($P = F / A$) (inHg)	22.55	15.033	11.280	7.517
Forces by the Atmospheric Pressure				
MSL Pressure (P)	101.325	kPa	101,325	Pa
Force ($F = PA$)	3.546	kN	3,546.375	N

The ratio of the densities can be used to calculate the forces due to the use of mercury versus air—the tube cross section does not vary between the two scenarios. The force, equal to the weight, may be converted to mass assuming a constant gravitational acceleration. Maximum loads that may be lifted for different atmospheric pressures are presented in Table 9. Pressure versus the luggage mass is shown in Figure 13. Note that $1 \text{ kPa} = 0.2953 \text{ inHg}$.

TABLE 9 Mass calculations given pressure.

Case	Pressure (inHg)	Pressure (kPa)	Force (N)	Mass (kg)
1	3.76	12.73	49.00	5.00
2	7.52	25.46	98.00	10.00
3	11.28	38.20	147.00	15.00
4	15.04	50.93	196.00	20.00
5	18.80	63.66	245.00	25.00
6	22.56	76.39	294.00	30.00
7	25.69	87.00	334.82	34.16
8	29.92	101.90	392.16	40.02
9	30.78	104.25	401.20	40.94
10	32.00	108.37	417.05	42.56

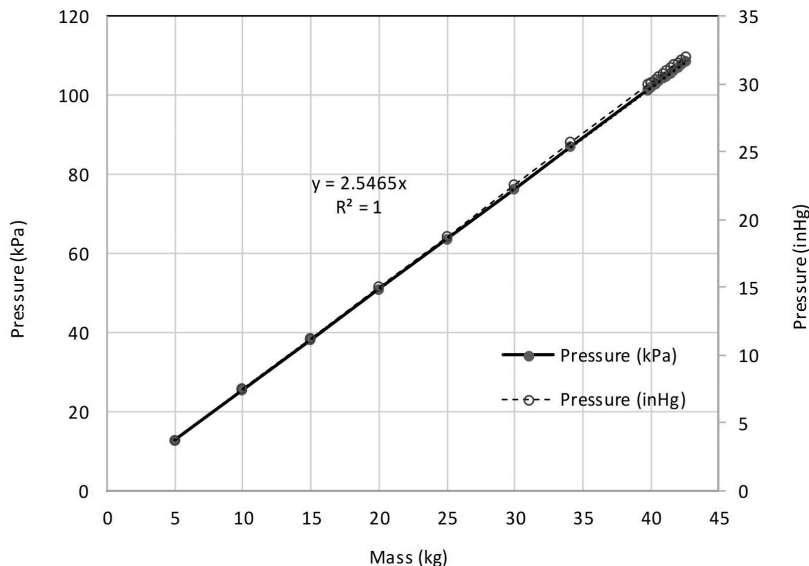


FIGURE 13 Pressure as a function of the luggage mass.

CHAPTER 4

THERMODYNAMICS OF AIR

In a thermodynamic system, a boundary—an imaginary or real, fixed or movable border—encloses a volume of particles. The boundary separates the system from its surroundings. It is understood that mass and energy (the latter as heat and work) flow across the boundary. There is no limit to the number of particles that may form a system; one atom (as suggested by Max Planck’s definition in 1900), one nucleus (as used in quantum thermodynamics), or a body of air mass (as suggested by Sadi Carnot in 1824) may each form a thermodynamic system. For a horizontally opposed piston engine, a piston moving inside the cylinder defines a moving boundary, while a piston at rest is a fixed boundary. The boundaries in both cases are real; on the other hand, for gases leaving the exhaust manifold, an imaginary boundary may be created to calculate the energy released to the atmosphere in the form of heat that may be used; for example, for cabin heating [82,83,84,85,86].

Depending on the level of interaction of a system with the surroundings, further categorizations of open, closed, and isolated systems have been introduced. Isolation may be thermal, mechanical, or as a combination of both. When an equilibrium state within a parcel of air is reached, differences in properties such as internal temperature, pressure, and density approach zero. When traveling from point A to point B in a three-dimensional space, the path taken may vary. The same is applicable in a thermodynamic process, where one may “travel” from state A to state B, along different paths. Each path represents different process. As one travels along this

path, the internal energy level of the system changes from the initial state (A) to the final state (B).

Atmospheric properties also affect other phenomena in which the atmosphere acts as a carrier. Sound pressure level (SPL) is a logarithm of the effective pressure with respect to a reference value (20 μPa). In other words, it is the difference between the average local pressure and pressure of the sound wave. An aircraft flying overhead generates sound with adverse health effects starting from 40 dBA; the sound is required to be managed to fall within the recommended intensity to comply with environmental conditions. This recommended intensity is as low as 57 dBA in Australia and 65 dBA in the United States for noise-sensitive zones. Studies show that a nighttime average sound pressure level of 55 dBA may increase the risk of heart attacks by 66 percent in men and 139 percent in women. They also found that a 5 dBA increase in aircraft noise exposure resulted in a two-month increase in children's reading age in the United Kingdom and a one-month increase in the Netherlands. Isobels (lines of constant sound pressure level) will assist with identifying the areas in which the sound pressure levels exceed 40 dBA. For comparison, singing birds generate 44 dBA and whispers generate 20 dBA; on the other hand, a jet generates 150 dBA at takeoff at 25 m, with a potential for eardrum rupture. Live rock music generates about 108 dBA, which is the average human pain threshold. In addition to the SPL, the exposure time is important when setting acceptable noise threshold limits [87,88,89,90,91,92,93].

4.1 Thermodynamics of Dry Air

Additional thermodynamic process classifications are: (a) isothermal, where temperature remains constant for the entire process; (b) isobaric, where pressure remains constant throughout the process; (c) isovolumetric, also known as isochoric, which happens at constant volume; (d) adiabatic, where there is no transfer of energy in the form of heat between the system and the surroundings; (e) isentropic, where entropy remains constant for the entire time; (f) isenthalpic, where enthalpy remains unchanged during the process; and (g) steady-state, where changes over time are insignificant.

The first law of thermodynamics, briefly discussed previously, introduced the existence of a connection between the gas internal energy (related to translational, rotational, and oscillatory motions of the molecules) and its temperature. This law—regarding the conservation of energy—states that

this change of internal energy is associated with work and heat and is in fact the difference between the two terms as presented in equation (53). There are also other interpretations of this law that express one term in terms of two others. For example, the energy that is inputted into the system in the form of heat (dQ) causes its internal energy to increase (dU) and for work to take place on the system (dW).

$$dU = dQ - dW \quad (53)$$

The air parcel floating in the sky is exposed to forces in all directions—Figure 9. The work that causes the volume to compress from V_1 to V_2 presented as an integral of pressure over volume in equation (54) shows the change of the state variables—pressure versus the volume—and this work's magnitude is the area under the diagram representing this relation (Figure 14). There are different paths to take from the initial state A via the intermediate state B, and the final state C, each having their associated heat and work; however, the total energy is the same, meaning the energy is conserved. Assuming a constant pressure or volume for the process, the area under the diagram will either represent the heat given to or taken away from the system, the work done on the system, or the work that the system does on its surroundings.

$$W = \int_{V_1}^{V_2} P dV \quad (54)$$

There are a number of general thermodynamic relations that govern ideal gas behavior and that are adopted for describing the thermodynamics

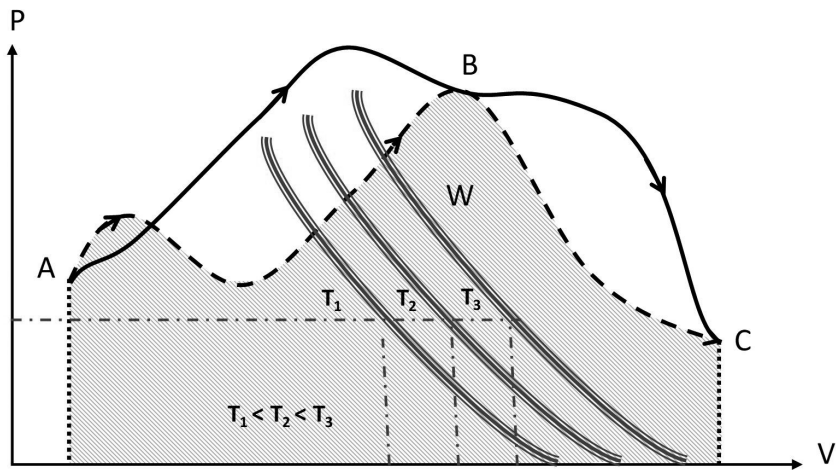


FIGURE 14 Pressure-volume diagram and representation of work.

of air. Ideal gas law explains the state of gas and is generally expressed with equation (55), where P is pressure, V is volume, n is the number of molecules (total mass per molar mass: m/M), R is the specific gas constant, and T is the absolute temperature. This equation is an empirical law and is calibrated for different gases, and therefore the specific gas constants vary depending on the gas. It is possible to reformulate this equation to include density (m/V)—equation (56)—where ρ is density and \bar{R} is the universal molar gas constant (8.314 kJ/kmolK). For air, the relevant constants required are the dry air molar mass (28.96 kg/kmol), the specific gas constant for dry air ($R_d = 287.0$ J/kgK), and the constant for saturated moist air ($R_d = 461.5$ J/kgK)—water molar mass is 18.02 kg/kmol). Recall Avogadro's law that volume increases with increasing the number of molecules, for a given temperature and pressure.

$$PV = n\bar{R}T \quad (55)$$

$$P = \rho RT \quad (56)$$

The previous relationship is intended to represent the status of gas at any stage of the process. Depending on the process, different expressions for these equations may be derived. Equation (55) is another form of Avogadro's law—equation (57)—where *cte* means it is constant.

$$\frac{V}{n} = cte \rightarrow \frac{V_1}{n_1} = \frac{V_2}{n_2} \quad (57)$$

A process is isothermal where temperature remains constant for the entire process (i.e., $T = \text{constant}$). For an isothermal process, equation (55) simplifies to equation (58)—also known as Boyle's and Mariotte's law, which states that the pressure and volume are inversely related at a constant temperature. At a constant temperature, the heat given to the system is purely used to create work on the system ($dQ = PdV = -Vdp = dW$).

$$PV = cte \rightarrow P_1V_1 = P_2V_2 \quad (58)$$

In an isobaric process, pressure remains constant throughout the process (i.e., $P = \text{constant}$). For an isobaric process, equation (55) simplifies to equation (59)—also known as Charles's law, which states that the absolute temperature and volume are inversely related at a constant pressure. At a constant pressure, the heat given to the system is purely used to increase the internal energy of the system ($dQ = Mc_p dT$), where c_p is heat capacity at constant pressure.

$$\frac{V}{T} = cte \rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (59)$$

A more general form of equation (59) in which thermal expansion coefficient (α) is taken into consideration is given in equation (60), where constant pressure has been identified in the form of subscript “ p ” for the expansion coefficient. A partial derivative of equation (58) results in the thermal expansion coefficient of $\alpha_p = 1/T$ for the gas at constant pressure. This coefficient shows that if the temperature of a gas at constant pressure increases by one Kelvin (or one degree Celsius), the volume increases by 0.37 percent ($1/273$) of the original volume. Note that here the temperature (T) is in Kelvin. Subscript “ θ ” represents the ISA conditions.

$$V = V_0 (1 + \alpha_p T) \quad (60)$$

Assuming that an ideal gas follows a process starting at an initial state 1 (P_1, V_1, T_1) and reaching a final state 2 (P_2, V_2, T_2), using Charles’s law, equation (61) is presented for the thermal expansion coefficient at constant pressure (α_p). In this relation, the gas constant (R) depends on gas, volume, and expansion coefficient, and absolute temperature (T'), resulting in equation (62). This equation is the derivation for converting the temperature unit from Celsius to Kelvin.

$$\alpha_p = \frac{1}{V} \left(\frac{dV}{dT} \right) \quad (61)$$

$$T' = \left(\frac{1}{\alpha_p} + T \right) = 273 + T \quad (62)$$

The isovolumetric (also known as isochoric) process takes place at constant volume (i.e., $V = \text{constant}$). For an isochoric process, equation (55) simplifies to equation (63)—also known as the Gay-Lussac’s law, which shows that the absolute temperature and pressure are inversely related at a constant volume. At a constant volume, the heat given to the system is purely used to increase the internal energy of the system ($dU = Mc_v dT$), where c_v is heat capacity at constant volume.

$$\frac{P}{T} = cte \rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad (63)$$

In an adiabatic process, there is no transfer of energy in the form of heat between the system and the surroundings. For this process, the

change of system internal energy is transformed only to compression or expansion work. Energy balance equation (53) is then transformed into equations (64) and (65). Processes occurring between the surface of the Earth and the atmosphere are usually of a non-adiabatic nature; however, short-term processes may be assumed to be adiabatic. Given the negative value of temperature gradient with altitude, the air parcel moving upward is expanded adiabatically, and the one moving downward is compressed adiabatically.

$$Mc_v dT = -PdV \quad (64)$$

$$Mc_p dT = VdP \quad (65)$$

Entropy ($\phi = s$) is a quantity introduced by the second law of thermodynamics. There is a relationship between entropy and the heat that is received or given back by the air parcel as a function of its absolute temperature—equation (66).

$$d\phi = \frac{dQ}{T} \quad (66)$$

An isentropic process, also known as reversible adiabatic, does not involve entropy change ($d\phi$), and equation (55) simplifies to equations (67) and (68)—which show that the absolute pressure is inversely related to the ratio of the volume raised to the power of the ratio of the specific heats ($\gamma = c_p/c_v$)—for a constant-temperature process. Combining equations (66) and (7)— $\theta = T(1000/P)^\gamma$ —results in equation (69).

$$PV^\gamma = cte \rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^\gamma \quad (67)$$

$$\frac{P^{\gamma-1}}{T^\gamma} = cte \rightarrow \frac{P_1^{\gamma-1}}{T_1^\gamma} = \frac{P_2^{\gamma-1}}{T_2^\gamma} \rightarrow \left(\frac{P_1}{P_2} \right)^{\gamma-1} = \left(\frac{T_1}{T_2} \right)^\gamma \quad (68)$$

$$\phi = C_p \ln \theta + cte \quad (69)$$

The general form of energy balance equation presented by equation (53) is then transformed to either forms of equations (70) or (71), taking into account that the heat capacity ratio ($\gamma = c_p/c_v$) for an ideal gas is a function of its degrees of freedom and is 1.4 for a diatomic gas with 5 degrees of freedom (the minimum number of coordinates required to identify the spatial position, three, plus direction, two).

$$dQ = Mc_v dT + PdV \quad (70)$$

$$dQ = Mc_p dT - VdP \quad (71)$$

4.2 Thermodynamics of Moist Air

Given that air is a mixture of gases, the single-component relationships presented previously are not directly applicable. Air may be treated as a combination of dry air and water vapor. Recall that dry air consists of 78 percent nitrogen, 21 percent oxygen, and 1 percent inert gases. When the air is saturated—where the rates of phase change from liquid to vapor and vice versa remain constant, and where temperature and pressure remain the same during the phase change process—the Clausius–Clapeyron relation is valid—equation (72), where e_s and e_{s0} are saturated partial vapor pressure for air and a reference air at T and T_0 , respectively. T_0 is the atmospheric reference temperature ($0.01^\circ\text{C} = 273.16\text{ K}$) whose partial pressure is 611 kPa (6 atm, 180.43 inHg, 88.62 psi)—also known as the triple point, where three phases of water are at equilibrium. L is the latent heat of evaporation in kJ/kg—the amount of heat required to change the phase of the unit mass of the liquid to vapor or vice versa, presented by equation (73). This value may be replaced with the latent heat of sublimation (L_s)—the amount of heat required to change the phase of the unit mass of the vapor to solid or vice versa, which occurs at a constant temperature. Temperature (T) is in Kelvin and R_v is the specific water vapor constant (461.5 J/kgK).

$$L_n \frac{e_s}{e_{s0}} = \frac{L}{R_v} \left(\frac{1}{T_0} - \frac{1}{T} \right) \quad (72)$$

$$L = 3331.3 - 2.9T \quad (73)$$

The heat capacities at constant pressure and volume for unsaturated parcels of air are defined using equations (74), (75), and (76)—where w is the mass ratio of the water content to that of the dry air, C_{pd} is the heat capacity at constant pressure for dry air, and C_{vd} is the heat capacity at a constant volume for dry air.

$$C_{pm} = C_{pd} (1 + 0.86w) \quad (74)$$

$$C_{vm} = C_{vd} (1 + 1.97w) \quad (75)$$

$$\frac{R_m}{C_{pm}} = \left(1 - \frac{1}{\gamma} \right) (1 - 0.26w) \quad (76)$$

If water and vapor remain in equilibrium at a temperature below zero degrees Celsius, water is called supercooled liquid. This unstable phase change triggers the fast freezing of supercooled water coming in contact with a nucleus, forming ice crystals. If the freezing nucleus is absent, pure water droplets in a supercooled state can reach temperatures of about -37°C , at which point they freeze uniformly.

4.3 Weather Models

Concepts of dry and moist adiabatic rates have been the foundation of theoretical and empirical models for atmospheric thermodynamics since 1860, with the invention of first widely used weather forecast diagram in 1927. Weather data such as dry-bulb and wet-bulb temperatures, pressure, and humidity are collected by different techniques such as a radiosonde attached to a latex balloon filled with a light gas (helium or hydrogen) that ascends through the atmosphere to the troposphere and lower stratosphere. As the pressure decreases with altitude, the balloon expands and its skin bursts at some point, stopping its ascent, and the apparatus then returns safely back to the ground by means of a parachute. This flight may take from 60 to 90 min. Radiosondes can also be dropped by an aircraft. They generally collect data such as dry-bulb and dew-point temperatures, pressure, wind magnitude, and direction as a function of the altitude. Weather is predicted by plotting data acquired by the radiosondes in different weather diagrams, to predict the changes of pressure and temperature with altitude as well as rain shower and thunderstorm forecasts, and also to facilitate calculation of convective stability or convective available potential energy. Wind barbs are often plotted on the far-right side of these diagrams to show the magnitude and direction of wind as a function of the altitude. The most widely used weather diagrams are the Stüve diagram, emagram, tephigram, skew-T log-P diagram, and Theta-Z-diagram, all discussed in the following sections. A general-purpose chart widely used to relate thermodynamic properties of the moist air, known as a psychrometric chart, is also presented. In the following descriptive comments, the inclination angle is the smallest angle between the process line and the ordinate (vertical axis). Each of these diagrams describes the thermodynamic state of the fluid (gas or liquid) and the effect of change of one property on the rest of the properties in order to predict the future trends based on the historical and current data.

The term *contour line* is used in weather maps to indicate the lines of equal property value (isopleths). These lines include the following: (a) isoclines (lines of constant gradient slope); (b) isotherms (lines of constant temperature); (c) isobars (lines of constant pressure); (d) isochores (lines of constant volume); (e) isohumes (lines of constant relative humidity); and (f) isodrosotherms (lines of constant dew-point temperature). Isopachs are lines of constant thickness and may be applied to any equal-distanced process lines or properties on a weather-related map or diagram.

Other lines associated with constant moist or dry air property values may be extracted using any of the two said properties (e.g., pressure and

temperature). The following are the possible scenarios: (a) isallobars (lines of constant pressure gradient for a specific time interval); (b) isopycnals (lines of constant density); (c) isogeotherms (lines of constant mean annual temperature); (d) isotheres (lines of constant mean summer temperature); (e) isohyets also known as isohyetals (lines of constant rainfall for a specific time interval); (f) isonephs (lines of constant cloud coverage); (g) isochalaz (lines of constant frequency of hail storms); (h) isobronts (lines of constant phases of thunderstorm activity that occur simultaneously); (i) isotachs (lines of constant wind speed); (j) isogons (lines of constant wind direction); (k) isopectics (lines of constant ice formation in each winter); (l) isotacs (lines of constant dates of thawing in each spring); (m) isohypses (lines of constant height for a given pressure level); (n) isopotentials (lines of constant electrostatic potential in space, also known as electrostatic charges); and (o) isophote (lines of constant illuminance or light intensity—total luminous flux incident on a surface per area, such as precipitation rate supplied per unit target area) [94,95].

In oceanography and meteorology-related sciences, additional isopleth names are used: (a) isobaths (lines of constant depth under the water); (b) isobathytherms (lines of constant depth of water with equal temperature); (c) isopycnals (lines of constant planes of equal water density); and (d) isohalines (lines of equal ocean salinity) [96].

4.3.1 Stüve Diagram

Figure 15 shows a Stüve Diagram [97]. It is a diagram based on an adiabatic process obtained using equation (7)— $\theta = T(1000/P)^\gamma$, and it shows that for a fixed potential temperature (θ), there is a linear relationship between the temperature (T) and pressure (P) to the power of specific heat capacity ratio ($\gamma = c_p/c_v$). Pressure to the power of the heat capacity ratio (P^γ) (ordinate) decreases upward as a function of the reference point (100 kPa) versus the temperature (abscissa) increasing to the right. The far-right ordinate represents the altitude increasing upward linearly. Lines of constant potential temperature (θ) are called adiabats—either moist or dry adiabats [98].

Isotherms are solid-straight-vertical lines, isobars are solid-straight-horizontal lines, and dry adiabats are solid-straight-left-oblique lines with about a 45-degree inclination. Dry adiabats converge to a point at zero pressure and absolute zero temperature and moist adiabats are dashed-left-oblique lines with roughly a 55-degree inclination, curving upward. Isohumes are dotted-left-oblique lines, curving upward with a 20-degree to 10-degree inclination, decreasing from the right side to the left side.

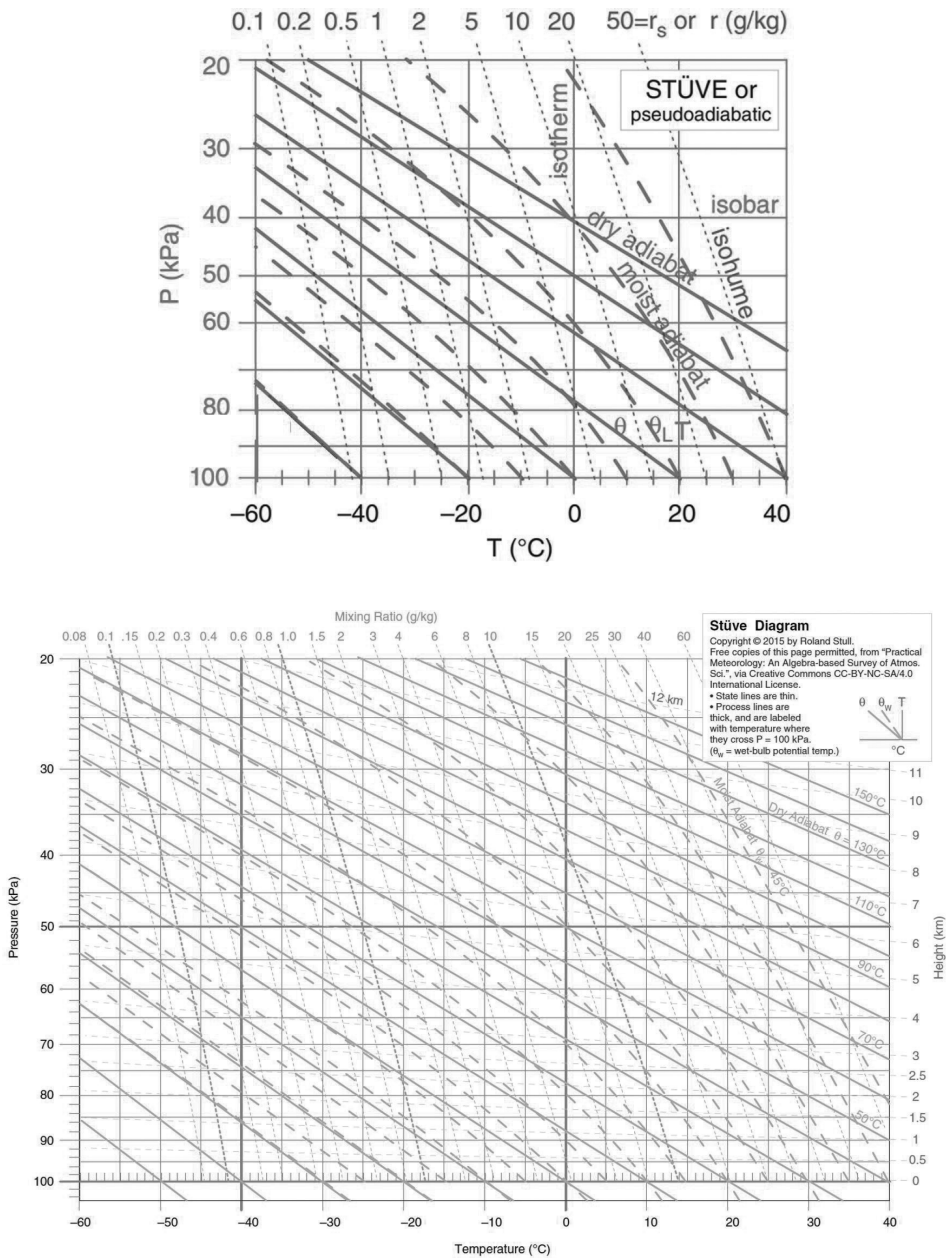


FIGURE 15 Stüve Diagram as a means to calculate properties of the moist air with a pressure range of 20 kPa to 100 kPa [97].

The pseudo-adiabatic assumption to generate the moist adiabats has resulted in the diagram being known as a pseudo-adiabatic chart as well, which calls for equal surface areas between the temperature of the surroundings and the moving air parcel. The stability and cloud base are addressed but not the CAPE.

4.3.2 Emagram

The energy-per-unit-mass diagram, also known as an emagram (Figure 16), presents a logarithm of pressure (ordinate) decreasing upward versus the temperature (abscissa) increasing to the right [97]. The far-right ordinate represents the altitude increasing upward linearly. The area under this diagram is associated with the energy or work. It is designed to present the temperature lapse rate and moisture content profiles in the atmosphere [99]. Isotherms are solid-straight-vertical lines and isobars are solid-straight-horizontal lines. Dry adiabats are solid-left-oblique lines with a 70-degree to 50-degree inclination decreasing from the right side to the left side and slightly curving downward, and moist adiabats are dashed-left-oblique lines with a 30-degree to 50-degree inclination, curving downward. Isohumes are dotted-left-oblique lines, curving upward with a 20-degree to 10-degree inclination, decreasing from the right side to the left side. Therefore, isotherms and isobars are perpendicular to one another. Dry and moist adiabats become parallel at low temperatures and high altitudes. This diagram is very similar to the Stüve Diagram.

4.3.3 Tephigram

The $T - \varphi$ diagram, also known as the tephigram (Figure 17), is based on temperature-entropy relations [97]. As shown earlier, a logarithm of potential temperature (θ)—the ordinate—is related to change in entropy (φ)—the abscissa [100]. The far-right ordinate represents the altitude increasing upward linearly. In this diagram, isotherms are presented with solid-straight-right-oblique lines with a 45-degree inclination while isobars are solid horizontal lines. Dry adiabats are solid-left-oblique lines with a 50-degree inclination downward, and moist adiabats have dashed-left-oblique profiles with inclination varying from zero to 45 degrees—increasing left to right and curving upward. Dry and moist adiabats become parallel at low temperatures and high altitudes. Isohumes are dotted-straight-right-oblique lines with an inclination varying from 25 to 30 degrees, increasing left to right. It is possible to compare CAPE and convective systems more accurately by measuring areas contained by the curves that have equal energies for equal areas.

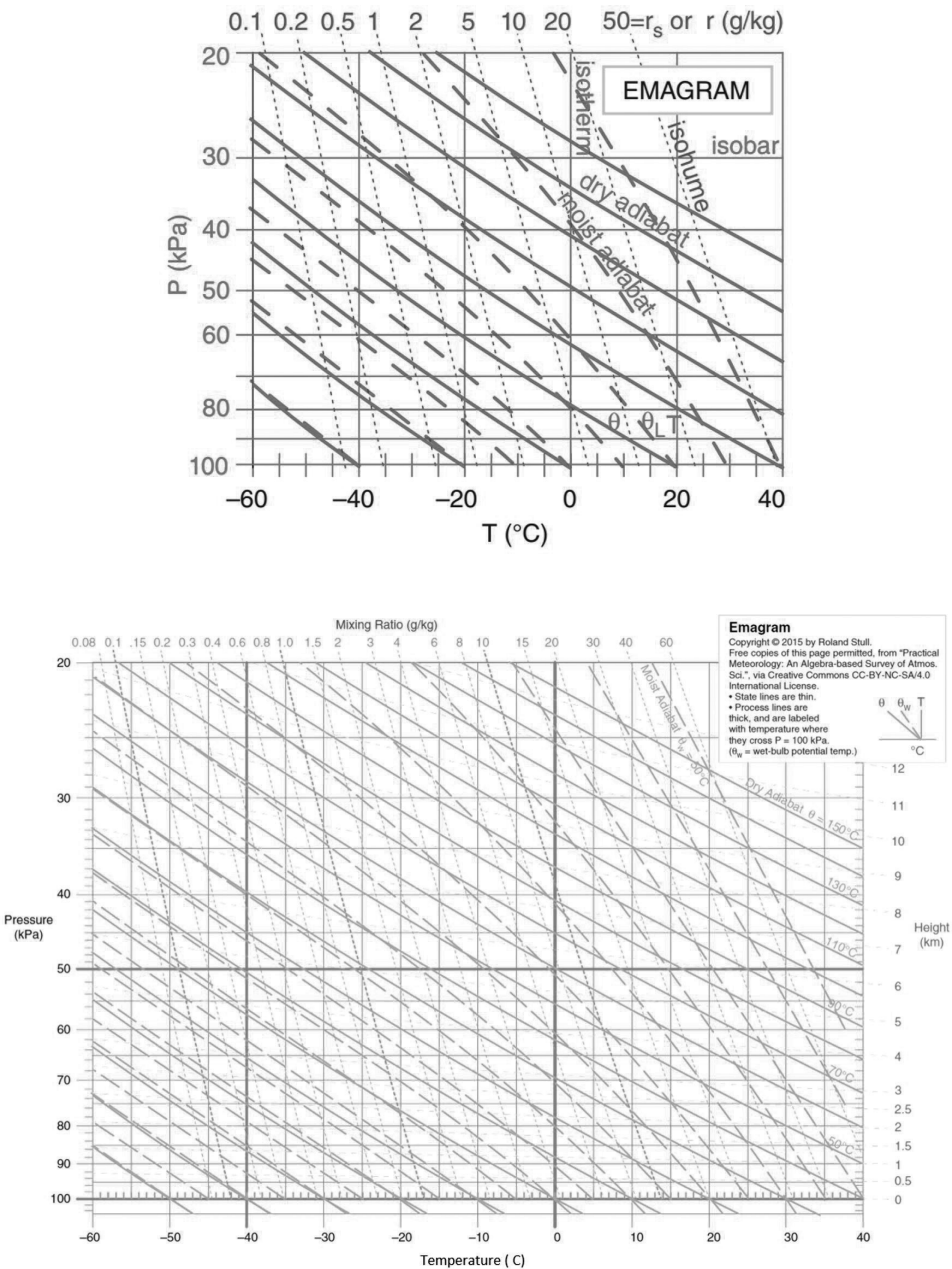


FIGURE 16 Emagram as a means to calculate properties of the moist air with a pressure range of 20 kPa to 100 kPa [97].

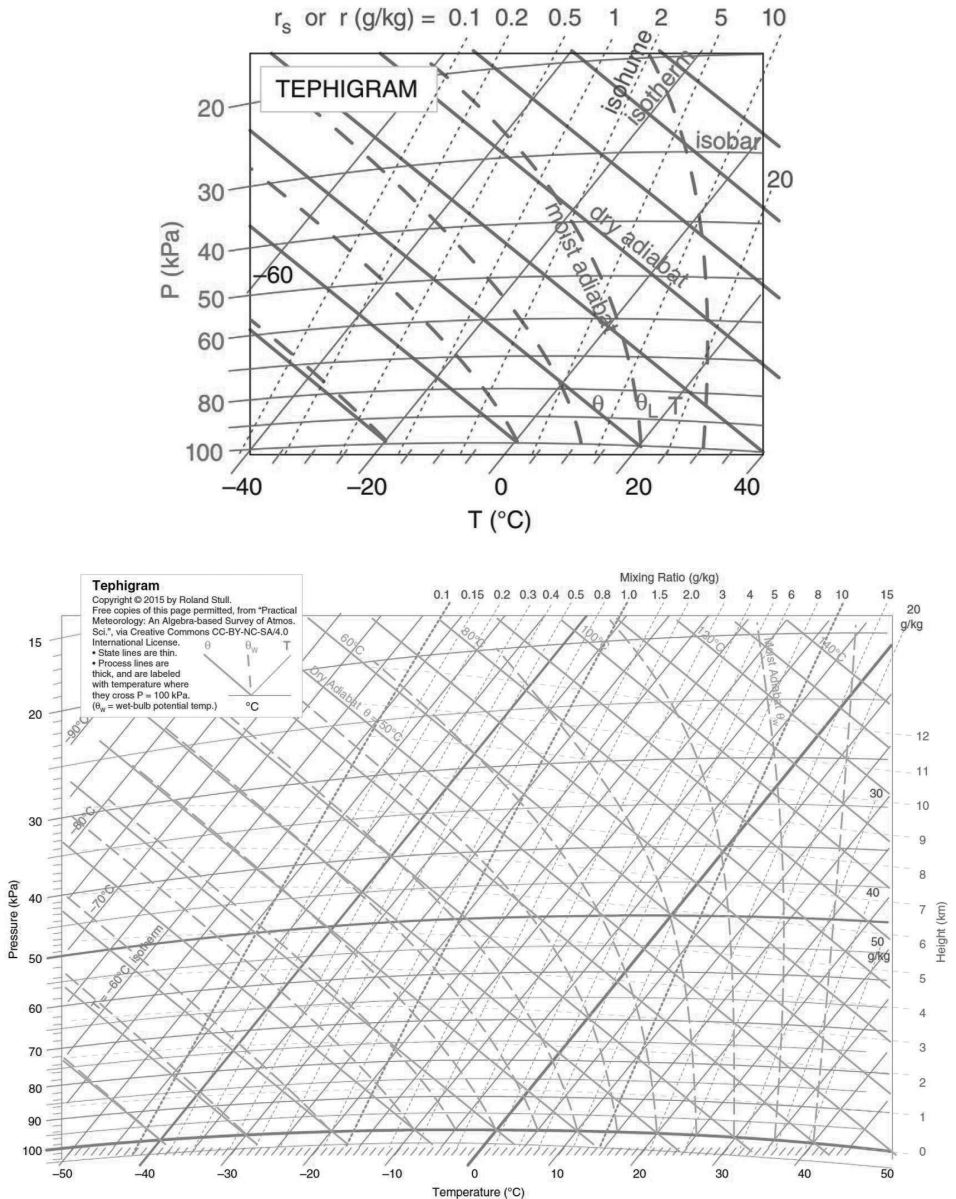


FIGURE 17 Tephigram as a means to calculate properties of the moist air with a pressure range of 15 kPa to 100 kPa [97].

4.3.4 Skew-T Log-P Diagram

Figure 18 and Figure 19 show a Skew-T log-P diagram [97]. This diagram is used to predict weather information such as saturation, stability, and wind shear based on visual analysis. It shows the logarithmic

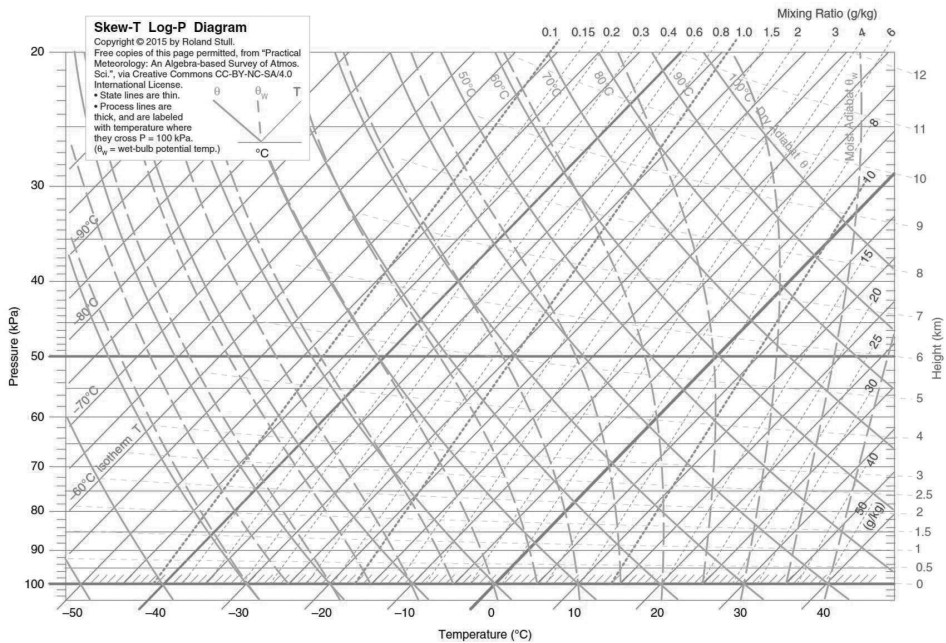


FIGURE 18 Skew-T log-P diagram as a means to calculate properties of the moist air with a pressure range of 20 kPa to 100 kPa [97].

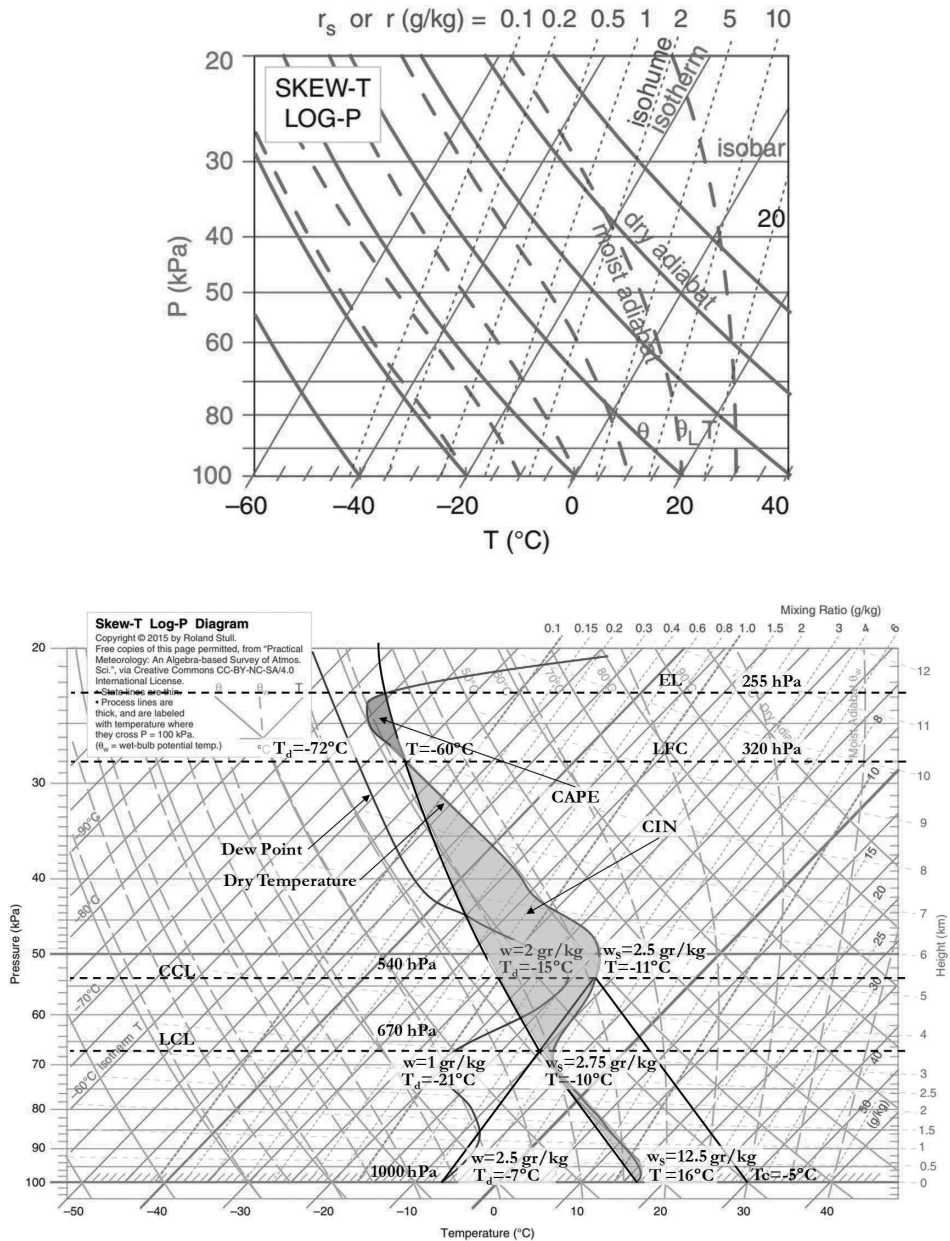


FIGURE 19 Example of the use of a Skew-T log-P diagram as a means to calculate properties of the moist air with a pressure range from 20 kPa to 100 kPa [97].

pressure (ordinate) versus the temperature (abscissa) for dry adiabats and saturated scenarios. The far-left ordinate shows pressure decreasing logarithmically; the far-right ordinate shows the altitude increasing linearly. The temperature increases to the right. Isotherms are solid-straight-right-oblique (i.e., skewed) lines, crossing the vertical lines at a 30-degree angle, corresponding to the decrease in temperature with increasing altitude and pressure. Isobars are solid-straight horizontal lines. Two main groups of isopleths are moist and dry adiabats. The moist adiabats are dashed-left-oblique lines, crossing the vertical lines at angles that vary between 5 and 30 degrees, concaving downward in lower temperatures and curving upward in higher temperatures. The dry adiabats are solid-left-oblique lines, crossing the vertical lines at 30-degree to 40-degree angles, concaving downward. Dry and moist adiabats converge to parallel lines at colder temperatures and higher altitudes. Isohumes are dotted-straight-right-oblique lines with a 20-degree inclination. Isotherms and dry adiabats in this diagram are almost perpendicular to one another. The Skew-T log-P diagram is similar to the tephigram. Using this diagram, it is possible to calculate the Lifting Condensation Level (LCL), Convective Condensation Level (CCL), Level of Free Convection (LFC), Equilibrium Level (EL), Convective Available Potential Energy (CAPE), and Convective Inhibition (CIN). A weatherperson plots the dry-bulb temperature (T) and dew-point (T_d) temperatures on the diagram and, based on their variation with altitude, predicts the aforementioned factors as a function of the altitude. Any prediction is to start from the current elevation on the diagram and is identified by the (T , T_d). Dry properties (e.g., dry adiabats) are to start from the dry-bulb temperature while the saturated properties (e.g., moist adiabats) are to start from the dew-point temperature.

For instance, to predict the LCL, which is the pressure level that an air parcel needs to reach for it to rise in dry adiabatic fashion in order to become saturated, you may follow the dry adiabats from the dry-bulb temperature, follow a saturation mixing ratio (isohumes) from the dew-point temperature, and intersect the two—where the LCL is located. In this location, the air parcel's lapse rate changes from the dry adiabats to the moist adiabats.

CCL is the pressure level of a parcel when it is heated to the convective temperature (a temperature that the parcel needs to reach to freely rise), and it rises to form cumulus clouds. The intersection of the environment temperature and the line of saturation mixing ratio (isohumes) passing from the dew-point temperature is the CCL. To obtain the convective temperature, you are to warm the parcel in a dry adiabatic process, starting from the CCL until you cross the current isobar. The intersection of the

isobar and the dry adiabatic line identifies the convective temperature. The LCL and CCL are used to determine the height of the cloud base, with the former mainly used for non-convective clouds and the latter used for convective clouds—the cloud bases are usually between the two condensation levels.

LFC is the pressure level of an air parcel that it is to reach so that its temperature equalizes to that of the environment, and it is obtained by intersecting the moist adiabats at the LCL with the environment temperature. Note that not all weather conditions result in the LFC—this is seen where the moist adiabats does not intersect the dry-environmental temperatures, and this is when the temperature is stable enough not to decrease sharply with increasing altitude. Additionally, having the LFC during the day does not guarantee also having one during the night in more stable conditions.

EL only exists where the LFC exists. This is where the moist adiabats intersect the air parcel environmental temperature again. This is the level at which the temperature of the air parcel is the same as that of its environment, and above this level the parcel is colder and denser. This identifies the top boundary of the anvil clouds, where they look as if they are creating an umbrella for their high momentum and not because the air above is buoyant.

CAPE is the area in between the environmental temperature and temperature of the parcel as it undergoes a moist adiabatic process. Therefore, its lower boundary is the LFC, and the upper boundary is the EL. CAPE determines the positive buoyancy of an air parcel relative to its environment and therefore identifies the strength of an updraft.

CIN measures the negative buoyancy—resistance to convection—with its upper boundary being the LFC and its lower boundary being the level of the environment temperature—namely the surface. Early morning has the highest CIN, and it decreases during the day with the warming up of the surface.

Examples for determining the LCL (670 hPa), CCL (540 hPa), LFC (320 hPa), and EL (255 hPa) are shown in Figure 19. The convective temperature is -5°C . The CAPE and CIN are indicated by blue and gray-filled areas.

4.3.5 Theta-Z Diagram

Figure 20 and Figure 21 show a Theta-Z diagram— $\theta - Z$ (with greater and smaller pressure ranges, respectively) [97]. This diagram shows the

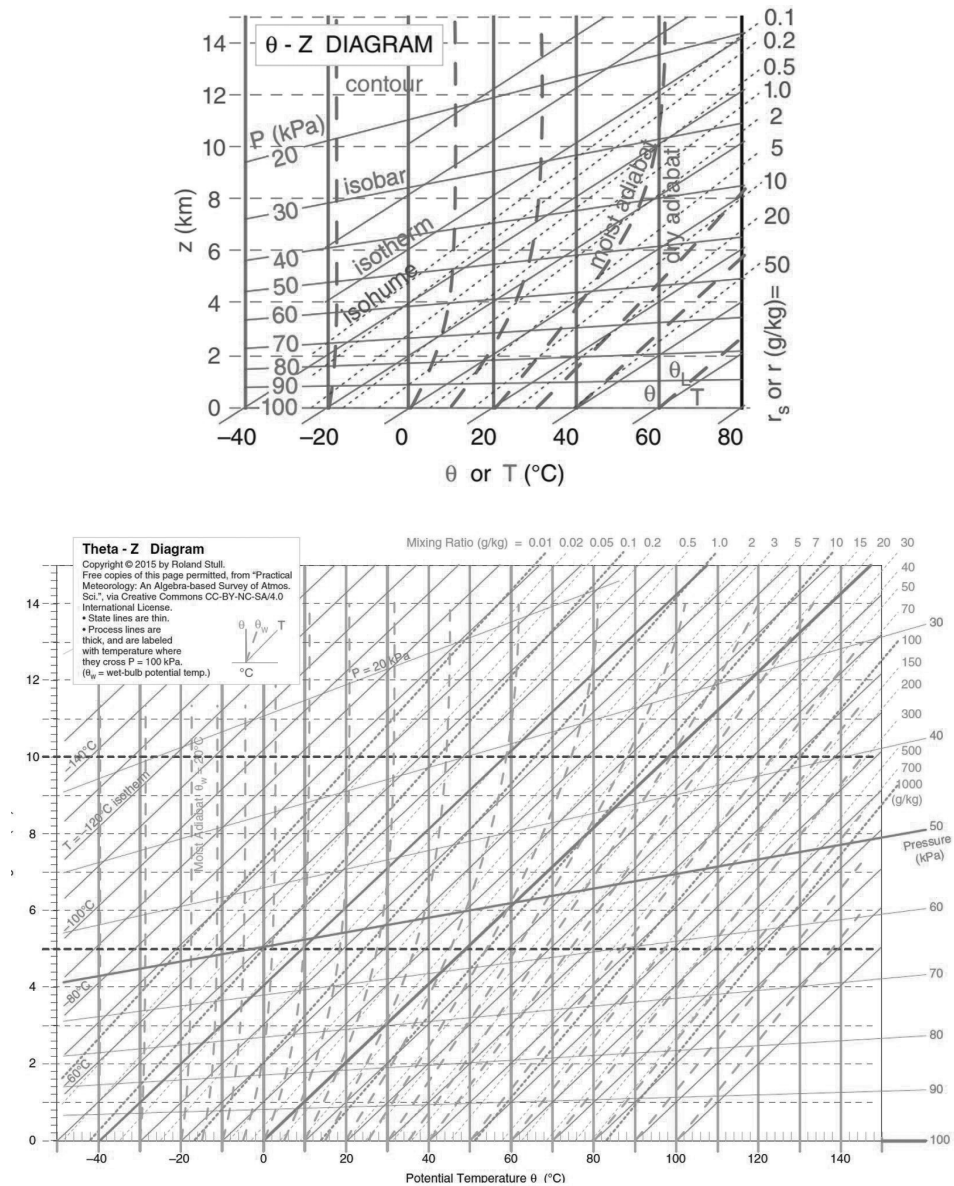


FIGURE 20 Theta-Z diagram as a means to calculate properties of the moist air with a height range of zero km to 15 km [97].

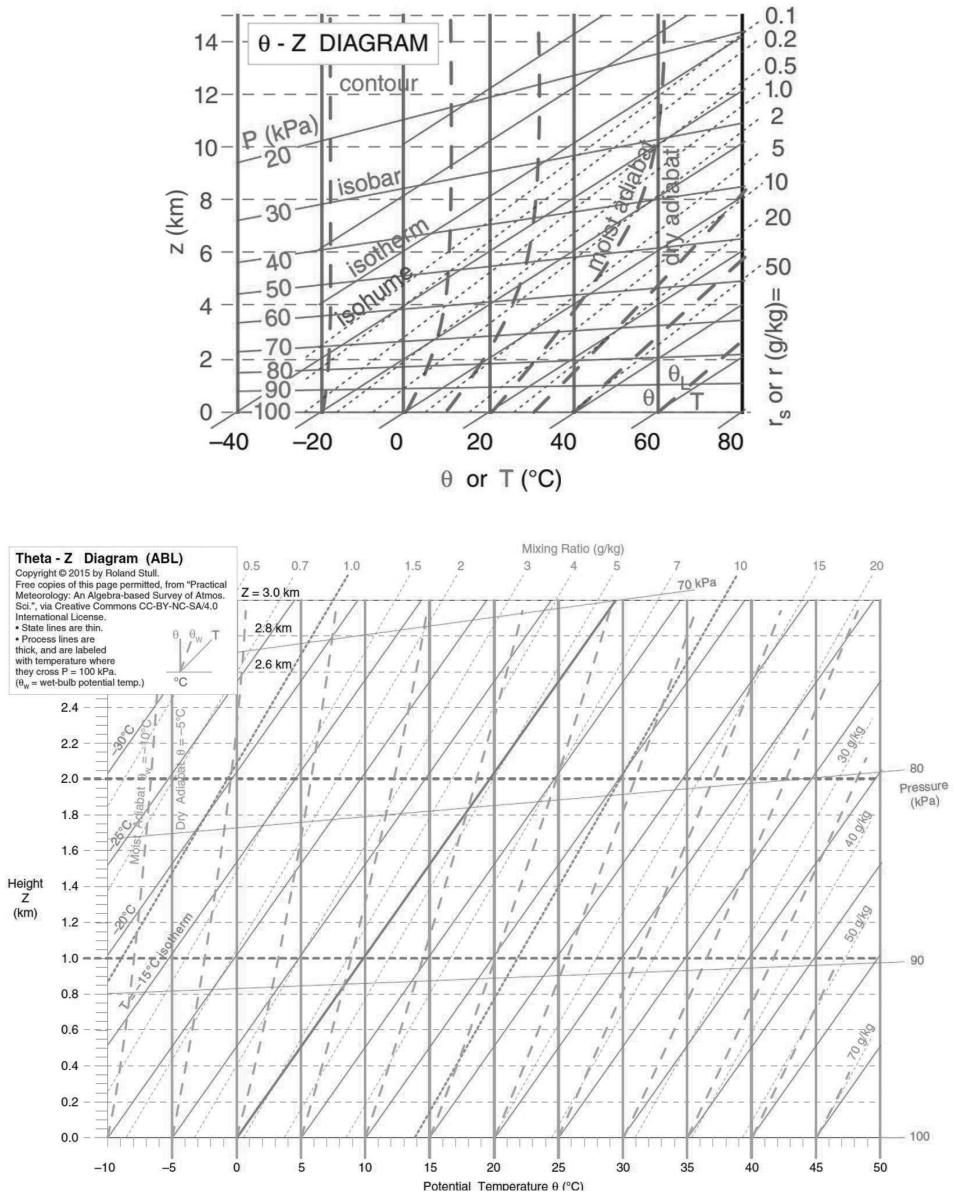


FIGURE 21 Theta-Z diagram as a means to calculate properties of the moist air with a height range of zero km to 3 km [97].

height (Z) along the ordinate increasing upward versus the potential temperature (θ) along the abscissa increasing to the right [97]. In this diagram, isotherms are presented with solid-straight-right-oblique lines with a 60-degree inclination, and isobars are presented with solid-straight-right-oblique lines with a 90-degree to 75-degree inclination.

Dry adiabats are solid-straight-vertical lines, and moist adiabats are dashed-right-oblique lines with an inclination varying from zero to 50 degrees, increasing from the left side to the right side, concaving downward. Dry and moist adiabats become almost parallel at low temperatures and high altitudes. Isohumes are dotted-straight-right-oblique lines with a 55-degree inclination. Altitude is accurate but isobars are an approximate representation of pressure lines.

4.3.6 Psychrometric Chart

Figure 22 and Figure 23 show psychrometric charts. These are among the most widely used general-purpose thermodynamic charts [101]. They identify properties such as dry-bulb, wet-bulb, and dew-point temperatures, relative humidity, specific enthalpy, and specific volume. Three independent parameters including pressure level are required to obtain other properties of the moist air. It is also possible to combine the independent and derived variables to obtain the rest of the variables. Changes of state due to mixing of air masses may be predicted using the chart as a function of the altitude that is converted to the sea-level chart at altitudes above 2,000 ft. In Figure 22 and Figure 23, dry-bulb temperatures are short-dashed vertical lines, while the humidity-ratio lines (g of water per kg of dry air) are presented by short-dashed straight-horizontal lines. Specific enthalpies of moist air (kJ per kg of dry air) are shown by solid straight-left-oblique lines with a 55-degree inclination, the same as for the wet-bulb temperatures. The slopes of the lines with a constant wet-bulb-temperature are related to the heat of vaporization of the water required to saturate the air with a given relative humidity. Specific volumes (m^3 per kg of dry air) are presented by dashed straight-left-oblique lines with a 15-degree inclination. Note that this chart is valid for a specific barometric pressure or elevation above the MSL. The relative humidity lines in percent (%) are presented by solid-right-oblique lines, curving downward, with decreasing space from the bottom to the top and inclination that varies from 55 to 20 degrees—increasing from the left side to right side [102].

Psychrometric charts have found extensive applications in heating and ventilation, air conditioning, meteorology, and anywhere that finding the properties of the gas-vapor mixtures is of interest. As a result, there are number of visual tools that present the relationship between the properties

presented in psychrometric charts. Figure 22 is an example of one of these numerical tools that shows the properties of a point selected on the chart with its corresponding data presented in the far-left table [103]. If you are interested in a thermodynamic process or a sequence of them, you may choose multiple processes and combine them to obtain information such as heat flow and loads (Figure 23) [103].

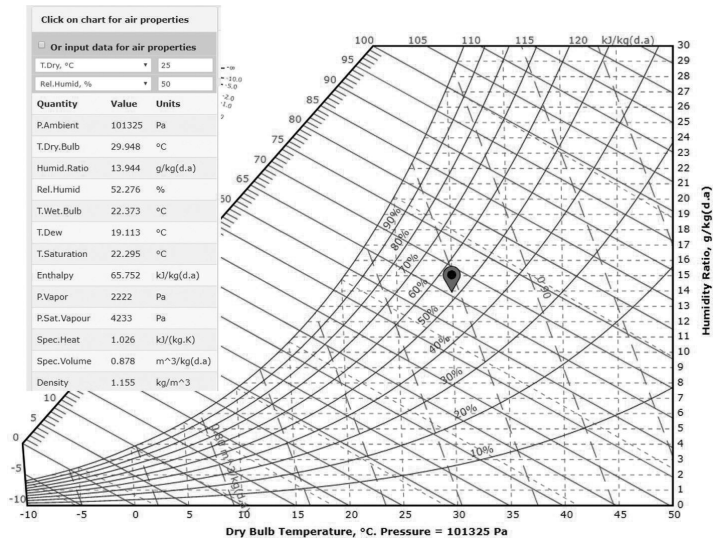


FIGURE 22 Psychrometric chart for determining the properties of the moist air at standard atmospheric conditions [103].

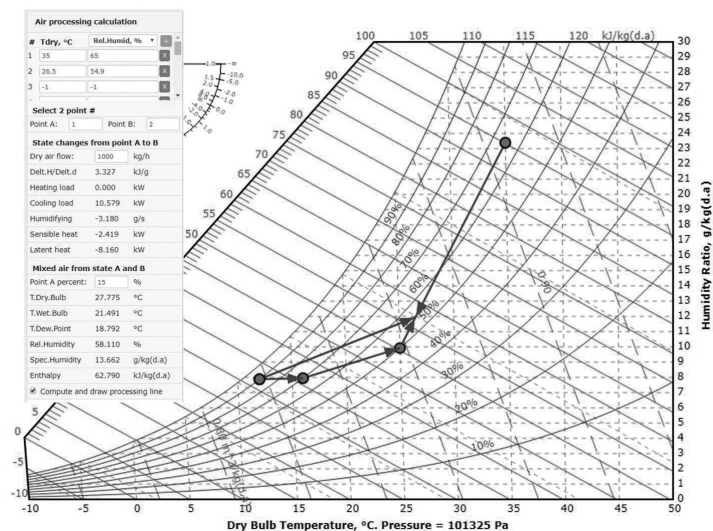


FIGURE 23 Example of the use of a psychrometric chart for determining the properties of the moist air at standard atmospheric conditions [103].

4.4 Weather Reports

Measuring atmospheric thermodynamic properties is the foundation for many weather forecast applications. These applications are based on feeding real-time data obtained from surface measurements, buoyant objects, or radiosondes into thermodynamic relationships. The result of this data acquisition (DAQ) and manipulation is the numerical weather forecast information in the form of short-term and long-term weather predictions that include clouds and weather reports, icing and turbulence conditions, temperature and pressure data, fronts formation, and movement of fronts over geographical areas.

Weather radars, also known as Doppler weather radars, have been employed since the 1950s as surveillance and warning methods. The output of a modern pulse-Doppler radar consists of three measurements known as: (a) radar reflectivity factor (related to scattered intensity); (b) mean Doppler velocity within the radar sampling volume (related to the mean component of scattered motion in the radial direction from the radar); and (c) spectrum width (related to the variability of Doppler velocity due to the turbulence within and across the sampling volume). Doppler velocity—the radial variation of distance between the radar and the target with respect to time—along with the reflectivity data are used within dedicated computer algorithms to recognize patterns, also known as signatures, to locate precipitation as well as characterize it by identifying its intensity, acceleration, direction, and path. These data as well as isodops (lines of constant Doppler velocity in radars) and isoechos (lines of constant radar reflectivity) are among the radar data that assist with identifying the potential hazards associated with precipitation [104,105].

4.4.1 METAR

METAR is the French translation for the Aviation Routine Weather Report based on the Federal Aviation Administration (FAA) and the Aerodrome Routine Meteorological Report based on WMO [106]. It is most commonly used by pilots as part of their flight planning for short or long hauls. METAR is also employed by meteorologists to predict long-term weather trends and forecasts. There is an ICAO standard language, known and used internationally, for METAR reporting that consists of raw data. There is also an easier-to-understand version that does not require knowledge of its terminology available to less experienced weather readers. METARs are normally issued every hour. If the conditions change significantly between the regular reports, a special weather report (SPECI) is issued with similar standards to those of ICAO and WMO.

4.4.2 TAF

TAF is the acronym for the Terminal Aerodrome Forecast that is issued every 6 hrs (or 3 hrs for military fields) starting at 0000 Universal Coordinated Time (UTC), also known as Zulu time, for every major airfield, applicable for a period of 24 to 30 hrs (or 3 to 30 hrs for latter) for an area with a radius of 5 statute miles surrounding the aerodrome where the data are published [107]. In Canada, data covers a circle with a radius of 5 nautical miles (NM) whose center coincides with that of the aerodrome. In North America, TAFs are usually written by on-duty personnel and on-site. However, in the United Kingdom, most TAFs are produced at the Meteorological Office located in Exeter, except for the ones generated at military aerodromes, which are produced on-site.

4.4.3 TTF

TTF is the acronym for Trend Type Forecast. It is similar to the TAF and is issued by a person who is located at the associated aerodrome and is valid for 3 hrs [108]. It is similar to the TAF and is enclosed at the end of a METAR report. This report supersedes the TAF.

4.4.4 SIGMET

Significant Meteorological Report (SIGMET) is issued as required and applies to all types of aircraft [109]. The two types of SIGMET are known as convective and non-convective: (a) Non-convective SIGMET is issued in cases of severe turbulence and icing for an area that exceeds 3,000 square miles, or instrument meteorological conditions (IMC) due to dust, sand, and ash covering about 3,000 square miles. These are valid for 4 hrs and are issued as required, although the ones issued for volcanic ash and hurricanes are valid for up to 6 hrs; (b) Convective SIGMET is issued in cases of severe thunderstorms or tornadoes affecting over 40 percent of a 3,000 square mile area, hail that exceeds 3/4 in diameter, or surface winds exceeding 50 kt. These are valid for 2 hrs and are issued as required.

4.4.5 PIREP

Pilot Report (PIREP) is a weather report that is relayed to the closest ground station by the pilots after encountering unforeseen or unknown weather conditions in flight in a format recognizable to the other pilots so that they can make more informed decisions [110]. The ground station then broadcasts the report to inform other pilots in the area. The submission may also be done in electronic format. Whether relayed by radio or submitted electronically, the report follows a special format for consistency that

is shared internationally. There are occasions when collision of cold and warm fronts results in sudden gusts or turbulence which are not detected by any remote weather observation means. A UA suffix is included at the beginning of the report to identify it as urgent.

4.4.6 AIRMET

Airmen's Meteorological Information (AIRMET), also known as weather advisories, are the occasional weather reports announced along with METARs and TAFs to inform pilots of occasional weather conditions that may jeopardize the safety of the aircraft and passengers [111]. They are issued every 6 hrs (beginning 0145 UTC and 0245 UTC during daylight savings and standard time, respectively) and only when an area of at least 3,000 square miles is affected. AIRMETs are less severe than SIGMETs and apply to icing and turbulence conditions as well as wind up to 30 kt. The three types of AIRMETs are known as Sierra, Tango, and Zulu (STZ) and they apply to: (a) ceilings lower than 100 ft, visibility under 3 miles, or mountain obstructions where visibility is affected in at least 50 percent of the area; (b) light to moderate turbulence and continuous surface winds of up to 30 kt; and (c) light to moderate icing with freezing levels. When the severity of SIGMET is reduced, the report changes to AIRMET and applies to all types of aircraft.

4.4.7 IWXXM

The ICAO Meteorological Information Exchange Model (IWXXM) is a weather report in Extensible Markup Language (XML) or Geographical Markup Language (GML) [112,113,114]. It consists of a number of ICAO and WMO standardized reports used in aviation, and it is not directly used by pilots. It is designed to be used by the software that pilots use, such as one where meteorological data are displayed.

4.5 Wind

Parcels of air in a stable atmosphere move horizontally, resisting the vertical movements of air fronts. In unstable air, the air fronts move in a vertical direction, either up or down depending on the currents, creating turbulent air. Such flows of parcels of air over the Earth's surface, chemical elements and gases between planets, or gases and charged particles from the Sun are known as wind, planetary wind, and solar wind, respectively. Winds are categorized based on their magnitude, direction, sources originating them, location they are originated from, and their effect on their

surroundings as they occur—usually related to wind velocity (i.e., direction and speed) and energy.

Gusts are the high-speed winds that last about 10 min or more and change direction over an angle of 30° and a magnitude of over 10 kt. Squalls are the winds that suddenly increase in magnitude by 15 kt to 21 kt for a short period (about one minute to several minutes). Multiple gusts may occur for the duration of squalls. Regular winds usually last twice as long as squalls. Listed in the order of decreasing intensity, winds can be categorized as: (a) tornadoes (recorded maximum gusts of 220 kt [115]); (b) hurricanes (recorded maximum gusts of 220 kt, the longest duration of 31 days, and the longest track of 8,250 mi [116]); (c) gales (recorded average of 52 kt); and (d) breezes [117,118,119,120,121,122,123].

Wind is measured as a function of the altitude by radiosondes every 60 to 90 min, relayed to the surface, and reported every 60 min (METAR and TAF), and sometimes in shorter intervals in case of special conditions. Wind speed measurement on the ground is done using an anemometer that consists of a vertical column and four attached cups capable of rotating freely about the column's vertical axis, capturing the magnitude of the horizontal wind.

The wind vector can be described in terms of its components—crosswind (u) and headwind (v). They are related to the magnitude (U) by equation (77), the wind direction, and the angle between the wind and the aircraft longitudinal axis (λ) by equation (78)—Figure 24.

$$U = \sqrt{u^2 + v^2} \quad (77)$$

$$\lambda = \frac{180}{\pi} \arctan\left(\frac{u}{v}\right) \quad (78)$$

As mentioned earlier, wind is an element of weather and an important factor in aviation meteorology. It is a vector quantity; therefore, it is to be identified by its magnitude and direction. These quantities on their own and also in combination with other factors such as cloud levels and visibility status are employed to evaluate aircraft airworthiness in given conditions.

Taking off into the wind (also known as headwind) is recommended, as it expedites the buoyancy of an aircraft. Flying into the headwind reduces the ground speed and thus increases the flight time and fuel required. Flying with the wind—also known as tailwind—results in the shortest flight time, and therefore less fuel is required to complete the same flight. Note

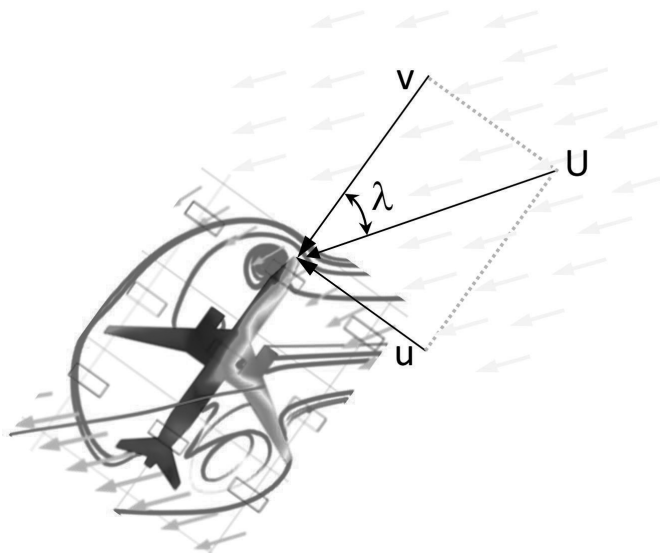


FIGURE 24 Headwind and crosswind components and the resultant vector.
The flow model was created using the COMSOL Multiphysics®
Computational Fluid Dynamics (CFD) Module.

that the total air time (time the aircraft can stay in the air) requires the same amount of fuel given similar performances.

The magnitude of the wind in crosswind situations is a determining factor for a plane's ability to land or take off safely. Tests are conducted to predict safe crosswind magnitude when designing an aircraft. For the Cessna 172M, the maximum demonstrated crosswind magnitude is 15 kt, meaning that the manufacturer certifies that the aircraft is capable of taking off and landing given this crosswind magnitude without structural damage.

In North America, wind direction is given in true north format. The runway heading, however, is specified relative to magnetic north. To convert the wind data into useful values for flight plans, the wind directions need to be converted to magnetic data, and therefore magnetic variations are to be incorporated into wind directions. A variation or magnetic declination is the angle in degrees between the direction of the North Magnetic Pole (as indicated by the compass needle) and the direction of the meridian passing through the same point and going toward the geographic North Pole. On the left side of the agonic line—where the variation is zero—the westerly variations are to be added to the wind data, while on the right side of the agonic line, the easterly variations are to be deducted from the wind direction data. To help you remember when to add and when to subtract, you can

think of the following: the Sun travels *FROM* east *TO* west. So, you subtract the easterly variation *FROM* the true heading to get the magnetic one and add the westerly variation *TO* the true heading to get the magnetic one.

4.6 Examples

4.6.1 Case Study – Temperature

For the case of saturated and unsaturated air using the Skew-T log-P diagram (Figure 19), for the pressure levels provided in Table 10, determine:

(a) dry-bulb temperature, (b) dew-point temperature, (c) mixing ratio in grams of moisture per kg of dry air, (d) saturated mixing ratio in grams of moisture per kg of dry air, (e) relative humidity, (f) saturated vapor pressure, (g) vapor pressure, (h) ratio of the vapor pressure to the pressure, (i) virtual temperature, (j) potential temperature, and (k) equivalent temperature.

TABLE 10 Properties of the dry and moist air extracted from the Skew-T log-P diagram chart in Figure 19.

Case	P (hPa = bar)	P (kPa)	RH (%)	e / e_s	T (°C)	T_d (°C)	w (gr/kg _{dry air})	w_s (gr/kg _{dry air})
1	1,000	100.00	20.00	0.20	16.00	-7.00	2.50	12.50
2	670	67.00	36.36	0.36	-10.00	-21.00	1.00	2.75
3	540	54.00	50.00	0.50	-11.00	-15.00	2.00	2.50
Case	w / w_s	RH (%)	e / P	T_v (°C)	T_p (°C)	T_e (°C)	e (kPa)	e_s (kPa)
1	0.20	20.00	0.80	141.52	16.00	23.50	80.08	1.80
2	0.36	36.36	0.62	69.96	21.93	24.93	41.31	0.29
3	0.80	80.00	0.76	95.21	39.52	45.52	41.19	0.27

4.6.2 Case Study – METAR – Kingston, Ontario, Canada

Interpret the following METAR published for a Canadian aerodrome (“C”), which is equipped with the weather measuring equipment (“Y”), located at Norman Rogers Airport (CYGK) in Kingston (Canada)—Table 11.

METAR CYGK 111300Z 19010KT 10SM FEW012 BKN250 02/M01 A3009 RMK SF1CI5 SLP193=

- Location: Kingston, Ontario, Canada—CYGK.
- Date and Time: The report has been published on the 11th of the month at 1300 Zulu time (UTC)—111300Z.

- **Period of Report:** The report is valid for the duration of one hour or until the next METAR is issued.
- **Wind Direction and Magnitude:** The wind direction is true south-south west from 190° with the magnitude of 10 kt–19010KT.
- **Visibility:** The visibility is 10 statute miles–10SM.
- **Cloud Levels and Coverage:** There are few clouds at 1,200 ft AGL; meaning the cloud coverage is between 1/8 and 2/8 at that level–FEW012. The ceiling is at 25,000 ft AGL; meaning the cloud coverage is between 5/8 and 7/8 at that level. Ceiling is determined by the lowest altitude of either broken or overcast cloud layer–BKN250.
- **Cloud Types and Coverage (in Remarks (RMK)):** Stratus Fractus clouds with coverage of 1/8 and cirrus clouds with coverage of 5/8–SF1CI5.
- **Temperature:** Dry-bulb temperature is 2 °C and dew-point temperature is -01 °C–02/M01.
- **Altimeter:** Altimeter is 30.09 inches of mercury–A3009.
- **Pressure:** MSL pressure is 1019.3 mbar–SLP193.
- **Remarks:** Describes the cloud types and their levels as well as the MSL pressure and frost information–RMK SF1CI5 SLP193.

TABLE 11 Encoded METAR data [124].

METAR CYGK 111300Z 19010KT 10SM FEW012 BKN250 02/M01 A3009 RMK SF1CI5 SLP193=		
METAR	LOCATION CYGK - KINGSTON/ON	DATE - TIME 11 APRIL 2018 - 1300 UTC
WIND 190 TRUE @ 10 KNOTS	VISIBILITY 10 STAT. MILES	RUNWAY VISUAL RANGE
WEATHER	CLOUDINESS FEW CLOUDS (1/8 - 2/8) 1200 FT BROKEN CLOUDS (5/8 - 7/8) 25000 FT	TEMP / DEWPOINT 2 C / -1 C
ALTIMETER 30.09 IN HG	RECENT WEATHER	WIND SHEAR

4.6.3 Case Study – TAF – Kingston, Ontario, Canada

Interpret the following TAF published for a Canadian aerodrome (“C”), which is equipped with the weather measuring equipment (“Y”), located at Norman Rogers Airport (CYGK) in Kingston (Canada)—Table 12.

TAF CYGK 111342Z 1114/1202 20005KT P6SM FEW010 BKN160
 PROB30 1114/1115 4SM BR BKN009 FM111500 20010G20KT P6SM
 BKN080 FM112000 21012KT P6SM -SHRA OVC030 FM112300
 22012KT P6SM -SHRA OVC020 RMK NXT FCST BY 112000Z=

- Location: Kingston, Ontario, Canada–CYGK.
- Date and Time: The report has been published on the 11th of the month at 1342 Zulu time (UTC)–111342Z.
- Period of Report: The report is valid for the duration of 1114 UTC to 1202 UTC on the 11th of the month–1114/1202.
- Wind Direction and Magnitude: The wind direction is south west from 200° with the magnitude of 5 kt–20005KT.
- Visibility: The visibility is over 6 statute miles–P6SM.

TABLE 12 Encoded TAF data [125].

TAF CYGK 111342Z 1114/1202 20005KT P6SM FEW010 BKN160 PROB30 1114/1115 4SM BR BKN009 FM111500 20010G20KT P6SM BKN080 FM112000 21012KT P6SM -SHRA OVC030 FM112300 22012KT P6SM -SHRA OVC020 RMK NXT FCST BY 112000Z=				
TAF		LOCATION CYGK - KINGSTON/ON		VALIDITY [UTC] 11 APR - 1342 to 12 APR - 0200
TIMES [UTC]	WIND [DEGREES TRUE]	VISIBILITY [STAT. MILES]	WEATHER	CLOUDINESS [FEET AGL]
11 APR - 1400 to 11 APR - 1500	200 @ 5 KNOTS	6+	NIL	1000 FEW 16000 BROKEN
30% PROBABILITY 11 APR - 1400 to 11 APR - 1500		4	MIST	900 BROKEN
11 APR - 1500 to 11 APR - 2000	200 @ 10 KNOTS GUSTS 20 KNOTS	6+	NIL	8000 BROKEN
11 APR - 2000 to 11 APR - 2300	210 @ 12 KNOTS	6+	LIGHT RAIN SHOWER	3000 OVERCAST
11 APR - 2300 to 12 APR - 0200	220 @ 12 KNOTS	6+	LIGHT RAIN SHOWER	2000 OVERCAST
REMARKS THE NEXT FORECAST WILL BE ISSUED BY 2000 UTC ON DAY 11				

- Cloud Levels and Coverage: There are few clouds at 1,000 ft AGL, meaning the cloud coverage is between 1/8 and 2/8 at that level—FEW010. The ceiling is at 16,000 ft AGL, meaning the cloud coverage is between 5/8 and 7/8 at that level. Ceiling is determined by the lowest altitude of either broken or overcast cloud layer—BKN160.
- The probability for the rest of the forecast is 30 percent plus minus 10 percent—PROB30 1114/1115 4SM BR BKN009.
 - Period of Report: The report is valid for the duration of 1400 UTC to 1500 UTC on the 11th of the month—1114/1115.
 - Visibility: The visibility is 4 statute miles—4 SM.
 - Precipitation and Frost: Mist (Brume)—BR.
 - Cloud Levels and Coverage: Broken clouds at 900 ft AGL (the ceiling), meaning the cloud coverage is forecast between 5/8 and 7/8—BKN009.
- Time of Change of Report: From 1500 UTC on the 11th of the month—FM111500.
- Wind Direction and Magnitude: Wind direction is from 200° with the magnitude of 10 kt gusting 20 kt—20010G20KT.
- Visibility: The visibility is over 6 statute miles—P6SM.
- Cloud Levels and Coverage: The ceiling changes to 8,000 ft AGL (the ceiling), meaning the cloud coverage is between 5/8 and 7/8 at that level—BKN080.
- Time of Change of Report: From 2000 UTC on the 11th of the month—FM112000.
- Wind Direction and Magnitude: Wind direction is from 210° with the magnitude of 12 kt—21012KT.
- Visibility: The visibility is over 6 statute miles—P6SM.
- Precipitation and Frost: Light (-) rain (RA) showers (SH)—SHRA.
- Overcast clouds at 3,000 ft AGL (the ceiling); meaning the cloud coverage is 8/8 at that level—OVC030.
- Time of Change of Report: From 2300 UTC on the 11th of the month—FM112300.

- Wind Direction and Magnitude: Wind direction is from 220° with the magnitude of 12 kt–22012KT.
- Visibility: The visibility is over 6 statute miles–P6SM.
- Precipitation and Frost: Light (-) rain (RA) showers (SH)– -SHRA.
- Overcast clouds at 2,000 ft AGL (the ceiling); meaning the cloud coverage is 8/8 at that level–OVC020.
- Remarks: Gives the next forecast time as at or before 2000 UTC on the 11th of the month–RMK NXT FCST BY 112000Z.

4.6.4 Case Study – Weather Report Time Zones

You are traveling from Kingston in Ontario (Canada) to St. John's in Newfoundland and Labrador (Canada) to visit your friend. Your reference weather report (METAR) is issued at 1500 UTC for May 3. Determine the corresponding time at your destination.

St. John's, Newfoundland is in the Northern Hemisphere and is UTC – (minus) 2.5 hrs. The time in which the report has been issued falls within daylight savings time.

The longitude is at W52°45.15' W. Each 15° of longitude averages to one time zone. Hence $(W52^{\circ}45.15'W / 15^{\circ} = 3.5W)$ equals theoretically to -3.5 zones, and this number is associated with the Newfoundland time zone (equivalent to the alpha zone presented in the NST column). Newfoundland Daylight Time (NDT) is associated with -2.5 ($UTC - 2.5 = 15 - 2.5 = 12.5 = 12:30$ NDT).

CHAPTER 5

DYNAMICS OF AIR AND FORCES

5.1 Airfoil Shape

Molecules consist of atoms, which are harmonic oscillators vibrating in their equilibrium positions as interdependent, semi-dependent, or independent entities. The interdependent entities are provoked by the energy of the surrounding neighbors. Nevertheless, statistical thermodynamic assumptions rely heavily on semi-independent or independent particles. On the microscopic level, matter is made of molecules and atoms. In their normal state, their molecules carry on their task of defining the state of material—gas, liquid, or solid. Gaseous molecules enjoy a high level of freedom due to the level of proximity of their molecules—the space in between the molecules. They have a longer mean free path, meaning that they can travel farther inside their environment without colliding with other molecules. There are several factors that can limit this freedom; for example, when the same number of molecules are transferred to a smaller container where the volume decreases. In thermo-fluid sciences, the Knudsen number (Kn)—the ratio of the mean free path (l) to the characteristic length (l'), the scale of a physical system—quantifies the level of freedom enjoyed by the molecules and hence their regime. A Knudsen number less than one is associated with a continuum flow, a value equal to one is associated with a slip flow, and a value greater than one defines free molecular flow. Assuming a continuum flow passes over a wall, similar to the air flow over a wing, the magnitude of the flow velocity adjacent to the wall is zero, while for a slip flow (fluid viscosity is absent), this value is nonzero due to

the formation of slip flow in the vicinity of the wall. The parabolic velocity profile associated with the flow passing over the wall changes to a linear profile in a free molecular flow, meaning that the flow velocity magnitude, a nonzero value at the wall, changes linearly with increasing distance from the wall. The previously described conditions happen within a thin layer of the flow in the proximity of the wall called the boundary layer. The flow is fully developed when the flow velocity equals 99 percent of the freestream velocity at a distance away from the wall, which is called the boundary layer thickness. The *wall* does not need to be a literal wall but can be a fluid-submerged solid object of any geometry and material. In fluid mechanics, the solid surfaces of a propeller, aircraft body, or a wing are all treated as *walls*.

Any object moving inside a fluid will influence and redirect the fluid flow and as a result will experience a force exerted upon it. The component of this force perpendicular to the direction of motion is defined as *lift*. The component of this force opposite to the direction of the motion, and thus resisting the motion, is defined as *drag*. In an aircraft, it is desirable to maximize the lift (which maximizes the plane's ability to carry a useful load) while minimizing the drag (which reduces the energy consumption and the required engine power). Thus, for an aircraft's wing cross section, a special shape called an airfoil has been designed so as to maximize the ratio of the lift to the drag, among other properties (Figure 25).

A straight chord line connects the leading edge to the trailing edge of an airfoil. The leading and trailing edges are located at the points of the highest curvature, respectively, at the front (right side in Figure 25) and the back (left side) of the airfoil. A camber line (or mean line) also connects the leading and trailing edges but traces along the airfoil at the midpoint between its upper and lower surfaces. Camber quantifies the asymmetry between the upper and lower surfaces of the airfoil. Most airfoils have a positive camber (as shown in Figure 25). Camber is defined as the maximum distance between the camber line and the chord line. Positive camber

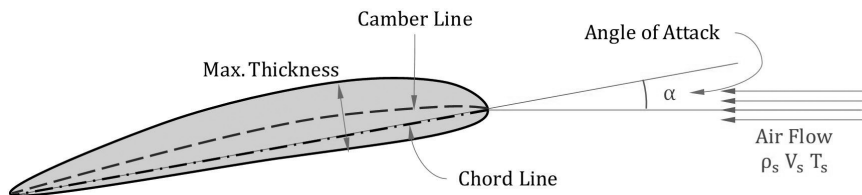


FIGURE 25 Airfoil diagram (drawings created using Solid Edge CAD tool).

means that the camber line is curved so that it is convex upward. Airfoil which is not cambered is called symmetrical. Positive camber is added to increase the maximum lift coefficient and thus reduce the stall speed.

Many different airfoil shapes have been developed since the dawn of aviation. A significant contribution to this field was made in 1929 by the United States National Advisory Committee for Aeronautics (NACA), which developed a standard way of describing the airfoil shape as well as a number of new shapes. The NACA number defines the airfoil shape by a set of digits, which can vary from four up to eight digits [126]. The values of these digits can be used in equations to generate the corresponding airfoil shape. For example, the Spitfire, a famous World War II (WWII) British fighter plane, used the NACA2213 profile near the wing root, transitioning to the NACA2209.4 profile near the tip. These were at that time the latest profiles designed to produce smoother (laminar) air flow over the wing and thus to reduce drag [127,128,129]. Another famous airfoil profile, called *Clark Y*, was designed in 1922, and was used in several famous aircraft of that era, such as the Ryan NYP *Spirit of St. Louis*, flown in 1927 across the Atlantic by Charles Augustus Lindbergh, and the Lockheed Vega, used for the transatlantic solo flight by Amelia Mary Earhart [130]. The Boeing 777 or a similar category aircraft does not use particular NACA airfoils. These companies perform numerous analyses using Computational Fluid Dynamics (CFD) methods in order to determine the wing profiles that generate the greatest lift force with minimum drag for the specific aircraft while complying with the structural integrity requirements. The wings they design are usually an amalgamation of different airfoils with changing rotation from the wing root to tip.

5.2 Angle of Attack

Bernoulli's law is applicable to the air flow around an airfoil. The law states that the combination of pressure head and kinetic energy at any location within the flow is constant. Thus, if the air flow is faster on one side of the airfoil versus the other, it will have greater kinetic energy, and as a consequence there will be lower pressure head there. The difference between the pressures at the upper and lower surfaces will contribute to the lift force. The same law also explains why low pressure is generated in a venturi tube, where the air flow is accelerated by progressively reducing the flow cross section.

Another way to explain the lift force is by considering that the wing deflects the airflow downward. This gives the air a vertical velocity that it

did not have before impacting the wing. Added velocity means the air was accelerated and, according to Newton's third law, a force that produced that acceleration had to also apply in the opposite direction on the wing. The magnitude of this force can be estimated using Newton's second law, which states that the force equals the rate of change of momentum ($F = m\dot{v}$), where (\dot{v}) is the time derivative of velocity (i.e., acceleration) and equals the rate of change in velocity over time.

Both lift (F_L) and drag (F_D) forces on the wing moving through the air are proportional to the wing area (A), air density (ρ), the square of the relative wind velocity (v), and a coefficient of lift (C_L) or drag (C_D), as appropriate—as in equation (79).

$$F_L = C_L A \frac{\rho v^2}{2} \quad \text{and} \quad F_D = C_D A \frac{\rho v^2}{2} \quad (79)$$

The lift and drag coefficients are both affected by the angle of attack of an airfoil (Figure 25). Angle of attack (AOA) is the angle between the airfoil's velocity vector and its chord line. An example of the relationship between AOA and the lift and drag coefficients is shown in Figure 26 for the *Clark Y* airfoil profile.

The lift coefficient is linearly related to the angle of attack over a significant range. Typically, airfoils with a positive camber will generate nonzero lift at a zero angle of attack (as seen in Figure 26). Reducing the

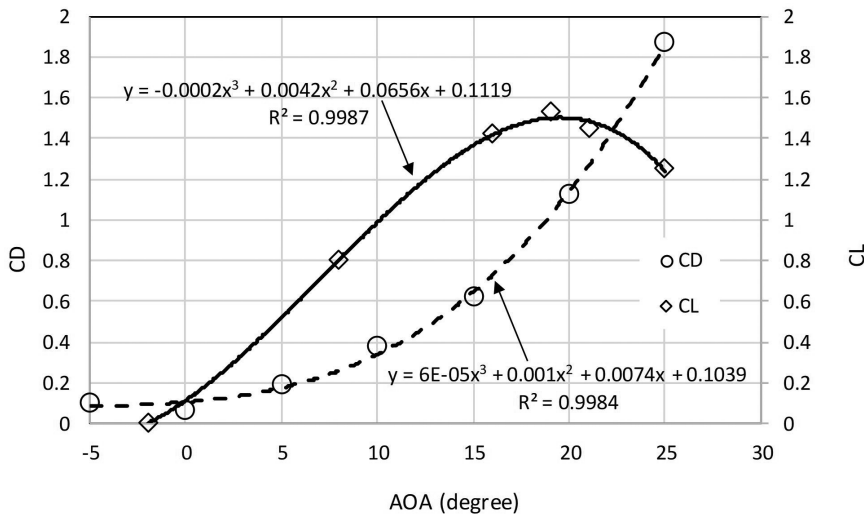


FIGURE 26 Example of lift and drag curves versus the angle of attack for the *Clark Y* airfoil design at an aspect ratio of 6 (raw data extracted from [131]).

angle further will result in the zero-lift condition at a negative AOA. As the AOA is increased, the lift coefficient will eventually reach a maximum value beyond which it will start decreasing. The angle of attack where maximum lift is generated is called the critical angle of attack. Its value is typically about 15° and is independent of the weight of the aircraft or the power. When this angle is reached, the wing is considered to have reached the stall condition.

At normal cruise speeds, the aircraft is designed to operate close to a minimum drag point with an AOA of 2° to 3° . The aircraft's pitch angle at cruise will also be affected by the angle of incidence, which is the angle between the airfoil's chord line and the aircraft's longitudinal axis. It is thus a fixed feature, a part of the aircraft design. If the aircraft speed is reduced, the air stream speed around the wing reduces as well, and this leads to less lift being generated, the lift being proportional to the square of the airspeed, as in equation (79). To keep the aircraft in balance with the weight, the angle of attack needs to increase to maintain the same lift with reduced airspeed. However, this only works up to the critical AOA. Beyond this angle, the lift force decreases sharply. This point is the threshold of the stall. A buffeting sound may be heard and vibrations may be felt at the onset of the stall. Beyond the critical angle of attack, separation of airflow begins at the leading edge and progresses to the trailing edge of the wing if the angle is not reduced. When in flight, this is experienced by feeling sluggishness—also known as sloppy or mushy response—of the controls accompanied by the stall horn warning sound. An increased load factor increases the stall speed and that is why the approach speed needs to be increased to compensate for the wind gusts or for greater aircraft weight.

The wings are usually designed so that the stall starts from the wing root (inboard portion of the wing) and continues to extend to the wing tip. This is achieved by designing the wings so that their angle of incidence decreases from the wing root to the tip. As the ailerons are located in the outboard section of the wing, this helps the pilot to retain some roll control at the onset of the stall, minimize the chance of entering into a spin, and thus recover from the stall faster. This characteristic of the aircraft design is called wash-out (decrease of the angle of incidence from the root to the tip—e.g., CF-18 Hornet) or wash-in (increase of the angle of incidence from the root to the tip—e.g., X29).

In some cases, wing twist may be due to the wing deflection caused by compressibility effects when the wing structure is not sufficiently stiff to resist bending by the flow passing over it. Insects are examples from nature

equipped with this aerodynamic feature. Wing twist is also known as *Sutter twist* after Joseph Frederick Sutter, an American engineer, the head of the Boeing 747 program, who promoted the idea of one of his engineers, Jim Hoy, in 1966 to address the high-load distribution at the outboard area of the 747's wing. Analysis showed that partially twisting the outer wing after the outboard engine pylon versus twisting the entire wing would provide 80 to 90 percent of the lift distribution, results they would have obtained if the entire wing were to be twisted by about 3° . This redesign was in response to the claims by the Boeing 727 and Boeing 737 program heads (directed by management to remedy the Boeing 747 wing problem) that the wing was "unsalvageable." If the entire wing had to be redesigned, the whole 747 program and possibly the company would have been jeopardized, considering that Boeing had its finances stretched to the limit at that time [132].

If the incipient stall is not quickly addressed by pushing forward the controls (down elevator to reduce the angle of attack), a spin may ensue, which is the autorotation of the aircraft to its left or right during which both wings stall. Since the control surfaces such as ailerons are not very responsive during the spin, the most responsive controls (opposite rudder to the spin rotation) is to be applied first to stop the spin (with a neutral control column) and then the yoke/stick is to be pulled back gently to reduce the airspeed as needed.

For a single-propeller aircraft, another consequence of increasing the angle of attack is that the thrust on one side of the propeller increases, and as a result the aircraft tends to yaw more to the left. In other words, the propeller blade with a higher angle of attack (descending or moving downward on the right side) creates additional thrust compared to the one on the left (ascending or moving upward on the left side). Note that in single-engine aircraft, the propeller normally rotates clockwise when seen from the cabin. However, in a multi-engine propeller-driven aircraft, the propellers may rotate in the same direction or the opposite direction, and that not only affects the load factor of the aircraft during the takeoff and landing but also flight characteristics of the aircraft after losing an engine, making it more sensitive to the loss of the left engine (critical engine) versus the right one due to the adverse effects of the asymmetric thrust and larger yawing moment generated by the functioning right engine. The slipstream creates similar effects on the load factors. When power is reduced, the elevator's downwash is reduced and so is the elevator effectiveness. The prop blast affects this load interaction as well, since it influences the angle of incidence of the horizontal stabilizer.

Note that the drag coefficient is a surjective function—meaning that instead of finding one dependent, $f(x)$, variable for every independent variable (x), as expected for an injective function, two or more variables are obtained for the dependent variable. For an aircraft, increasing the velocity decreases the lift-induced drag; however, this increases the parasite drag. Both curves meet at a point—also known as the optimum point—where the total drag is at its minimum value. To the left and right of this optimum point, the aircraft speed is lower and higher, respectively, and you find two different speeds producing the same total drag. That makes the total drag curve versus the aircraft speed a surjective function; on the contrary, individual lift-induced and parasite drag diagrams versus the aircraft speed are injective functions (Figure 27).

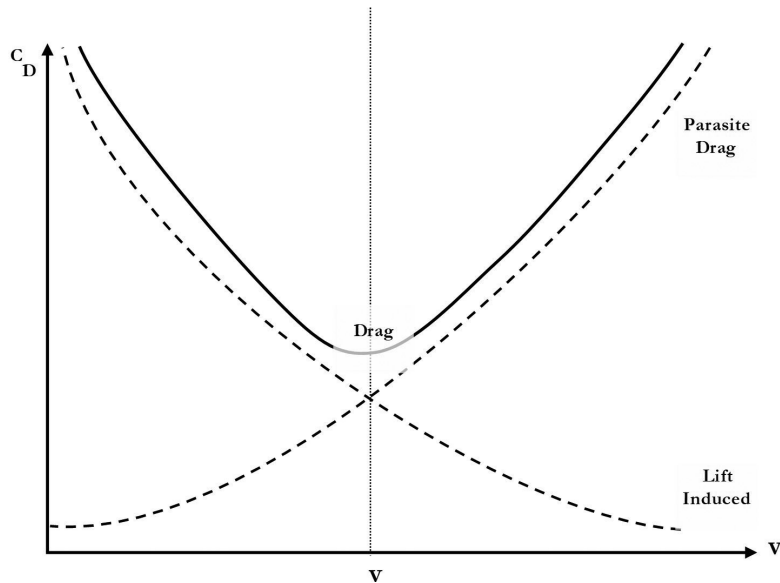


FIGURE 27 Drag coefficient versus the velocity.

5.3 Forces in Flight

Aerodynamic forces applied to an aircraft at different locations may be simplified by one force applied to a point on the aircraft, which is called the Center of Pressure (COP). The COP changes as the angle of attack changes and as the aircraft performs different maneuvers. In other words, aircraft aerodynamic balance is affected. One can also imagine replacing all the forces due to the weight in a straight-and-level flight with one force applied

at a single point on the fuselage—known as the Center of Gravity (COG). You can visualize this as the aircraft being suspended by a rope attached at the COG and staying horizontal.

The COG depends on the payload and therefore varies for each flight; during the flight it may also shift to some extent due to the reduction of fuel weight. The relative location of the COG with respect to that of the pressure or lift determines the longitudinal stability: the COG in front of the COP presents positive stability (Figure 28, left) while their colocation or rearward position of the COG presents neutral or negative stability, respectively (Figure 28, right). The latter case is particularly undesirable, as it makes it harder to recover from stalls and spins. Other aspects of aircraft design affect the stability as well. For example, dihedral wings (i.e., those shaped like a very shallow “V” when looking from the front) increase lateral (roll) stability (Figure 29). An aircraft is easier to control and tends to become stable after the source of its destabilization is eliminated. A neutrally stable aircraft keeps the condition at which it was left after the source of destabilization is eliminated. A negatively stable aircraft does not return to its original stable condition when the stability disturbance is removed.

During a straight-and-level flight, the weight and lift are equal and so are the thrust and drag. When in a coordinated turn, the aircraft banks (rotates about the roll axis). This causes the lift force to turn away from the vertical-axis by the angle of the bank. You can resolve the lift force vector into the vertical component balancing the weight and the horizontal component causing the aircraft to turn by creating a centripetal force pointing toward the turn center (Figure 29). Thus, the wing load for an aircraft of weight W and at θ degrees angle of bank is given by $F = W/\cos\theta = W\sec\theta$. For example, solving for an aircraft weight of $3,000g$ N (where g is the gravity acceleration) and an angle of bank of 15° , the required lift is $3,105.8g$ N. Assuming that the same aircraft structure can tolerate a

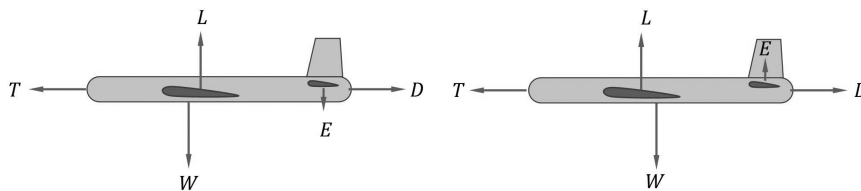


FIGURE 28 The relative location of the COG with respect to the COP or lift:
(a) the COG in front of the COP (left), (b) the COG behind the COP (right)
(drawings created using Solid Edge CAD tool).

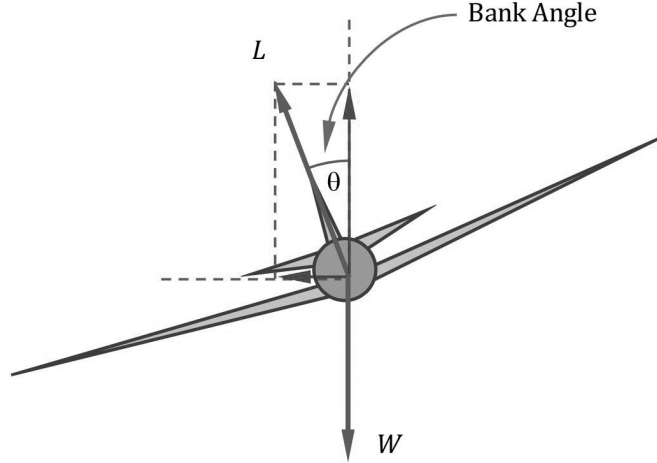


FIGURE 29 Front view of dihedral shallow V-shaped wings to increase lateral (roll) stability (drawings created using Solid Edge CAD tool).

load factor up to 1.5, a maximum wing load of 4,500g N is obtained, which corresponds to the angle of bank of 48.2°.

A powered aircraft moves forward only when the thrust generated by the engine can balance the drag. If the forces are equal, the aircraft does not experience any acceleration; it moves in a straight line at a constant speed. The engine power required to provide the thrust force (F_T) can be roughly estimated by applying the relationship which says that power (P) is the product of force and velocity (v) at which the body to which the force is applied is moving— P/v . Reducing the thrust to zero makes a plane into a glider. To fly, the component of the gravity force (weight) is to balance the drag. The pitch of the aircraft is then to be adjusted so as to generate enough of this component for sufficient lift.

The balance of forces for straight-and-level flying aircraft is depicted in Figure 30. The thrust force is balanced by and equals the drag force, as in equation (80). The lift force is mostly balanced by the weight, as in equation (81). Note that for the case of the COG forward of the COP, there must be an additional small downward force generated at the elevator position to balance the moment between the gravity force and the lift force.

$$-\frac{P}{v} + F_D = 0 \quad (80)$$

$$F_L - mg - F_{\text{elevator}} = 0 \quad (81)$$

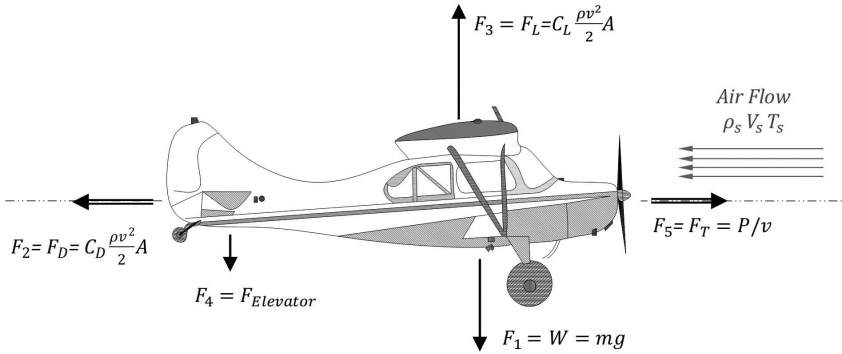


FIGURE 30 Forces acting on the aircraft during a straight-and-level flight (drawings created using Solid Edge CAD tool).

An aircraft in a positive pitch attitude is shown in Figure 31. Note that even though the aircraft is pitched up, it does not mean that the AOA is high. It is important to remember that the AOA is with respect to the relative wind, not the horizontal plane. If the aircraft is climbing, its AOA will likely be greater than that in level flight, but it will not be equal to the pitch angle.

The thrust force in this case needs to balance the drag and the component of the weight parallel to the direction of motion, as in equation (82). The lift force balances the component of the weight perpendicular to the

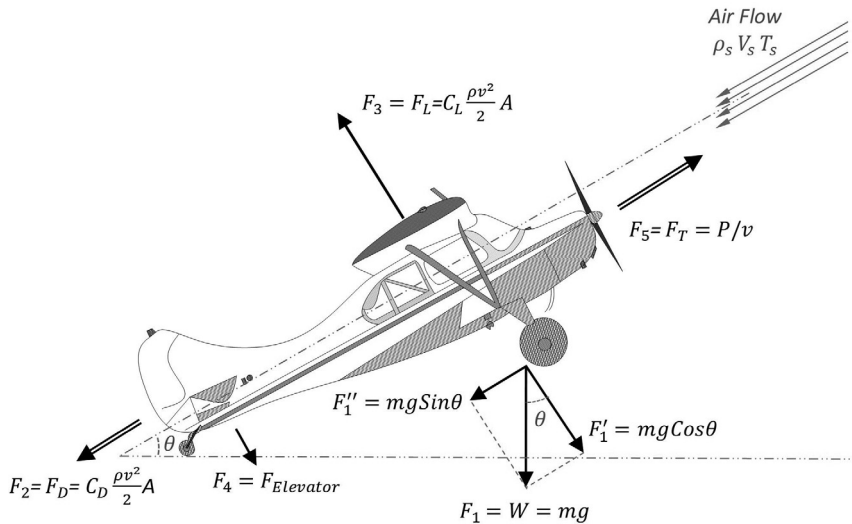


FIGURE 31 Forces acting on the aircraft during a climb (drawings created using Solid Edge CAD tool).

direction of motion plus a small elevator force equation, as in (83). The net force in both directions should be near zero if the aircraft is flying at a steady speed (even while it is climbing).

$$-\frac{P}{v} + mg\sin\theta + F_D = 0 \quad (82)$$

$$F_L - mg\cos\theta - F_{\text{elevator}} = 0 \quad (83)$$

An aircraft in a negative pitch attitude (nose down) is shown in Figure 32. In this attitude, there will be a component of the weight parallel to the direction of motion ($mg\sin\theta$), which can compensate for a fraction of drag force (in a powered case) or all of the drag (for a glider), as in equation (84). The weight component perpendicular to the motion direction will balance the lift force, as in equation (85).

$$\frac{P}{v} + mg\sin\theta - F_D = 0 \quad (84)$$

$$F_L - mg\cos\theta - F_{\text{elevator}} = 0 \quad (85)$$

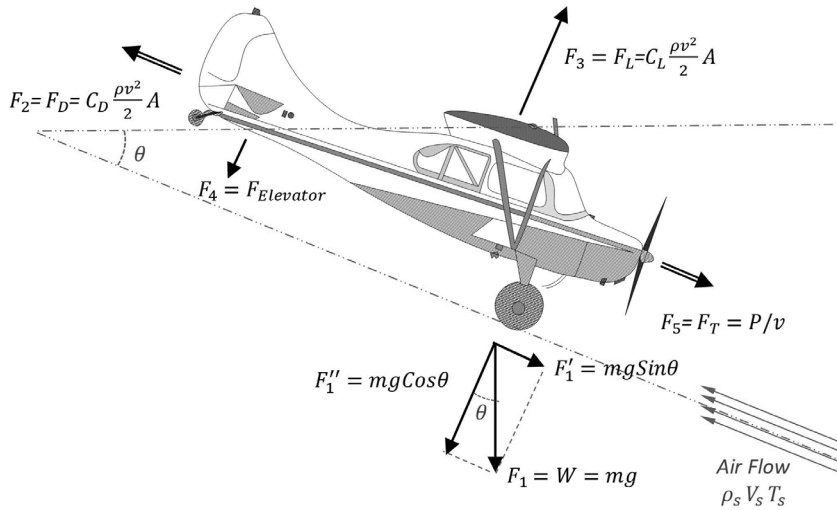


FIGURE 32 Forces acting on the aircraft during a descent
(drawings created using Solid Edge CAD tool).

Table 13 summarizes the forces acting on the aircraft for the three types of straight flight: level, ascent, and descent. Drag (C_D) and lift coefficients (C_L) are required to calculate the forces of drag (F_D) and lift (F_L). These coefficients can be obtained using the correlation factors from wind tunnel experiments or by calibrating numerical results from CFD analysis using experimental data.

TABLE 13 Flight types along with drag and lift forces.

Flight Type	Drag Force	Lift Force
	$F_D = C_D \frac{\rho v^2}{2} A$	$F_L = C_L \frac{\rho v^2}{2} A$
Straight-and-Level	$\frac{P}{v}$	$mg + F_{\text{elevator}}$
Climb/Ascent	$\frac{P}{v} - mg \sin \theta$	$mg \cos \theta + F_{\text{elevator}}$
Descent	$\frac{P}{v} + mg \sin \theta$	$mg \cos \theta + F_{\text{elevator}}$

5.4 Examples

5.4.1 Case Study – Drag and Lift Coefficients

Wind tunnel experiments were performed to find the lift and drag forces of an aircraft at its cruising speed at standard atmospheric conditions. Data are presented in Table 14 and Table 15.

The forces then were used to calculate the lift and drag coefficients using wing surface area as the reference surface. The average drag coefficient results are curve-fitted into a quadratic relationship $C_D = 0.0628C_L^2 + 0.0484$ plotted in Figure 33. Using this relationship ($y = ax + c$), derive a quadratic equation for drag coefficient C_D and identify coefficients of a , b , and c for three given scenarios where the Angle of Attack (AOA) varies from from 0° to 30° . Using the data presented in Table 14 and Table 15.

Calculate: (a) drag coefficient, (b) lift coefficient, (c) ratio of the lift to the drag coefficient, (d) ratios for the three scenarios of AOA with that of the average drag, (e) lift and drag pressures based on the average lift and drag coefficients over different AOA, and (f) lift and drag forces given the reference area of 19.46 m^2 (174 ft^2), TAS of 67 m/s (130 kt), and an air density of 1.196 kg/m^3 .

Figure 33 presents the relation between the drag and lift coefficients. This relation is curve-fitted and the suggested equation ($C_D = 0.0628C_L^2 + 0.0484$) is presented as a result. The equation is to be rewritten to include only one of the lift or drag coefficients using the relation $C_L = fC_D$. The result is a quadratic equation in the form of $af^2C_D^2 + bC_D + c = 0$. This equation is to be solved using the algebraic relations. The steps taken to

obtain the solution are presented in Table 14 and Table 15. Steps 20 and 21 (Table 15) are for verification to ensure the ratios of the calculated lift to drag force (coefficients) are equal.

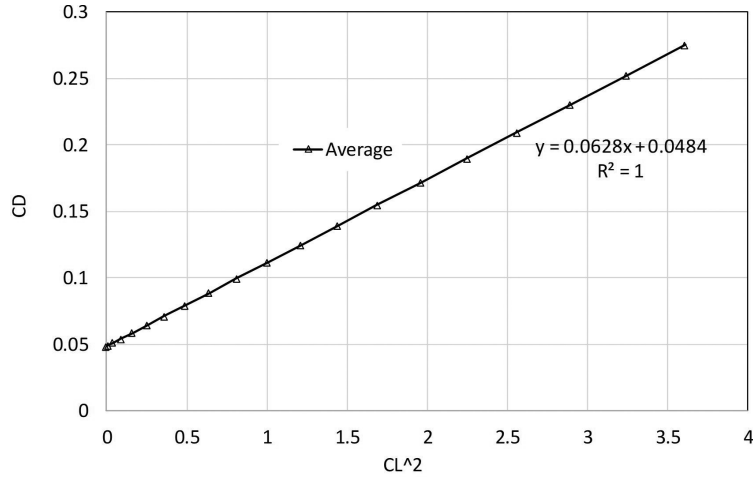


FIGURE 33 Drag and Lift coefficients.

TABLE 14 Drag and lift coefficients versus the angle of attack.

Step	Function	Average	0°	20°	30°
1	$af^2C_D^2 + bC_D + c = 0$				
2	$f = C_L / C_D$	7.702	9.313	7.469	6.880
3	C_D	0.207	0.140	0.220	0.260
4	C_L	1.592	1.304	1.643	1.789
5	C_L^2	2.533	1.700	2.700	3.200
6	f	7.702	9.313	7.469	6.880
7	a	0.063	0.064	0.062	0.062
8	af^2	3.726	5.569	3.445	2.958
9	b	-1.000	-1.000	-1.000	-1.000
10	c	0.048	0.037	0.049	0.059
11	$4af^2c$	0.722	0.832	0.680	0.693
12	$b^2 - 4af^2c$	0.278	0.168	0.320	0.307
13	$\sqrt{b^2 - 4af^2c}$	0.528	0.409	0.566	0.554
14	$-b + \sqrt{b^2 - 4af^2c}$	1.528	1.409	1.566	1.554
15	$-b - \sqrt{b^2 - 4af^2c}$	0.472	0.591	0.434	0.446

TABLE 15 Drag and lift pressure and forces versus the angle of attack.

16	$C_{D1} = \frac{-b + \sqrt{b^2 - 4af^2c}}{2af^2}$	0.205	0.127	0.227	0.263
17	$C_{D2} = \frac{-b - \sqrt{b^2 - 4af^2c}}{2af^2}$	0.063	0.053	0.063	0.075
18	$C_{L1} = fC_{D1}$	1.579	1.178	1.697	1.808
19	$C_{L2} = fC_{D2}$	0.488	0.494	0.471	0.518
20	Test Step : $f_1 = C_{L1} / C_{D1}$	7.702	9.313	7.469	6.880
21	Test Step : $f_2 = C_{L2} / C_{D2}$	7.702	9.313	7.469	6.880
22	$(P_{Di}, P_{Li}) = \frac{1}{2}(CD_i, CL_i) \rho mv^2$				
23	P_{D1} (kPa)	0.551	0.340	0.611	0.707
24	P_{D2} (kPa)	0.170	0.143	0.169	0.203
25	P_{L1} (kPa)	4.245	3.168	4.564	4.861
26	P_{L2} (kPa)	1.313	1.328	1.266	1.393
27	$(F_D, F_L) = (P_{Di}, P_{Li})A$				
28	F_{D1} (kN)	10.729	6.621	11.893	13.752
29	F_{D2} (kN)	3.317	2.775	3.299	3.942
30	F_{L1} (kN)	82.630	61.665	88.829	94.620
31	F_{L2} (kN)	25.549	25.848	24.640	27.119

5.4.2 Case Study – Balance of Forces and Wind Interference

Performance data for a number of aircraft are provided in Table 16. These data include mass, TAS, coefficients of lift and drag, thrust per mass, and the component of thrust along the x-coordinate. Wind data are also provided. The aircraft are to fly a given course, at specified altitudes, atmospheric conditions (i.e., temperature, pressure, and wind), and angles of attack that may vary depending on their maneuvering positions. Ignore magnetic variations in this problem. Using the data presented in Table 16, for the cases 1 to 10 provided, calculate:

(a) angle of attack in radians, (b) angle of wind with respect to the aircraft heading in degrees and radians, (c) wind magnitude including half of the gust factor, (d) crosswind component of the wind, (e) headwind component

of the wind, (f) aircraft crab angle, (g) true heading, (h) ground speed, (i) air pressure in inHg and kPa, (j) density, (k) assuming the standard-adiabatic lapse rate (1.98 °C/1,000 ft), calculate the temperature for the given altitudes, (l) ratio of the lift to the drag, assuming wing surface area as the reference area, (m) drag force per unit area, (n) lift force per unit area, (o) drag force given wing surface area, (p) lift force given wing surface area, (q) total thrust along the x-coordinate using its dimensionless value (i.e., thrust per $0.5\rho v^2A$, (r) total thrust, (s) required velocity for the aircraft to remain balanced, and (t) compare calculated and given thrusts and conclude if the calculated thrust (maximum available thrust) is sufficient for the aircraft to remain aloft given the specific scenarios.

Table 16 presents the performance data for the list of the aircraft in this case study. To calculate the wind magnitude to be used to find the vertical and horizontal components of the wind, half of the gust factor— $G_f = (-w + G)$ —is to be added to the given wind magnitude ($w' = w + G_f/2$), where G is the gust. The Angle of Attack (AOA) is the angle between the relative air with respect to the aircraft's chord line of its airfoil (α). As the aircraft pitch angle increases (positive and negative pitch angles may represent ascending and descending positions, respectively), the component of weight parallel to the motion of the aircraft ($mg\sin\alpha$) resists or encourages the forward motion of the aircraft, respectively. The crosswind ($w\sin\phi$) and headwind ($w\cos\phi$) components of the wind are the cosine and sine of the angle between the nose and the wind ($\lambda = \phi - \beta$) multiplied by the wind magnitude (w')—considering the gust factor (G_f)—where β is the true course. The crab angle is the arc(sine) of the ratio of the crosswind component to the TAS ($CA = \delta = \arcsin(w\sin\lambda/TAS)$) using vector algebra (Table 17).

TABLE 16 Aircraft angle of attack, mass, and wind direction.

Case	Aircraft	AOA α (°)	AOA α (rad)	Mass m (lb)	Mass m (kg)	Wind Speed w (kt)	TAS (kt)	True Course β (°)	Angle of Wind ϕ (°)
1	SIAT-Marchetti S-211	1	0.0175	6,063	2,750	25@2.5ktG0	265.1	18	25
2	Electro.Aero [133]	1	0.0175	1,213	550	35@4.5ktG5	51.6	43	35
3	Learjet 24	5	0.0873	13,499	6,123	45@7ktG7.25	230.8	40	45
4	F-4 Phantom II (subsonic)	9	0.1571	61,796	28,030	55@9.5ktG10.25	272.4	62	55
5	Beach 99	13	0.2269	10,421	4,727	65@12ktG14	101.6	59	65
6	F-104 Starfighter	18	0.3142	29,026	13,166	75@14.5ktG18.5	228.3	83	75
7	Cessna 172/182	20	0.3491	2,300	1,043.3	85@16.5ktG23.75	43.6	79	85
8	Cessna 620	24	0.4189	13,650	6,191.5	95@18.5ktG29.75	63.0	99	95
9	Boeing 747	28	0.4887	833,348	378,000	105@20.5ktG30.65	158.3	90	105
10	Cessna 310	30	0.5236	4,600	2,086.5	115@23ktG31.65	77.4	130	115

Depending on the direction of the wind (easterly or westerly), the aircraft is to crab into the wind; therefore, to calculate the aircraft crab angle, the wind direction is to be added to or deducted from the true course to obtain the crab angle— $THdg = \beta \pm \delta$. Crosswinds from the right or left sides are to be added to or deducted from the true course to result in the true heading.

If the magnetic variation were to be included, the magnetic heading would have been calculated by adding it to (westerly variations) or deducting it from (easterly variations) the true heading ($THdg \pm Var = MagHdg$), and the results would also have been used for landing and takeoff calculations. The effect of geographic variation is ignored in this exercise. The magnetic variation may also be added to (westerly variations) or subtracted from (easterly variations) the difference between the two angles as well ($\lambda \pm Var$). Note that this data may then be used to obtain the compass heading. When selecting the runway to take off or land, however, the variation is to be combined with the true wind data to generate the wind magnetic heading, which can then be used to identify the crosswind and headwind components given the runway magnetic heading.

The ground speed (GS) is the speed of the aircraft relative to the ground. Depending on the wind direction, headwind, or tailwind, the ground speed will be less (headwind) or more (tailwind) than the TAS. Note that $GS = TAS \pm \omega \cos \lambda$ —the headwind/tailwind component is calculated based on the angle between the wind direction and longitudinal axis along the fuselage.

The pressure altitude (P_{AL}) may be assumed the same as the flight level (FL) in this scenario. Given the elevation (H) and the standard atmospheric condition for pressure altitude (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi), pressure (P) at the given altitude (H) may be calculated in inches of mercury (inHg) and then converted to kPa— $PA = H + 1000(29.92 - P_{inHg})$ (Table 18).

Knowing this pressure (P_{inHg}), the outside air temperature ($T = OAT$), and the air specific gas constant for dry air ($R_a = 287.0 \text{ J/kgK}$), the air density may be calculated— $\rho = P/RT$. This density is then used in combination with the speed of the aircraft (TAS) to find the multiplier for the lift and drag coefficients— $(0.5 \rho v^2)$ —to obtain the force per unit area or pressure ($P = F/A$) for both drag and lift forces. Note that the wind tunnel experiments and the related data are based on a common reference plane when deducing the lift and drag coefficients. Once the wind tunnel tests are conducted and the lift (F_L) and drag (F_D) forces are measured based on the wing surface

area (A) as the reference area, the lift and drag coefficients can then be calculated. For this problem, the lift and drag coefficients are already provided (Table 19). Drag and lift coefficients may also be adopted from scholarly sources or obtained from wind tunnel experiments [133].

The remaining steps are as follows: (a) using the lift coefficient, assuming that the lift force equals the vertical component of weight, calculate the minimum velocity magnitude required to ensure the force balance, $v_e = \sqrt{2mg \cos \alpha / \rho A C_L}$; (b) lift force is essentially the same as the vertical component of the weight and is a function of the lift coefficient, $F_L = 0.5 A \rho v_e^2 C_L$; (c) given the calculated balancing velocity and using the given drag coefficient, the drag force can be calculated, $F_D = 0.5 A \rho v_e^2 C_D$; (d) thrust is essentially the same as the drag plus the horizontal component of the weight, and therefore it depends on the pitch angle, $F_T = F_D mg \sin \alpha$; (e) thrust per mass is obtained as F_T/m ; (f) minimum available thrust is provided but also can be calculated using the total power per cruise speed (assume TAS), $F_{Tlimit} = \eta Power / TAS$, where η is the efficiency of the engine; (g) the horizontal component of the thrust is calculated using the dimensionless horizontal component of the thrust multiplied by the equivalent lift or drag forces per associated lift and drag coefficients ($0.5 A \rho v^2$), but based on the TAS, $F'_x = 0.5 F_{Tx} \rho TAS^2 A$; and (h) eventually, by balancing the forces applied along the aircraft motion (i.e., components of weight, thrust, and drag parallel to the fuselage) ($F = F_T - F_D \pm F_w$), and dividing the result by the total takeoff mass, the acceleration ($a = F/m$, if any, can be estimated—it equals zero if the forces are correctly balanced (Table 20).

This example considers the balance of the forces along the direction of the aircraft motion. The total thrust obtained from these calculations may be compared with the available thrust for the given cases. If the calculated value is under the available thrust for the specific angle of attack, performance parameters, and given atmospheric conditions, the aircraft is able to generate the thrust required to remain aloft.

TABLE 17 Relative angle of wind, wind components, crab angle, and ground speed.

Case	Relative Angle of Wind λ (°)	Relative Angle of Wind λ (rad)	Wind Magnitude w' (kt)	Wind Headwind $w' \cos \lambda$ (kt)	Wind Crosswind $w' \sin \lambda$ (kt)	Crab Angle δ (rad)	Crab Angle δ (°)	$THdg$ (°)	Ground Speed GS (kt)
1	7.0	0.122	2.5	2.5	0.3	0.001	0.1	18.1	262.7
2	-8.0	-0.140	4.8	4.7	-0.7	-0.013	-0.7	42.3	46.9
3	5.0	0.087	7.1	7.1	0.6	0.003	0.2	40.2	223.7
4	-7.0	-0.122	9.9	9.8	-1.2	-0.004	-0.3	61.7	262.6
5	6.0	0.105	13.0	12.9	1.4	0.013	0.8	59.8	88.7
6	-8.0	-0.140	16.5	16.3	-2.3	-0.010	-0.6	82.4	211.9
7	6.0	0.105	20.1	20.0	2.1	0.048	2.7	81.7	23.6
8	-4.0	-0.070	24.1	24.1	-1.7	-0.027	-1.5	97.5	38.9
9	15.0	0.262	25.6	24.7	6.4	0.040	2.3	92.3	133.6
10	-15.0	-0.262	27.3	26.4	-6.8	-0.088	-5.1	124.9	51.0

TABLE 18 Outside air properties, pressure altitude, and wing surface area.

Case	Ground Speed GS (m/s)	Outside Air Temperature (OAT) T (°C)	Pressure Altitude PA (ft)	Altitude H (ft)	Pressure P (inHg)	Pressure P (kPa)	Density ρ (kg/m³)	Wing Surface Area A (ft²)	Wing Surface Area A (m²)
1	135.12	-44.301	FL300	29,950	29.87	101.15	1.540	136.0	12.63
2	24.14	11.238	FL20	1,900	29.82	100.98	1.237	102.4	9.51
3	115.08	7.377	FL40	3,850	29.77	100.81	1.252	230.0	21.37
4	135.07	-36.084	FL260	25,800	29.72	100.64	1.479	530.0	49.24
5	45.61	-0.345	FL80	7,750	29.67	100.47	1.283	280.0	26.01
6	109.03	-20.046	FL180	17,700	29.62	100.30	1.381	196.0	18.21
7	12.15	-4.107	FL100	9,650	29.57	100.14	1.297	174.0	16.17
8	20.03	11.832	FL20	1,600	29.52	99.97	1.222	340.0	31.59
9	68.73	-27.669	FL220	21,550	29.47	99.80	1.416	5,500.0	510.97
10	26.22	8.07	FL40	3,500	29.42	99.63	1.234	175.0	16.26

TABLE 19 Aircraft lift, drag, and thrust ratio coefficients.

Case	Drag Coefficient C_D	Lift Coefficient C_L	$\rho v^2/2$	Lift to Drag $\frac{F_L}{F_D} = \frac{C_L}{C_D}$	Drag Pressure F_D/A (Pa)	Lift Pressure F_L/A (Pa)	Lift F_L (kN)	Drag F_D (kN)	Thrust Limit F_T (kN)
1	0.021	0.15	14,322.9	7.3	293.6	2,134.1	27.0	3.7	11.1
2	0.140	1.30	436.2	9.3	61.1	567.1	5.4	0.6	2.3
3	0.046	0.32	8,824.6	7.0	402.5	2,799.4	59.8	8.6	26.2
4	0.077	0.38	1,4517.9	4.9	1,120.1	5,513.9	271.5	55.2	158.8
5	0.165	0.99	1,752.4	6.0	289.8	1,736.4	45.2	7.5	10.2
6	0.507	0.71	9,519.4	1.4	4,827.3	6,743.7	122.8	87.9	69.0
7	0.352	1.82	326.5	5.2	115.1	594.7	9.6	1.9	7.3
8	0.640	2.74	641.8	4.3	410.5	1,756.1	55.5	13.0	30.9
9	0.918	1.36	4,696.5	1.5	4,309.4	6,405.5	3,273.0	2,202.0	1,012.0
10	1.070	1.12	977.3	1.0	1,045.9	1,090.0	17.7	17.0	12.2

TABLE 20 Aircraft lift, drag, thrust, and balance velocity magnitude.

Case	Thrust F_T (kN)	Weight Horizontal Component $mg\sin\alpha$ (kN)	Weight Vertical Component $mg\cos\alpha$ (kN)	Total x-axis Dimensionless Thrust F'_x	Total x-axis Thrust F_{T_x} (kN)	Thrust/Mass F_{T_x}/m (N/kg)	v_e (m/s)	v_e (kt)	v_e (MPH)
1	4.2	0.5	27.0	25.0	4,524.2	1.5	136.4	265.1	305.1
2	0.7	0.1	5.4	-	-	1.2	26.6	51.6	59.4
3	13.8	5.2	59.8	27.9	5,260.8	2.3	118.7	230.8	265.6
4	98.2	43.0	271.5	30.0	21,445.0	3.5	140.1	272.4	313.4
5	18.0	10.4	45.2	29.8	1,358.5	3.8	52.3	101.6	116.9
6	127.8	39.9	122.8	55.3	9,585.6	9.7	117.4	228.3	262.7
7	5.4	3.5	9.6	32.0	168.9	5.1	22.4	43.6	50.2
8	37.7	24.7	55.5	42.0	851.4	6.1	32.4	63.0	72.5
9	3,942.2	1,740.3	3,273.0	25.0	59,992.9	10.4	81.4	158.3	182.2
10	27.2	10.2	17.7	31.0	492.6	13.1	39.8	77.4	89.0

CHAPTER 6

PISTON ENGINE AND COMBUSTION PROCESS

Aircraft systems are categorized into subsystems based on their functionality. From the conception stage to the planning, execution, and implementation, there are many steps required to complete and then certify the aircraft design. For the aircraft to be airworthy, the subsystems are to meet specific engineering and customer requirements. This section focuses on the piston engine and the chemistry of the combustion process.

6.1 Piston Engine

For the case of a four-stroke piston engine (Figure 34 and Figure 35), the four stages are intake, compression, combustion/expansion, and exhaust. Two of these strokes (i.e., compression and expansion) involve work being done; in compression the work is done on the fluid, and in expansion the work is done by the fluid. The net effect of the two strokes is the work generated by the engine; the maximum rate at which this work can be carried out is known as the engine power (expressed in horsepower or Watts).

6.1.1 Alternator

In a smaller, piston-engine aircraft, such as the Cessna 172M, an alternator provides the Direct Current (DC) electrical power for charging the battery and operation of the electronics and electrical systems. In the

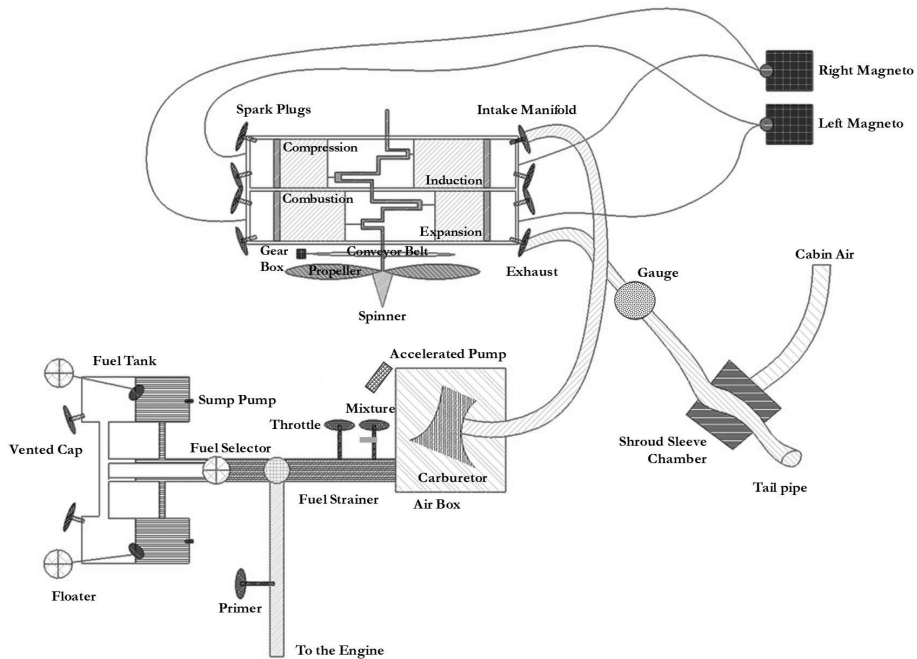


FIGURE 34 Example of a four-cylinder horizontally opposed engine, fuel, magnetos, cylinders, and exhaust systems (drawings created using Solid Edge CAD tool).

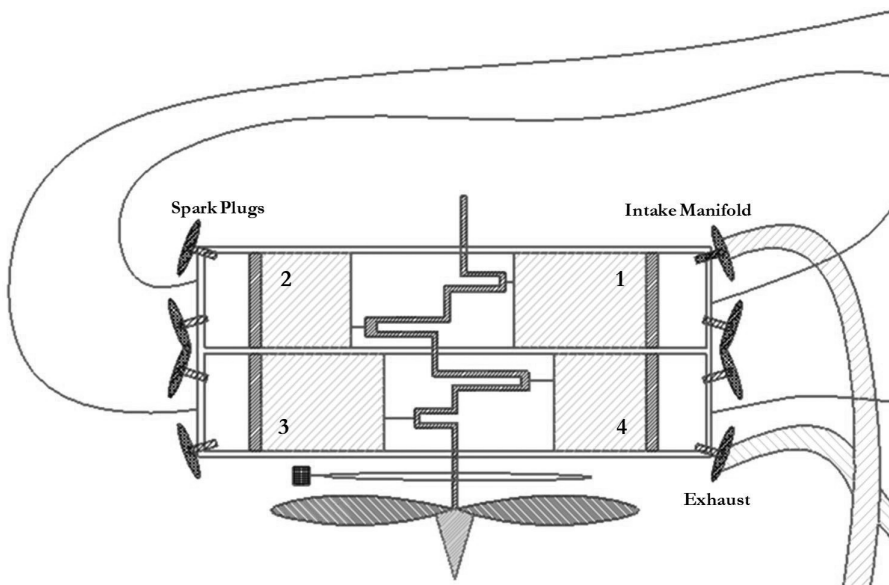


FIGURE 35 Example of a four-cylinder horizontally opposed engine, cylinders, valves, and propeller systems (drawings created using Solid Edge CAD tool).

Cessna 172M, it is capable of generating 14 V and 60 A of DC power. A starter motor is connected to the engine crankshaft by a belt; the shaft is connected to the pistons via crankpins and connecting rods. The crankshaft moves the piston head inside the cylinder as well as the camshaft assembled on top of the central shaft, which operates the intake and exhaust valves synchronously with the piston motion, as in Figure 34 and Figure 35. As in all internal combustion engines, the four stages of intake, expansion, combustion, and exhaust take place. During the intake stage, the atomized fuel combined with air enters through the intake valves via a manifold. The Cessna 172M is equipped with four horizontally opposed cylinders that are air-cooled and drive the propeller directly. Each piston is capable of displacing air inside the cylinder by 320 in³. The engine generates about 150 to 165 hp (Lycoming O-320-E2D engine). The main shaft rotates the propeller at the desired Revolutions Per Minute (RPM). There are two different situations possible for engine RPM values: (a) static, in which the aircraft is stationary while on the ground (2,300 RPM–2,420 RPM); and (b) dynamic, which is the maximum RPM demonstrated during flight, where the fuel-to-air mixture ratio has been set to its maximum performance level (2,700 RPM).

6.1.2 Fuel System

The fuel system for a small aircraft (i.e., takeoff weight under 1,500 lb) typically consists of two internal wing tanks that are connected by a tube for venting air and keeping the fuel level on both sides, in addition to air vents that are built in the fuel caps, securing the fuel from exiting the tanks when performing maneuvers. Early versions of the Cessna 172 used a single air-vented fuel cap for the left tank; however, the right-tank fuel cap has also been air-vented since the mid-1980s. There is also a separate air vent located at the bottom of the left tank with an elongated L-shape; it is the main source of air venting. If the air passage is blocked for whatever reason, such as accumulation of ice and debris at the entrance of the air vent, fuel starvation may occur. Beryl Clutterbuck Markham's aircraft, Vega Gull, suffered fuel starvation due to icing of the fuel tank vents, causing her to crash-land at the end of her solo flight west across the Atlantic. Note that when performing maneuvers such as landing, takeoff, descent, and ascent, the fuel regulator is to be set on both tanks. An exception is in emergency situations such as if one wing is on fire and the fuel feeding from that wing is to be turned off to avoid fueling the fire and to do damage control to the extent possible. This is done in addition to turning off the electrical devices associated with that wing such as landing, navigation, and taxi lights.

Aircraft flight condition, such as level, ascending, or descending flight, is a determining factor in deciding on the optimum rate of fuel supply required to minimize fuel consumption. In a piston-engine aircraft, the internal combustion engine principles apply, and therefore the mixture of air and fuel is required in order for the combustion process to take place. Since the fuel is denser than the air by a factor of 589 on average at 15 °C (721 kg/m³ versus 1.225 kg/m³), one needs to vary the number of air molecules; for example, by leaning or enriching the fuel mixture, for the combustion process to proceed efficiently. The air-to-fuel mixture mass ratio may vary from 8:1 (rich) to 20:1 (lean). However, the complete combustion mixture ratio is approximately 12.5: 1—the chemically correct mixture for a stoichiometric process for an octane fuel.

The ideal process of burning fuel follows a stoichiometric combustion process in which the fuel belonging to a specific family—such as isooctane (2,2,4-trimethylpentane—C₈H₁₈) used as an Aviation Gasoline (AVGAS), versus the Motor Gasoline (MOGAS) used in some light aircraft—is mixed with air in a carburetor after being strained for small contaminant particles by means of gravity in high-wing aircraft and a fuel pump in low-wing aircraft.

100/100LL (blue) is the low-lead version of isooctane fuel types that is used in piston engines, and it performs better than the 80/87 (red) and 100/130 (green) fuel types due to the presence of lead, which improves the combustion process. Lead itself is poisonous to handle; therefore, wearing plastic gloves when filling the tanks or doing the walkaround fuel checks is recommended. During excess idling at the start of the flight while doing the routine checklist, the extra lead may deposit on the cylinder heads and therefore cause buildup and charring, which is the accumulation of the carbon black on the heads and spark plugs. The char may be formed in any part of the cylinder head, and due to its high heat absorption properties, it acts as a new spark plug, meaning that the created hot spots work as combustion nuclei, and as a result combustion takes place away from the spark plugs. This causes detonation, fouling of the spark plugs, and knocking to occur (a premature explosion which leads to banging noises due to the inappropriate contact between the piston and cylinder head). Tetraethyl Lead (TEL), first introduced in the 1920s, is the additive used to prevent engine knocking [134]. It allowed for an increase in engine compression and therefore engine performance. The United States Clean Air Act in the 1970s limited the use of TEL due to its negative environmental effects. 100LL (0.56 grams lead per liter, 1.2 to 2 grams TEL/US gallon) is allowed

for 0.5TEL as an additive used for 100/130 (1.12 grams lead per liter, TEL) AVGAS. Earlier aircraft models run on a lower grade of AVGAS fuel such as 80/87 (0.14 grams lead per liter, TEL). China remains the largest TEL producer, followed by Algeria, Yemen, and Iraq.

6.1.3 Carburetor and Venturi Tube

The fuel carburetor mixes fuel and air to the required portion for the engine combustion process to take place. The ratio of the fuel and air is the determining factor for whether the combustion process is efficient. As the air is drawn into the carburetor by the movement of the cylinders (which work like an air pump), it passes by a narrowed opening which creates a venturi tube where the air accelerates. This creates a low pressure, which draws the fuel through small holes (called jets). The atomization process happens by forcing the fuel through these small jet openings under high pressure to break it into a fine misted spray (by primer)—similar to a perfume spray bottle mechanism. This mist is then mixed with air and becomes usable for the internal combustion engine. The flow of the air-and-fuel mixture is then controlled by a butterfly valve (throttle) connected to the throttle control lever. It nearly blocks the air flow when closed at idle and provides unrestricted flow when open at full power.

Turbine engines as well as diesel piston engines use jet fuel (kerosene-based). For an optimum stoichiometric combustion process to occur, a mixture of 12.5 molecules of air mixture to 1 molecule of fuel is required for a pure octane fuel—meaning the air-to-fuel ratio is 12.5 to 1, which will be discussed later in this section. This equals a 0.08 fuel-to-air ratio that the reader may come across in other scholarly resources.

You may have noticed a Venturi tube—the horn-shaped system which is used to operate vacuum-driven gyros in non-engine-driven vacuum systems, located on the side of older generations of aircraft such as the 612 kg single-piston-engine semi-monocoque Aeronca-7DC built in 1946, the first stick-controlled aircraft the author flew (Figure 36). In the constricted cross section of the tube, due to reduction in area and conservation of mass, fluid speed is increased, resulting in lower pressure to comply with a simplified form of conservation of energy as described by Bernoulli's principle for cases of an incompressible flow. The flow then moves from the area of the higher pressure to that of the lower pressure (i.e., to the constricted area in a vacuum-driven gyro system). As the pressure drops, it will pull in the air from the gyro, causing the gyro to spin, as in Figure 37.

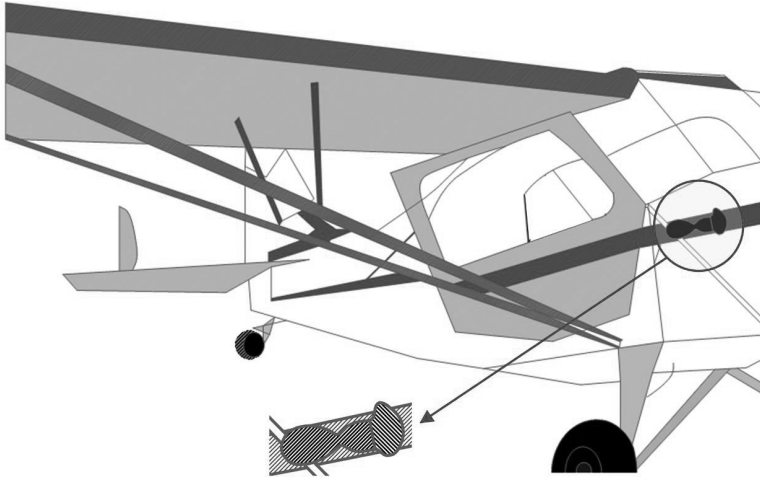


FIGURE 36 Single-piston-engine semi-monocoque Aeronca-7DC built in 1946 (drawings created using Solid Edge CAD tool).

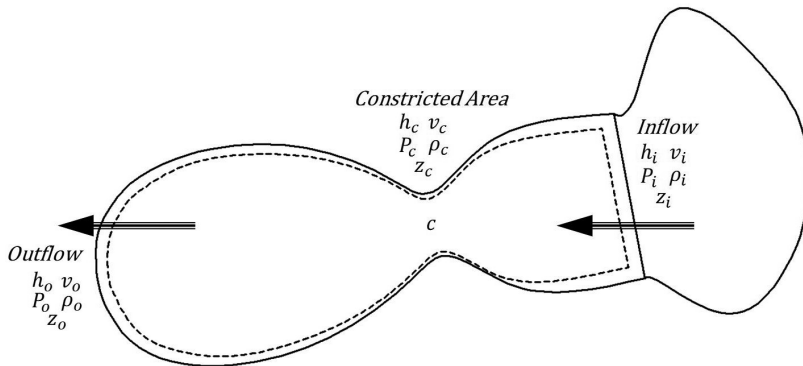


FIGURE 37 Venturi tube (drawings created using Solid Edge CAD tool).

Equation (86) shows the conservation of the mass inside a venturi tube (Figure 37). Mass flow is constant for each of the sections enclosed in a virtual control volume shown by dashed lines. If the flow is compressible (the density does not remain constant with temperature change or non-homogeneities), inviscid (viscosity is close to zero), and steady (the flow does not change with time), equation (87) is valid. Subscripts “i,” “c,” and “o” are related to the properties for the inflow (flow entering the tube), the flow at the constricted area, and the outflow (flow exiting the tube), respectively. ρ is fluid density, A is area, v is velocity, P is pressure, z is height with respect to a reference point, and \dot{m} is the mass flow rate that is assumed constant throughout the tube.

For an incompressible flow, where central locations of the inlet and constricted areas are at the same level, equation (87) may be simplified to equation (88)—assuming that the volumetric flow rate (Q) is constant throughout the tube.

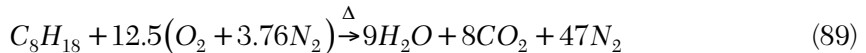
$$\dot{m} = \rho_i A_i v_i = \rho_c A_c v_c = \rho_o A_o v_o \quad (86)$$

$$\dot{m} = \rho_c A_c \sqrt{\frac{\left(\frac{P_i}{\rho_i} - \frac{P_c}{\rho_c}\right) - g(z_c - z_i)}{\frac{1}{2} \left[1 - \left(\frac{\rho_c A_c}{\rho_i A_i}\right)^2\right]}} \quad (87)$$

$$Q = A_c \sqrt{\frac{2(P_i - P_c)}{\left[1 - \left(\frac{A_c}{A_i}\right)^2\right]}} \quad (88)$$

6.2 Combustion Process

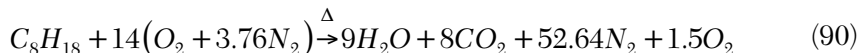
Combustion is the process of oxidation of the fuel components that are oxidable in which the component masses remain constant. Air is composed of nitrogen (78 percent), oxygen (21 percent), and noble gases (mainly argon, 1 percent). Argon gas is not involved in the combustion process, so it is assumed that air consists of 79 percent nitrogen and 21 percent oxygen with a nitrogen-to-oxygen ratio of 3.76. The minimum air required for the combustion process to happen in a complete fashion is called theoretical air, and the process is called a stoichiometric process. AVGAS is a combination of isooctanes: it could be represented by octane molecules consisting of eight carbon atoms as the basis (C_8H_{18}) and heptane molecules consisting of seven carbon atoms as the basis (C_7H_{16}). It is possible to find the theoretical air when burning AVGAS such as isooctane, octane, heptane, or their combinations. Equation (89) represents a complete combustion process for an octane fuel. It is seen that the combustion by-products consist of molecules of water (H_2O), carbon dioxide (CO_2), and nitrogen (N_2).



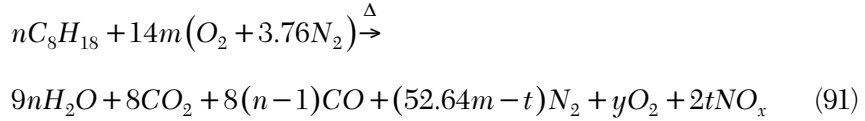
Octane is added to increase the compression ratio and therefore enhance combustion performance of gasoline engines, while heptane is more environmentally friendly. Octane content is a standard measurement of aviation fuel and engine performance and is known as the octane rating

of the fuel (also known as the octane number). In diesel engines, the lower the octane number and the higher the cetane number (inverse of the octane number) is, the better the performance is expected to be. These two numbers both indicate ignition readiness, if the fuel ignites spontaneously or not. Spark plugs ignite the air-fuel mixture at the end of the compression phase in a piston engine. An increased fuel compression ratio is necessary to reduce the knocking that is common for fuels with higher octane numbers. The optimum balance between octane and heptane numbers is 0.9 to 1. Any fuel mixture with a high likelihood of knocking has an ideal octane to heptane ratio—also known as an octane rating—of 90. Any combustion process using an amount of air that deviates from that of the stoichiometric one either includes excess air among the by-products or leads to an incomplete process with formation of carbon monoxide and nitrogen oxide as its by-products. The existence of contamination (e.g., water, debris, and color variation) in the fuel may be spotted by discharging some from the fuel tank sump into a clear bottle and investigating the content of the bottle.

In reality, more air molecules are needed for a combustion process to be complete for efficiencies that are less than ideal. For example, for gasoline piston engines to operate close to their peak, the air-to-fuel ratio of 14 to 1 is recommended for operation, which is 12 percent extra air than the theoretical air shown previously. This ratio is to be honored to comply with fuel economics. As the aircraft climbs, the air density decreases with increasing altitude, and fewer molecules are available per unit volume of air; therefore, to make the same air-to-fuel ratio possible you need to lean the mixture. This becomes particularly important for long haul flights—for example, your favorite cross-country. The chemical reaction for this combustion process is presented by equation (90), where fourteen molecules of oxygen are mixed with one octane molecule. It is seen that the combustion by-products consist of molecules of water (H_2O), carbon dioxide (CO_2), nitrogen (N_2), and oxygen (O_2). Note that in a complete combustion process with excess air, the possibility for creation of nitrogen oxides (NO_x) exists; however, it has been ignored in this case.

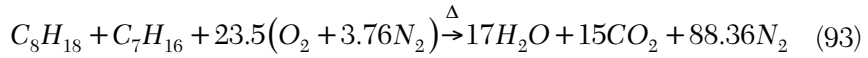


Assuming that the combustion process is incomplete, meaning that carbon monoxide (CO) and nitrogen oxides (NO_x) are also a by-product of the process, the chemical reaction may be represented by equation (91)—where y is the number of released oxygen molecules given by equation (92) and m and n are multipliers for the ideal process that is larger than one.

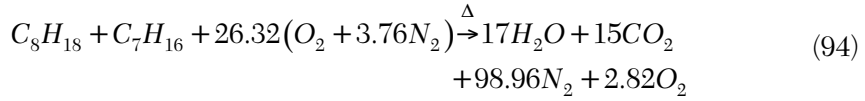


$$y = \frac{28m - 17n - 8 - 2tx}{2} \quad (92)$$

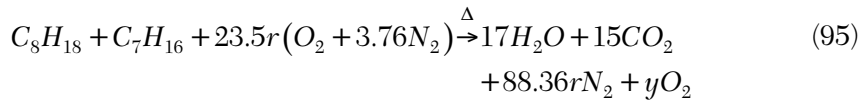
Theoretical air for a complete combustion process to occur when burning AVGAS consisting of an octane-heptane combination is presented by equation (93). It is seen that the combustion by-products consist of molecules of water (H_2O), carbon dioxide (CO_2), and nitrogen (N_2).



As mentioned earlier, more oxygen molecules are needed to make the combustion process a complete one. If the same excess air (112 percent) as used earlier for complete combustion of the octane is now employed for an octane-heptane combination, equation (94) is obtained.

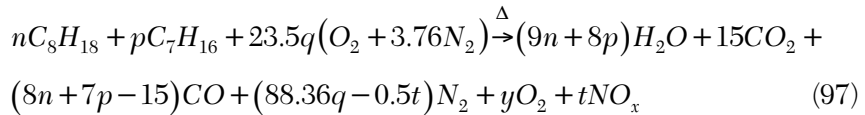


Assuming that you do not know the optimum value for the fuel-to-air ratio for a complete combustion process of octane-heptane, this chemical reaction can be presented by equation (95)—where y is the number of released oxygen molecules given by equation (96) and r is the number of oxygen and nitrogen molecules that is larger than one.



$$y = \frac{47(r-1)}{2} \quad (96)$$

If an incomplete combustion of octane-heptane is employed that affects both engine efficiency and the environmental impact of gasoline burning into the atmosphere, chemical reaction (97) may be written—where y is the number of released oxygen molecules given by equation (98), and n and p are multipliers for the ideal process that are larger than one.



$$y = \frac{47q - 17n - 15p - 15 - tx}{2} \quad (98)$$

Using any of the previous chemical reactions, you are able to calculate the heat of combustion using the enthalpy of the input and output products. Heats of combustion for octane ($44.427 \text{ MJ/kg} = 19,104 \text{ BTU/lb}$) and heptane ($44.566 \text{ MJ/kg} = 19,163 \text{ BTU/lb}$) are available in the literature—with the heptane generating more energy of about 139 kJ/kg (59.76 BTU/lb) of fuel. As a 2,2,4-trimethylpentane, isooctane generates about 44.310 MJ/kg ($19,053 \text{ BTU/lb}$) of energy, falling behind heptane by 260 kJ/kg (111.78 BTU/lb) of fuel. Heats of combustion for most commonly used AVGAS fuels are presented in Table 21 [135,136,137,138].

Let us discuss the energy exchange process that fuel is to achieve in order to ensure an effective and efficient combustion and hence an improved engine performance. The first law of thermodynamics expressed by equation (53)— $dU = dQ - dW$ —states the conservation of energy principle in the form of heat and work. For a flow moving inside and outside of a control volume, similar to the venturi flow shown in Figure 37, the inflow and outflow share this energy transfer. This energy is in the form of kinetic energy (due to the flow velocity), potential energy (due to the elevation), and internal energy. Change in internal energy is due to changes in temperature of the fluid molecules and also due to the changes in pressure and the boundaries of the control volume interacting with the fluid. The combination of internal, kinetic, and potential energy is the energy that is brought in or taken away by the flow. Conservation of energy then is defined using equation (99)—where “ R ” and “ P ” are the subscripts for reactants (inputs) and products (outputs), respectively.

$$Q_{c.v.} + \sum_R n_i \left[\bar{h}_f^\circ + \Delta \bar{h} \right]_i = W_{c.v.} + \sum_P n_o \left[\bar{h}_f^\circ + \Delta \bar{h} \right]_o \quad (99)$$

It is assumed that the enthalpy of formation for the elements at 25°C and 101.325 kPa is zero—this is the reference point where the element, gas, solute, or substance is at its pure one-phase form or the most stable state. Flow enthalpy at a desired temperature is the enthalpy value at the reference point and enthalpy difference between the desired state and the reference point. For example, when carbon dioxide is produced from elements of oxygen and carbon, the enthalpy of the formation of the carbon dioxide product is the amount of heat required to create it by joining the reactants. This enthalpy of formation may be negative or positive depending on whether the process is exothermic or endothermic—meaning that the standard enthalpy of the products is less or more than the standard enthalpy of the reactants. The formation of carbon dioxide (CO_2) from the original elements is an exothermic process, meaning that energy is released to the environment, and therefore its enthalpy of formation is

negative ($-393,522$ kJ/kmol); on the other hand, the formation of ozone is an endothermic process, meaning that energy is absorbed, and therefore its enthalpy of formation is positive ($142,700$ kJ/kmol). The third atom of oxygen in the ozone molecule (O_3) is the atom that is reattached to other molecules and as a result changes their chemical composition and their interaction with other molecules. Vegetation generally can absorb up to 20 percent of the ozone produced by the atmosphere, and that is the testament to their paramount role in keeping other living creatures safe. During heat waves where plants are stressed and close the small pores on their leaves (stomata) so that water evaporation is reduced, they do not absorb pollutants and ozone. The excess ozone in the atmosphere therefore creates adverse health conditions such as damaged lungs, chest pain, throat irritation, and different types of respiratory problems—an example is the loss of 460 lives in the United Kingdom during the heat waves of the summer of 2006 [139,140]. The enthalpies of formation of some aviation reactants and products are given in Table 21.

TABLE 21 Heat values of common AVGAS fuels
 (“l” and “g” stand for liquid and gas) [135,136,137,138].

Fuel	Molecular Weight m (g)	Enthalpy of Formation h_f (kJ/kmol)	Heat Value h_v (MJ/kg)	Heat Value h_v (BTU/lb)	Freezing Point T (°C)	Autoignition Temperature T (°C)	Flashpoint (°C)	Adiabatic Burn Temperature T (°C)
Octane (g) Octane (l)	114.23	-208,447 -249,952	44.427	19,104	-	280	-43	-
Isooctane	-	-223,800	44.310	19,053	-	-	-	1,477
Heptane	-	-187,900	44.566	19,163	-	-	-	-
Jet	-	-	43.71	18,800	-	-	-	-
Wide Cut	-	-	43.54	18,720	-	-	-	-
Kerosene	-	-	43.28	18,610	-	220	38-72	-
Kerosene JET A	-	-	42.8	18,404	-40	210	38	2,230 1,030 (open air)
Wide Cut JET B	-	-	42.8	18,404	-60	-	-	-
Wide Cut JP-4	-	-	42.8	18,404	-	-	-	-
Wide Cut JP-5	-	-	42.6	18,318	-	-	-	-
Wide Cut JP-8	-	-	42.8	18,404	-	-	-	-
Carbon Monoxide (g)	28.011	-110,529	-	-	-	-	-	-
Carbon Dioxide (g)	44.011	-393,522	-	-	-	-	-	-
Water (g) Water (l)	18.015	-241,827 -285,838	-	-	-	-	-	-

If the process of combustion is adiabatic, and there is neither work nor heat generated—that is, no heat transfer occurs—the products reach the adiabatic burn (flame) temperature, and only enthalpy terms remain in equation (99). This is the maximum temperature to which the reactants may be exposed, since there is no waste of heat from the initial elements or as the by-product of the incomplete combustion.

Figure 38 shows the enthalpy of formation versus the absolute temperature for water vapor. Three sets of curves are presented, with two labeled as modeled for heat capacities at constant pressure, both temperature-dependent and temperature-independent. The one from the published data in thermodynamic tables is somewhat closer to those of the temperature-dependent heat capacities at constant pressure; however, they underpredict the value of heat formation. For lower values of absolute temperature, the first and second order terms may be ignored, and therefore the predicted relationship shows a linear relationship between the enthalpy of formation and absolute temperature.

Figure 39 shows the mass ratio of the actual air to the theoretical air versus the absolute adiabatic flame temperature for the octane and octane-heptane fuels using equations (105) and (108), which will be discussed in the case studies. The temperatures for the given air mole mixture for these two fuels are very similar; therefore, the average of the two is curve-fitted and represents the adiabatic flame temperature versus the air ratios as a design criterium guidance for similar scenarios. The details of the models are presented in the case studies at the end of this section.

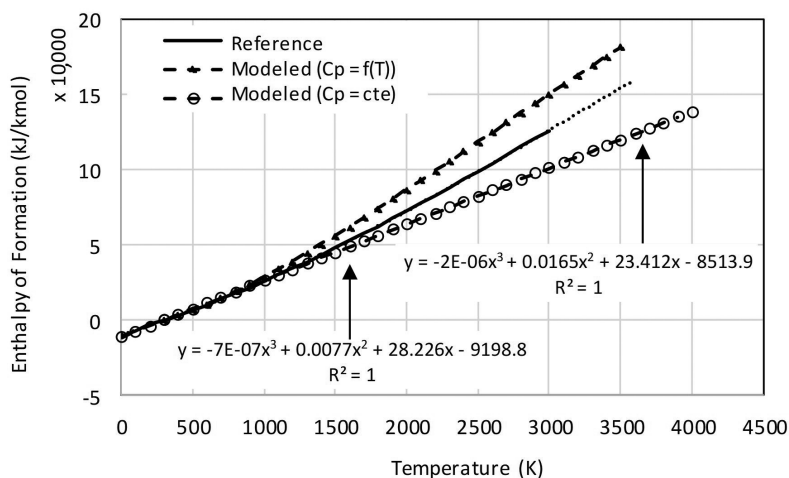


FIGURE 38 Enthalpy of formation for water vapor versus the absolute temperature for constant and temperature-dependent heat capacities.

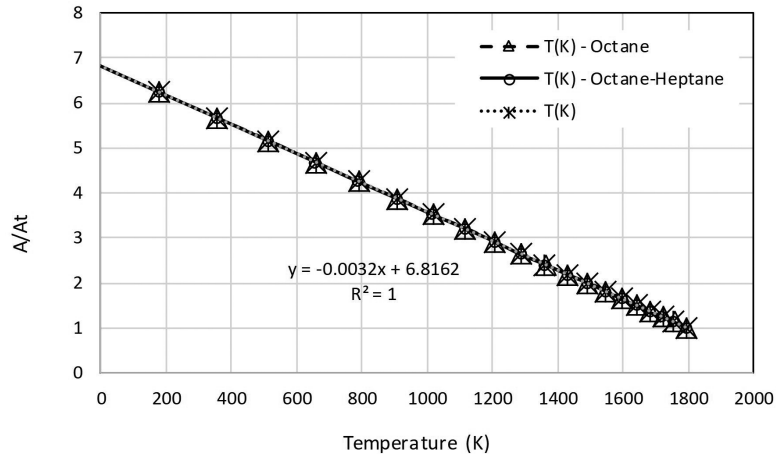


FIGURE 39 Mass ratio of the actual air to the theoretical air versus the absolute adiabatic temperature for octane and octane-heptane fuels.

6.3 Engine Performance

When reviewing the performance charts for a piston engine aircraft or any others such as turboprops and turbofans, you often encounter Brake Horsepower (BHP), Shaft Horsepower (SHP), Indicated Horsepower (IHP), Wheel Horsepower (WHP), and rated or Nominal Horsepower (NHP) as performance measures of the engine power.

Nominal horsepower was commonly used in the nineteenth century as a rule of thumb to estimate the power of steam engines (power equals force multiplied by the speed) assuming a pressure of 7 psi (48 kPa) to calculate the force. The indicated horsepower is theoretical horsepower for a frictionless engine where all the energy due to the gas expansion is transferred to work—it is the gross horsepower without the losses due to engine friction. Brake horsepower is the power measured at the crankshaft outside the engine when the engine is operating at a specific angular speed, including the friction of the piston inside the cylinders but excluding the power losses caused by the transmission. It measures the power generated by a piston engine. Shaft horsepower is the power delivered to a turbine, propeller, or an output shaft—the transmission power without the losses due to friction at the mechanical joints, links, nuts, and bolts. It measures the power generated by a turboprop and turbofan engine. Effective, true, or wheel horsepower is the actual power that causes the vehicle to accelerate after all losses have been taken into consideration. It is a performance

measure used in motor vehicles. The horsepower ratings are obtained by torque meters and dynamometers, or are estimated using diagrams.

The general form of conservation of energy is given by equation (100)—where simplifications may be made for the cases in which the mass or energy does not cross the boundaries in the case of closed or open systems.

$$Q_{c.v.} + \sum_R m_i \left[h + \frac{v^2}{2} + gz \right]_i = W_{c.v.} + \sum_P m_o \left[h + \frac{v^2}{2} + gz \right]_o \quad (100)$$

The four-stroke piston engine (Figure 35) may be described thermodynamically by an Otto cycle (Figure 40). Figure 40 shows pressure versus the volume and temperature versus the entropy for such a cycle. The areas under these curves represent the generated or consumed energy in the form of work and heat. The process consists of: (a) intake—where the mixture of air and fuel enters the cylinder at atmospheric pressure and temperature via the intake valve; (b) compression—where the fuel-mixture ratio is compressed from the Bottom-Dead-Center (BDC) position, where the volume under the piston is maximum, to the Top-Dead-Center (TDC) position, where the volume under the piston is minimum; (c) combustion/expansion—where the mixture completes the oxidization process initiated by the spark plugs; (d) expansion—where moving the piston from the TDC position to the BDC position produces mechanical work; and (e) exhaust—where the combustion products are evacuated to the atmosphere through the exhaust valve.

Both compression (1→2) and expansion (3→4) processes are isentropic processes, meaning that they happen in adiabatic conditions and are ideally reversible. They are followed by an increase and decrease in temperature and pressure of the fuel mixture inside the cylinder. The work is done on the fluid during the compression process. In ideal conditions, the difference between the upper and lower levels of temperature and pressure remains constant during the two processes; however, the upper and lower levels for the expansion process are generally higher and this is the cycle which generates useful work, which explains the overall positive work done by the fluid to make the engine work. The ratio of the largest volume in the combustion chamber to that of the smallest one is the compression ratio ($r = V_1/V_2$ in Figure 40). A higher compression ratio gives better performance of the engine. Engine efficiency (η) is given by equation (101), where γ is the ratio of the heat capacity at a constant pressure to that of the constant volume. Note that hatched areas enclosed within the thermodynamic cycles in Figure 40 equal the work and heat. Combustion is happening with every other stroke (e.g., 1-3-4-2) (Figure 35), where the

numbers represent the cylinder in which combustion occurs—combustion happens in cylinder 2 in this example.

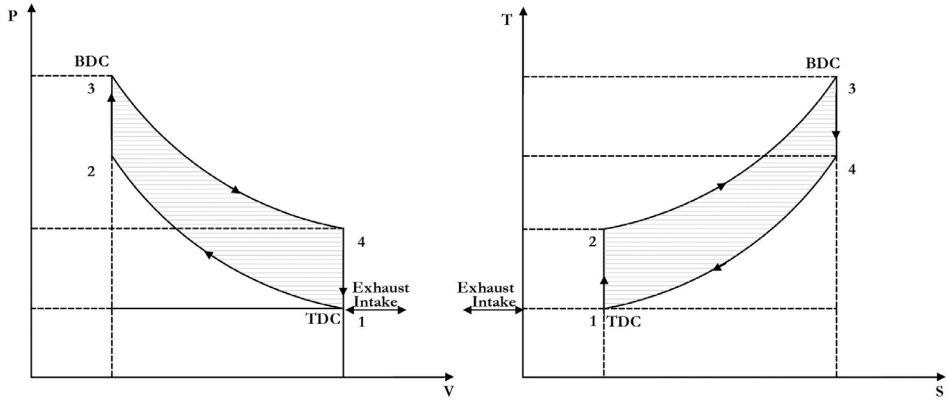


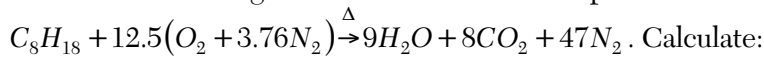
FIGURE 40 PV and TS diagrams for a four-stroke engine Otto cycle.

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^{\gamma-1}} \quad (101)$$

6.4 Examples

6.4.1 Case Study – Octane Fuel Analysis

A chemical reaction is presented by equation (89), where liquid octane is used in a complete combustion process with theoretical air (enthalpy of formation data along with the heat values are presented in Table 21)—



(a) adiabatic flame temperature, and (b) analyze the combustion process.

To calculate the adiabatic flame temperature, equation (102) is applicable, which is a simplified form of equations (99) and (100). This is an adiabatic process happening at standard combustion conditions (i.e., 25 °C, 101.325 kPa). The enthalpy of ideal gases as a function of the temperature is available in the literature. In this situation, the total enthalpy of formation of the products is the same as that of the reactants—equation (102)—meaning that the difference is zero when maximum flame temperature is reached. Employing the enthalpy for different temperatures results in the balance of energy expressed by equations (103) and (104).

$$\sum_R n_i [\bar{h}_f^\circ + \Delta \bar{h}]_i = \sum_P n_o [\bar{h}_f^\circ + \Delta \bar{h}]_o \quad (102)$$

$$\sum_R n_i [\bar{h}_f^\circ + \Delta \bar{h}]_i = 1(-249,952 + 0) + 12.5(0 + 0) + 47(0 + 0) \quad (103)$$

$$= -249,952 \frac{\text{kJ}}{\text{kmol}}$$

$$\sum_P n_o [\bar{h}_f^\circ + \Delta \bar{h}]_o = 9(-241,827 + \Delta \bar{h}_{H_2O}) + 8(-393,522 + \Delta \bar{h}_{CO_2}) +$$

$$47(0 + \Delta \bar{h}_{N_2}) + 0(0 + \Delta \bar{h}_{O_2}) = -5,324,619 + 9\Delta \bar{h}_{H_2O}$$

$$+ 8\Delta \bar{h}_{CO_2} + 47\Delta \bar{h}_{N_2} + 0\Delta \bar{h}_{O_2} \frac{\text{kJ}}{\text{kmol}} \quad (104)$$

Figure 41 shows temperature versus the total heat release in the chemical reaction presented by equation (89). Based on the previous discussion, the total heat release is to be zero ($x = 0$) for the adiabatic flame temperature to be calculated, meaning that the heat release associated with the reactants—equation (103) and equation (104)—should be equal. Using a trial and error technique, an adiabatic flame temperature of 2,412.1 K is found for theoretical air for the 12.5 moles of air mixture available for the combustion process. This is the maximum temperature that the engine may be exposed to; therefore, it can be used as a design criterion when selecting the material for the piston and cylinder in addition to making engine performance calculations. For 12 percent excess air (i.e., 14 moles of air mixture), an adiabatic flame temperature of 2,225.7 K is obtained.

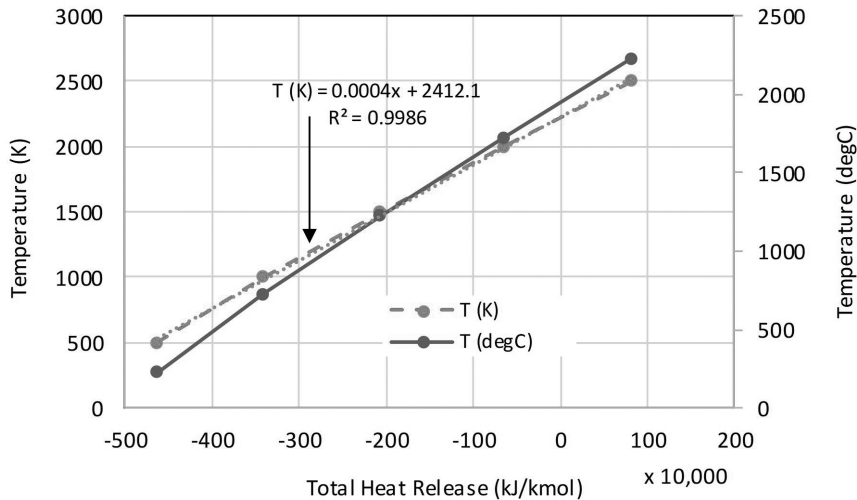


FIGURE 41 Absolute adiabatic flame temperature versus the total heat release for equation (89) for octane.

With the increase of excess air, the adiabatic flame temperature decreases as shown in Figure 42 and Figure 43—meaning the process does not require exerting itself for complete combustion to occur. For actual air that is four times the amount of theoretical air (A/At), an adiabatic flame temperature of 945.4 K is obtained. This means that if the fuel mixture is to be enriched in a piston engine, the fuel consumption will increase, and if the amount of air molecules decreases—for example, in higher

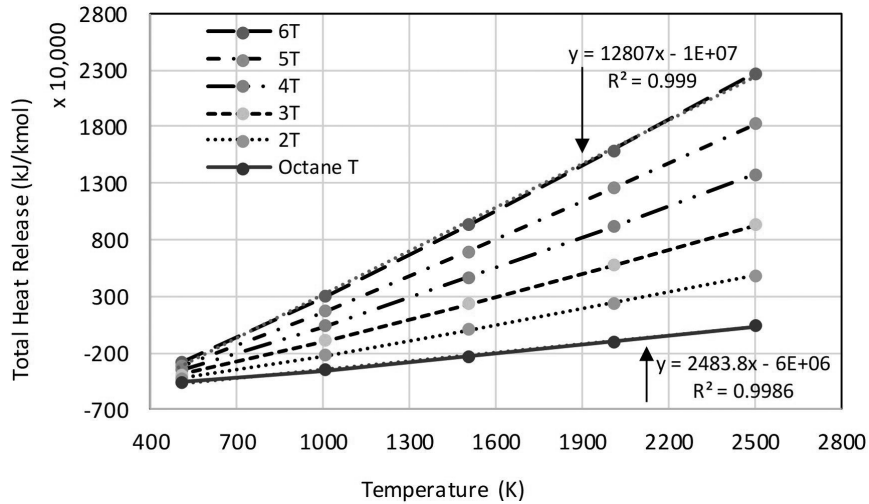


FIGURE 42 Total heat release versus the absolute adiabatic flame temperature for octane-heptane for different mass ratios of the actual air to the theoretical air.

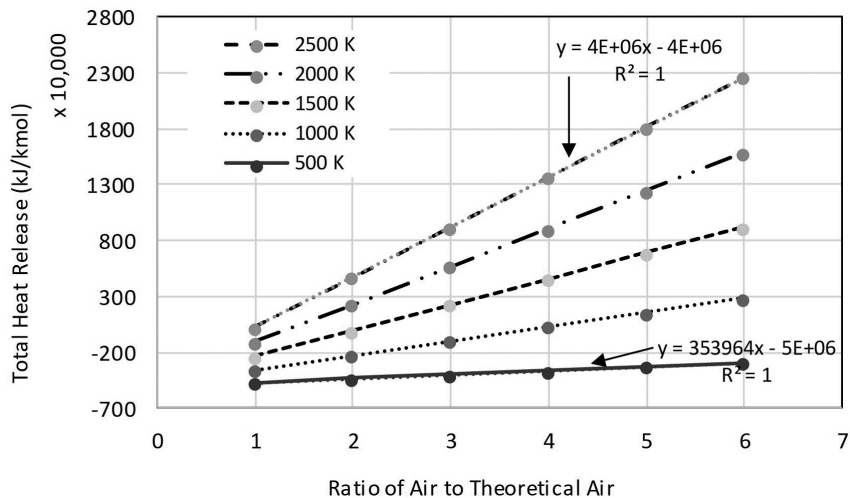


FIGURE 43 Total heat release versus the mass ratio of the actual air to the theoretical air for octane-heptane for different absolute adiabatic flame temperatures.

altitudes—the fuel consumption may be decreased by leaning the mixture in a piston engine. It is possible that a concave diagram curving downward is obtained, provided that more data points are added to the diagram of the mass ratio of the actual air to the theoretical air—Figure 44 and Figure 45. Nevertheless, it is possible to predict the adiabatic flame temperature trend by interpolating the three sets of data given a valid trend based on the linear relationship (Figure 44) or the power relationship (Figure 45) to present the mass ratio of the actual air to the theoretical air versus the heat release.

It is possible to present the total heat release as a function of the combustion temperature or the mass ratio of the actual air to the theoretical air (A/A_t) (Figure 44 and Figure 45). The individual models depicting the influence of each of these single variables on the heat release during combustion have been presented inside the diagrams for the upper and lower limits of either temperature or the mass ratio of the actual air to the theoretical air. Performing regression analysis, you may obtain a relationship that considers both variables as independent components into the expression for heat release of combustion presented by equation (105)—where T is the absolute temperature (K) and $Q_{c.v.}$ is the heat release during combustion (kJ/kmol).

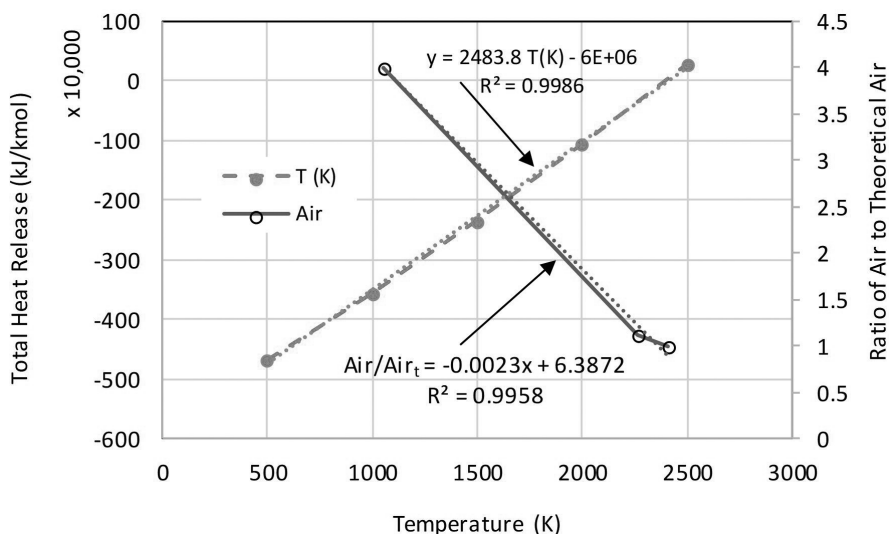


FIGURE 44 Total heat release and mass ratio of the actual air to the theoretical air versus the absolute adiabatic flame temperature for octane, linear curve-fit.

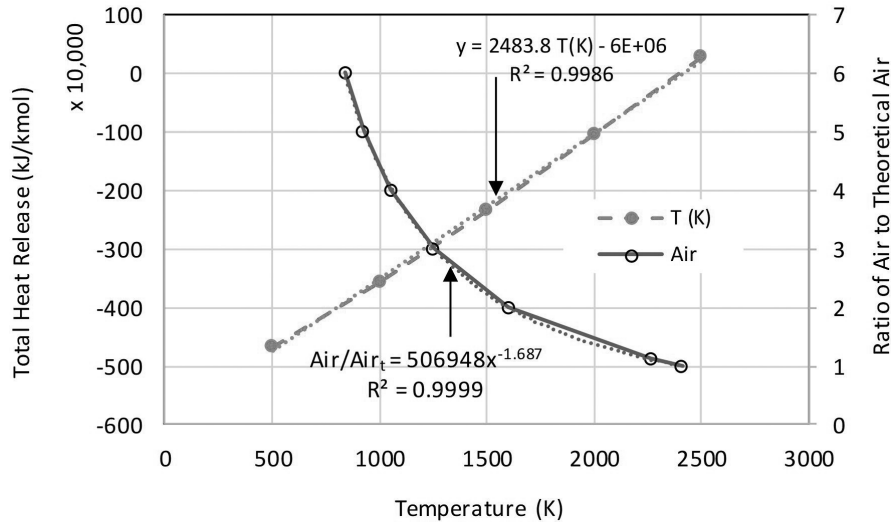


FIGURE 45 Total heat release and mass ratio of the actual air to the theoretical air versus the absolute adiabatic flame temperature for octane, power curve-fit.

Table 22 and Figure 46 present the analysis summary and normal probability plot for the regression analysis. It is seen that the relationship between the heat releases of combustion and air ratios as well as absolute temperatures are statistically significant and may be expressed with a confidence level above 87 percent. At adiabatic conditions, no heat enters or leaves the control volume, so term $Q_{c.v.}$ (kJ/kmol) is ignored and $T(K)$ would approximately be the adiabatic flame temperature, which varies with the mass ratio of the actual air to the theoretical air.

$$Q_{c.v.} = -15909509 + 2333822 \frac{A}{At} + 7554T \quad (105)$$

Heat release as a function of the temperature presented by equation (105) in kJ/kmol may be multiplied by the molecular mass in kmol/kg of the fuel to obtain the energy per mass of the fuel (kJ/kg)—also known as the specific energy. This energy then may be employed per duration of operation to represent the specific energy per time that is also known as horsepower per unit mass of the fuel (specific horsepower) in combination with the engine efficiency.

TABLE 22 Model summary for the regression analysis for the heat release combustion model for octane as a function of the mass ratio of the actual air to the theoretical air and adiabatic flame temperature presented by equation (105).

S	R-sq	R-sq(adj)	R-sq(pred)
2604100	87.92%	87.02%	83.33%

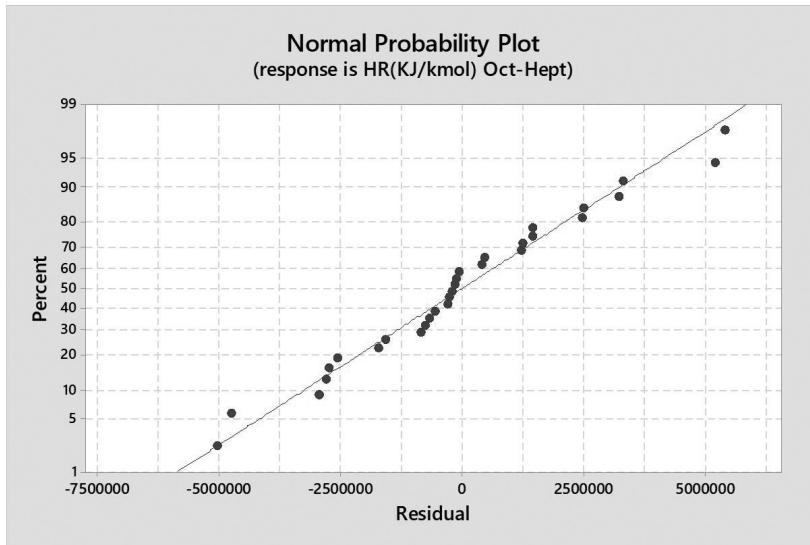


FIGURE 46 Normal probability plot for the regression analysis for the heat release combustion model for octane as a function of the mass ratio of the actual air to the theoretical air and adiabatic flame temperature presented by equation (105).

6.4.2 Case Study – Octane-Heptane Fuel Analysis

A chemical reaction is presented by equation (93), where liquid octane-heptane with an octane number of 90 is used in a complete combustion process with 12 percent excess air— $0.9C_8H_{18} + 0.1C_7H_{16} + 13.83(O_2 + 3.76N_2) \xrightarrow{\Delta} 8.9H_2O + 7.9CO_2 + 52.02N_2 + 1.48O_2$. Enthalpy of formation data along with the heat values are presented in Table 21. Calculate:

(a) adiabatic flame temperature, and (b) analyze the combustion process.

This example is somewhat similar to the previous one; however, there are multiple fuel types among the reactants. The methodology presented herein may be applied to any number of fuel reactants—either the combustion process is complete or incomplete. To calculate the adiabatic flame temperature, equation (106) is applicable. This is an adiabatic process happening at standard combustion conditions (i.e., 25 °C, 101.325 kPa). The enthalpy of ideal gases as a function of the temperature is available in the literature. In this situation, the total enthalpy of formation of the products is to be the same as that of the reactants—equation (102)—meaning that this difference is zero when the maximum temperature is reached. Employing the enthalpy for different temperatures results in the balance of energy expressed by equations (106) and (107).

$$\sum_R n_i [\bar{h}_f^\circ + \Delta \bar{h}]_i = 0.9(-249,952 + 0)_{C_8H_{18}} + 0.1(-187,900 + 0)_{C_7H_{16}} + 13.83(0 + 0)_{O_2} + 52.01(0 + 0)_{N_2} = -243,747 \frac{\text{kJ}}{\text{kmol}} \quad (106)$$

$$\sum_P n_o [\bar{h}_f^\circ + \Delta \bar{h}]_o = 8.9(-241,827 + \Delta \bar{h})_{H_2O} + 7.9(-393,522 + \Delta \bar{h})_{CO_2} + 52.02(0 + \Delta \bar{h})_{N_2} + 1.48(0 + \Delta \bar{h})_{O_2} = -5,261,084 + 8.9\Delta \bar{h}_{H_2O} + 7.9\Delta \bar{h}_{CO_2} + 52.02\Delta \bar{h}_{N_2} + 1.48\Delta \bar{h}_{O_2} \frac{\text{kJ}}{\text{kmol}} \quad (107)$$

Figure 47 shows temperature versus the total heat release expressed in the chemical reaction presented by equation (93). Based on the previous discussion, the total heat release is to be zero ($x = 0$) for the adiabatic flame temperature to be calculated—meaning that equations (106) and (107) are to be equal. Using a trial and error technique, an adiabatic flame temperature of 2,413.4 K is found for theoretical air—12.35 moles of air mixture available for the complete combustion process. This is the maximum temperature to which the engine can be exposed, so it may be used as a design criterion when selecting materials for the piston and cylinder in addition to engine performance calculations. Figure 47 also compares the results from using only octane versus the octane-heptane, and it is seen

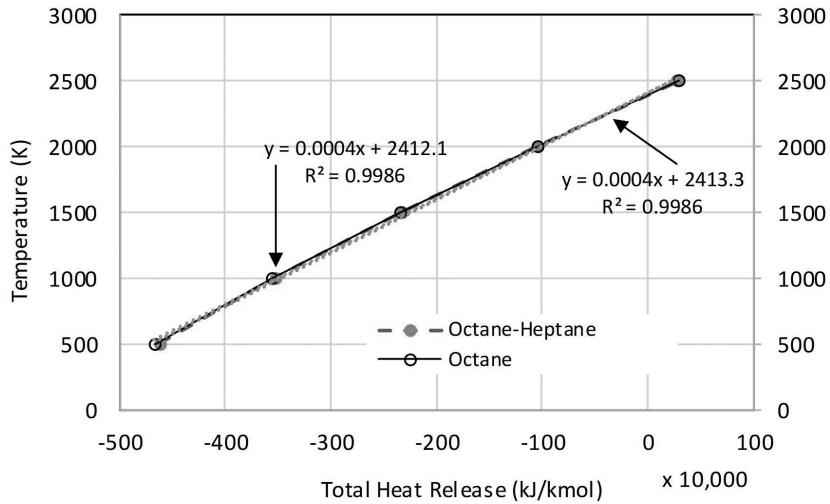


FIGURE 47 Absolute adiabatic flame temperature versus the total heat release for equation (93) for octane-heptane (octane number of 90).

that their trends closely follow each other for an octane number of 90; that said, the octane-heptane case is slightly underpredicting the adiabatic flame temperature as a function of the total heat release compared to its counterpart for the octane-only case. The results converge at higher values for excess air. For 12 percent excess air (i.e., 14 moles of air mixture), an adiabatic flame temperature of 2,221.4 K is obtained. With the increase of excess air, the adiabatic flame temperature decreases as shown in Figure 48 and Figure 49—meaning the process does not require exerting itself for complete combustion to occur. For actual air that is four times greater than the theoretical value, an adiabatic flame temperature of 945.8 K is obtained.

It is possible to present the total heat release as a function of the combustion temperature or the mass ratio of the actual air to the theoretical air (Figure 48, Figure 49, and Figure 50). The individual models depicting the influence of each of these single variables on the heat release during combustion has been presented inside the diagrams for upper and lower limits of either temperature or the mass ratio of the actual air to the theoretical air. Performing regression analysis, you may obtain a relationship that considers both variables as independent components into the expression for heat release of combustion presented by equation (108)—where T is absolute temperature (K) and $Q_{c.v.}$ is heat release during combustion (kJ/kmol). Heat release as a function of the temperature presented by equation (108) in kJ/kmol may be multiplied by the molecular mass in kmol/kg of the fuel to obtain the energy per mass of the fuel (kJ/kg)—also

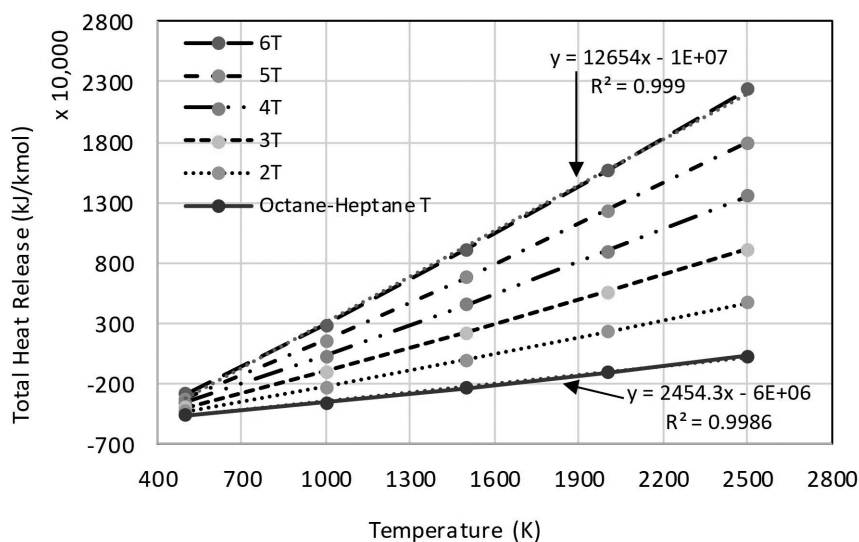


FIGURE 48 Total heat release versus the absolute adiabatic flame temperature for octane-heptane for different mass ratios of the actual air to the theoretical air.

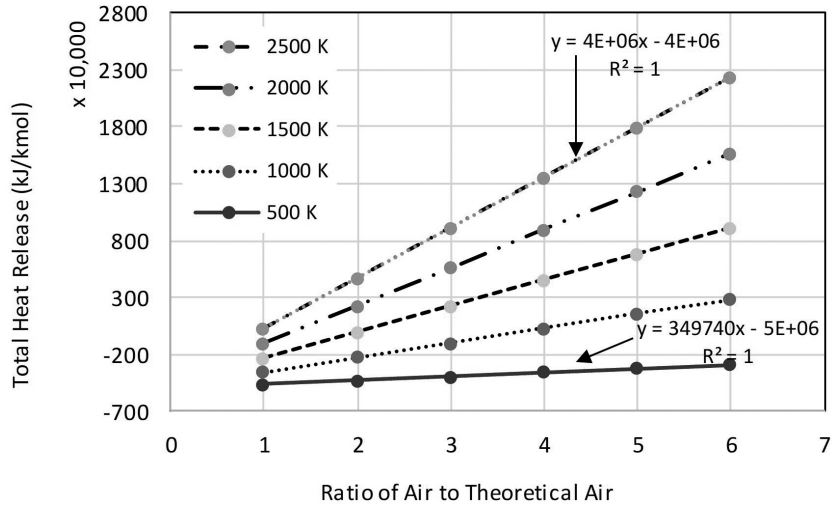


FIGURE 49 Total heat release versus the mass ratio of the actual air to the theoretical air for octane-heptane for different absolute adiabatic flame temperatures.

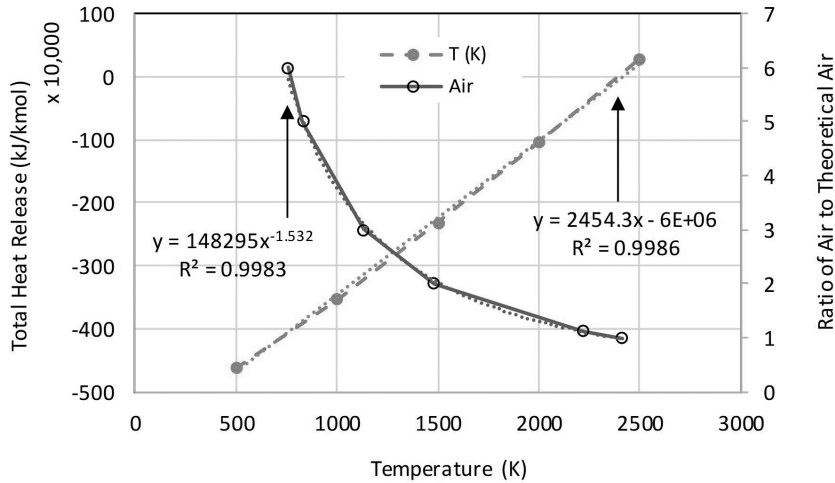


FIGURE 50 Total heat release and mass ratio of the actual air to the theoretical air versus the absolute adiabatic flame temperature for octane-heptane (octane number of 90).

known as the specific energy. This energy then can be employed to find the specific energy per duration of the operation to represent the specific horsepower, considering the engine efficiency.

Table 23 and Figure 51 present the analysis summary and normal probability plot for the regression analysis. It is seen that the relationship between the heat releases of combustion and the mass ratios as well as the absolute temperatures are statistically significant and may be expressed

with a confidence level above 87 percent. As stated earlier, at adiabatic conditions, term $Q_{c.v.}$ (kJ/kmol) is ignored and T (K) would approximately be the adiabatic flame temperature, which depends on the mass ratio of the actual air to the theoretical air (Figure 50).

$$Q_{c.v.} = -16098341 + 2362010 \frac{A}{At} + 7645T \quad (108)$$

TABLE 23 Model summary for the regression analysis for the heat release combustion model for octane-heptane as a function of the mass ratio of the actual air to the theoretical air and adiabatic flame temperature presented by equation (108).

S	R-sq	R-sq(adj)	R-sq(pred)
2635551	87.92%	87.02%	83.33%

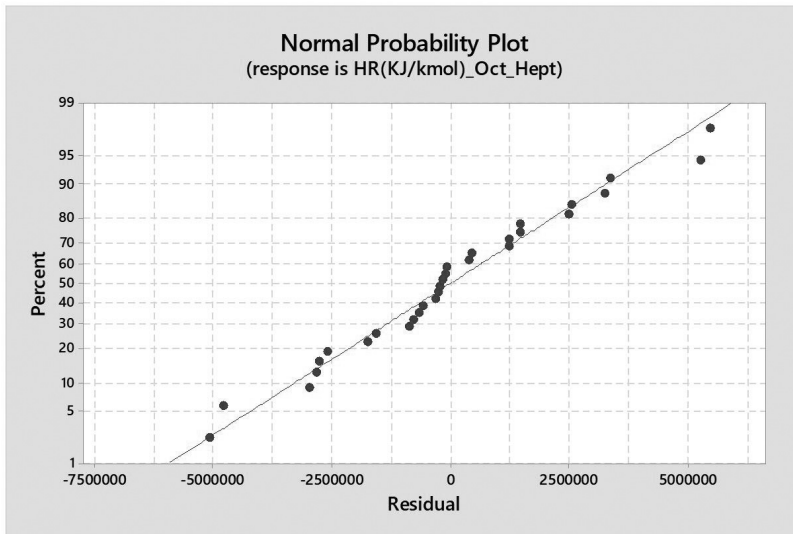


FIGURE 51 Normal probability plot for regression analysis for the heat release combustion model for octane-heptane as a function of the mass ratio of the actual air to the theoretical air and adiabatic flame temperature presented by equation (108).

6.4.3 Case Study – Exhaust Gas Temperature

Assume the exhaust gas temperature for the case studies presented for octane and octane-heptane examples when 112 percent excess air is employed at 1,500 K (1,227 °C). At this temperature, based on equations (105) and (108), estimate the heat release.

$$\begin{aligned} Q_{c.v.Oct} &= -15,909,509 + 23,338,22 \times 1.12 + 7,554 \times 1,500 \quad (109) \\ &= -1,964.6284 \text{ MJ/kmol} \end{aligned}$$

$$\begin{aligned} Q_{c.v.Oct-Hept} &= -16,098,341 + 23,620,10 \times 1.12 + 7,645 \times 1,500 \quad (110) \\ &= -1,985.3898 \text{ MJ/kmol} \end{aligned}$$

Using the individual linear relationship obtained for the heat release where only one independent variable is present, the values of $-2,065.546$ MJ/kmol and $-2,044.0156$ MJ/kmol are obtained for the combustion of octane and octane-heptane, respectively, which means these relationships are overpredicting the estimated values presented in the equations by 3.9 percent (Figure 52, Figure 53, and Figure 54).

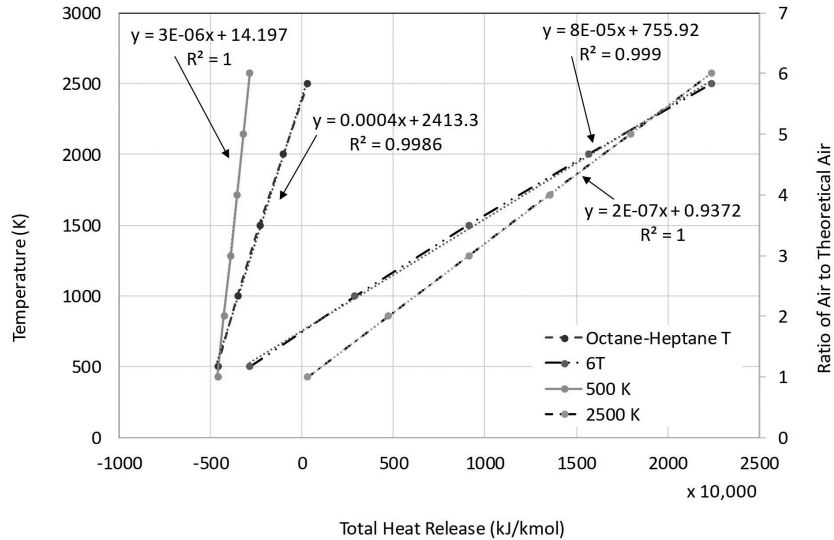


FIGURE 52 Absolute adiabatic flame temperature and mass ratio of the actual air to the theoretical air versus the total heat release for octane combustion.

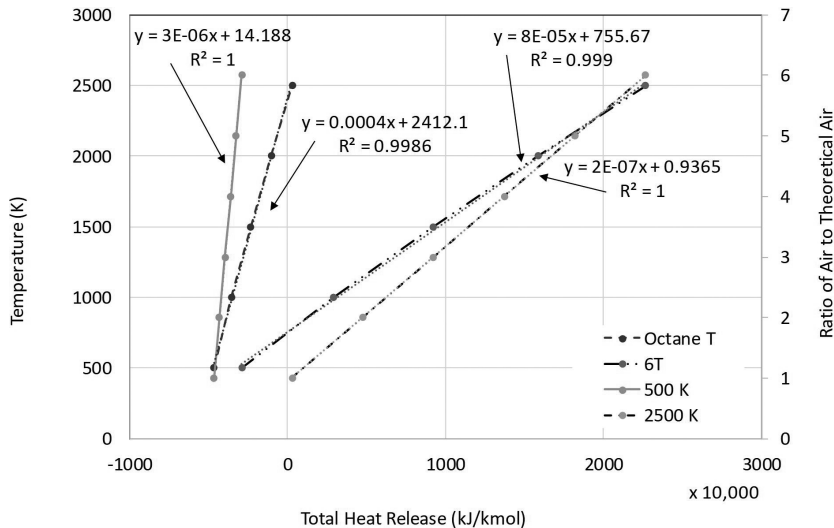


FIGURE 53 Absolute adiabatic flame temperature and mass ratio of the actual air to the theoretical air versus the total heat release for octane-heptane combustion.

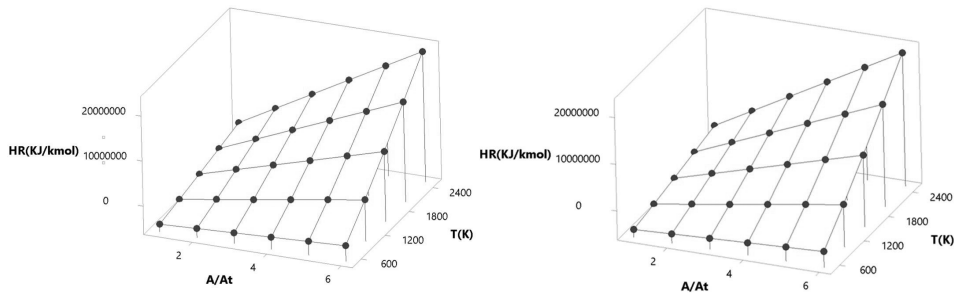


FIGURE 54 Total heat release versus the absolute adiabatic flame temperature and the mass ratio of the actual air to the theoretical air for: (a) octane combustion (left), and (b) octane-heptane combustion (right).

6.4.4 Case Study – Piston Engine

For Figure 55, Figure 56, and Figure 57, which present four-stroke engine Otto cycle PTV diagrams for one mole of air mixture (Figure 55) and 4.76 moles of air mixture (Figure 56), as well as the engine's sensitivity to the combination of mole numbers (Figure 57), provide a complete thermodynamic analysis of the cycles including the temperature and pressure at different stages of the cycle. Use data presented in Table 24 for the required input data.

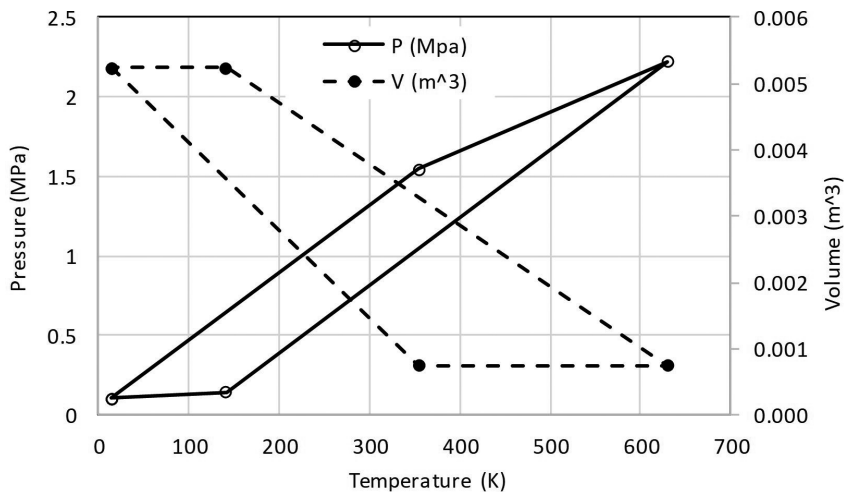


FIGURE 55 Four-stroke engine Otto cycle PTV diagram for one mole of air mixture.

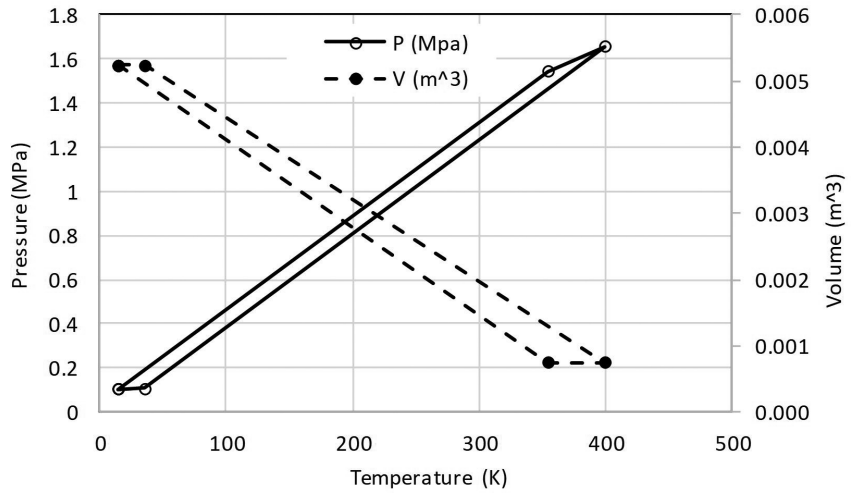


FIGURE 56 Four-stroke engine Otto cycle PTV diagram for 4.76 moles of air mixture.

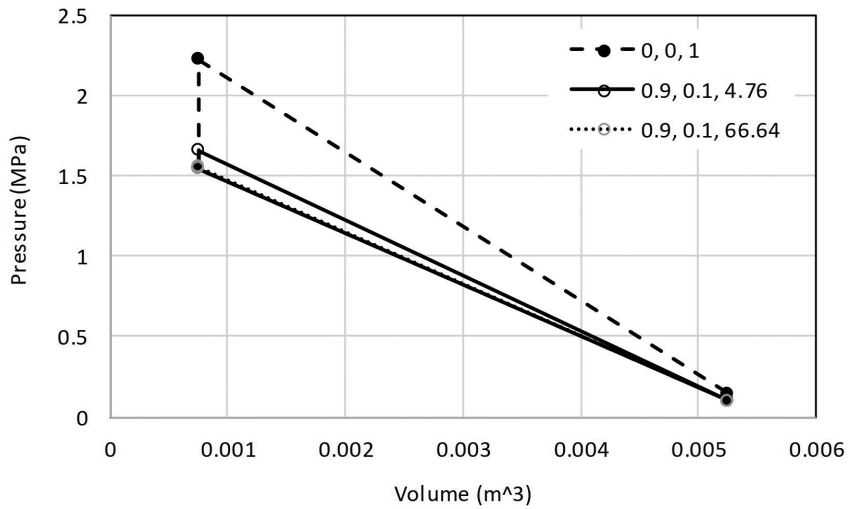


FIGURE 57 Sensitivity of four-stroke engine Otto cycle PV diagram to the combination of mole numbers of air mixture.

TABLE 24 Calculations for the data presented in Figure 55, Figure 56, and Figure 57.

R_universal	8,314.46	J/kmol.K						
Air Mass	28.84	kg/kmol						
$\gamma = C_p / C_v$	1.40							
Octane Number	90.00						Mass fraction	Mol Fraction
Fuel Mass (C8H18)	114.00	gr	0.90	Mol	102.60	gr	0.41	0.16
Fuel Mass (C7H16)	100.00	gr	0.10	Mol	10.00	gr	0.04	0.02
Air Mass (O2 + 3.76 N2) / 4.76	28.84	gr	4.76	Mol	137.28	gr	0.55	0.83
Total Fuel - Air Mass	249.88	gr	5.76	Mol			Total Fraction	1.00
Air Cv	720.73	J/kg.K	395.96	J/kg.K				
Fuel Cv (C8H18)	254.70	J/kg.K	104.58	J/kg.K				
Fuel Cv (C7H16)	224.64	J/kg.K	8.99	J/kg.K				
Total Cv			509.53	J/kg.K				
Air R	288.29	J/kg.K	52.50	J/kg.K				
Fuel R (C8H18)	72.93	J/kg.K	277.64	J/kg.K				
Fuel R (C7H16)	37.01	J/kg.K	5,613.30	J/kg.K				
Total R			5,943.44	J/kg.K	33.27	J/kg.K		
Cp - Cv	33.27	J/kg.K						
P1	101,325.00	N/m^2	101.33	kN/m^2	0.10	Mpa		
V1	320.00	in^3	0.00	m^3	0.01	m^3		
R	33.27	J/kg.K						
T1	15.00	C	288.15	K				
v	2,200.00	RPM	1,100.00	Workstroke	18.33	RPS		
M = PV / RT	0.06	kg						
r = V1 / V2	7.00							
$\eta = 1 - (T1 / T2) = 1 - 1 / (r^\gamma)$	2.18	0.45915655	0.54	54.08%				
P2 = P1 (r^ γ)	1,544,734.58	N/m^2	1,544.73	kN/m^2	1.54	Mpa		
P2 / P1	15.25							
T2 = T1 (r^ $(\gamma-1)$)	627.56	K	354.41	degC				
V2 = V1 / r	0.00	m^3						
V3 = V2	0.00	m^3						
MR	1.84	J/K						
T3 / P3 = V3 / (MR)	0.00	m^3.J/K						
V4 = V1	0.01	m^3						
T4 / P4 = V4 / (MR)	0.00	m^3.J/K						
T3 / T4 = r^ $(\gamma-1)$	2.18							
HP	150.00	W						
Cylinders	4.00							
W = HP / Cylinders	37.50	W						
Q = W / η	69.34	W						
E = Q (Workstroke)	1,271.16	J	1.27	kJ				
T3 - T2 = E / (MCv)	45.01	K						
T3 = (T3 - T2) + T2	672.57	K	399.42	degC				
T4 = T3 / (T3 / T4)	308.82	K	35.67	degC				
P4 = T4 / (T4 / P4)	108,592.22	N/m^2	108.59	kN/m^2	0.11	MPa		
P3 = T3 / (T3 / P3)	1,655,525.79	N/m^2	1,655.53	kN/m^2	1.66	Mpa		

UNMANNED FLYING OBJECTS

The objects that fall into the category of Unmanned Aerial Vehicles (UAV), sometimes called drones, are aircraft without a human pilot on board—perhaps only a smiling picture of one. If this vehicle is not completely autonomous, a ground-based controller and a communication system will also be required to operate it. These vehicles are deployed on missions that are either unsuitable or dangerous for human beings. Applications such as observation of combat zones, populations of rodents in a tight space, troops of quarreling monkeys, or dueling pilots are among these scenarios. These vehicles find use in applications in science and technology, military, recreation, agriculture, surveillance, photography, racing, and product deliveries. All of these drone applications have become popular over the recent years due to advances in control electronics, batteries, electric motors, and video technology. Unmanned flying objects (satellites) may also circle the heavenly bodies.

7.1 UAVs Through the Ages

The earliest mention of an unmanned flying object dates back to 1849, where balloons were used for combat. There were attempts to use balloons to carry bombs that were launched by Austrian forces from the sea and land; these experiments were only partially successful due to the wind blowing the balloons off target or even back to the Austrian side. The development continued in the 1900s and during the World War I (WWI), where the first aerial torpedo was patented in 1912 by Admiral Bradley A. Fiske—an American navy officer and technical innovator with over a

hundred electrical and mechanical inventions for both civilian and military applications. This method of combat was approved in 1917 by the United States Congress during WWI [141].

The concept of a powered UAV was first expressed in 1915 by Nikola Tesla—a Serbian-American inventor, engineer, and physicist, most famous for his role in designing alternating current electrical devices. In 1916, British engineer Archibald Montgomery Low, also known as the father of radio guidance systems, attempted (with very limited success) to implement the concept of the powered UAV. The first radio-controlled, remote-piloted UAV was made and flown by British actor and aviator Reginald Denny in 1935. Germany built different UAVs during WWII (e.g., the V-1 flying bomb, also known as the buzz bomb, maybug, cherrystone, and doodlebug). Jet engines were employed in UAVs after WWII in the Australian Government Factories (GAF) Jindivik, a target drone made by GAF in 1952; and in the Ryan Firebee, a target drone made in 1951 by Ryan Aeronautical Company, which merged with Teledyne in 1999. The Beechcraft introduced a target drone, the MQM-61 Cardinal, also known as Model 1001, in 1955 [141].

Drones were used extensively during the Vietnam War [142,143]. This initiative was part of the response to coping with the unknown territory and extensive loss of lives as part of a highly classified program called *Red Wagon*. Among the drones employed during this war were: (a) Lockheed D-21—a supersonic reconnaissance drone powered by a modified version of a ramjet engine constructed by the Marquardt Corporation for the Boeing CIM-10 Bomarc long-range surface-to-air missile; (b) Ryan AQM-91 Firefly, a target drone powered by a General Electric YJ97-GE-3 turbojet generating 17.8 kN (4,000 lb) of thrust, with the engine exhaust mixed with cool air to reduce its infrared signature; and (c) the Ryan 147 Lightning Bug, a jet-powered target reconnaissance drone whose Model T variant was equipped with a Teledyne CAE J100-CA-100 engine capable of providing 12.5 kN (2,800 lb) of thrust, which allowed the drone to operate at altitudes up to 75,000 feet.

During the War of Attrition (1967–1970) between Israel and Egypt, the use of drones was expanded by the addition of reconnaissance cameras. During the 1973 Yom Kippur War, drones were used by Israel as prey to distract and destroy (waste) the enemy's expensive anti-aircraft missiles. This conflict also saw the first deployment of drones with real-time surveillance capability. The use of drones in combat flight simulations for the testing of agility and post-stall-controlled flight steering was reported in Israel in 1987.

Deployment of drones with no loss of life to the flight crew was further expanded during the second Gulf War in 1991. The drones, which first were mainly used as surveillance aircraft, were later complemented by the armament-carrying types, such as the General Atomics MQ-1 Predator. This drone was powered by an internal combustion Rotax engine driving a propeller and could fly up to 740 km (400 NM) to about 852 km (460 NM) to a target, hover above it for up to 14 hrs, and then return to the base. The initial versions were for reconnaissance only, while later versions were armed with AGM-114 Hellfire air-to-surface missiles. Civil UAV Applications and Economic Effectivity of Potential Configuration Solutions (CAPECON) was a European Union project (2002–2005) to develop and expand the use of UAVs for civilian applications. Since 2013, the use of UAVs has expanded to over fifty countries [144], with applications including target and decoy, reconnaissance, combat, logistics, research, and development.

7.2 Satellites

In a baseball game, a pitcher throws the ball to a catcher. Depending on the pitcher's level of competency and the batter's skill, the batter may hit or miss the ball. The ultimate goal for the batter is to hit the ball and run the four bases counterclockwise to score runs. Assume the batter makes good contact with the ball, and the ball takes off and lands on the ground due to gravitational acceleration ($g = 9.8 \text{ m/s}^2$), finding a gap in the outfield. The batter may choose to stay on first base or go to second or third base, or even to home plate. But no matter how hard the ball was hit, even in record-breaking cases, the ball eventually came back to the Earth, as shocking as it seems, fulfilling the prophecy of *what goes up must come down* according to Newton's third law and the band Blood, Sweat, and Tears [145,146,147,148,149].

The previous example shows what happens to objects launched with insufficient velocity to escape the Earth's pull. With increasing launch velocity of an object, the orbit, also known as the trajectory, approaches an ellipse based on Kepler's first law of planetary motion, with the Earth being located at one of the foci. Kepler's second law of planetary motion states that the area of the orbit's segment for the object orbiting the heavenly body is the same for equal time intervals, as in Figure 58. Each segment of the elliptical orbit (a slice of the elliptical pie) (Figure 58) corresponds to 1/16 of the orbit's period (let us say around an imaginary planet called *DRLSM* [150]). Assuming that the period of this heavenly body is 301 days, each slice of the ellipse represents 18.8 Earth days (451.5 hrs, 27,090 min, or 1,625,400 s). If the angles shown in Figure 58 are incremented in small

intervals such that the increased area at each interval is equal, and this angle is plotted versus the cumulative area, the curve shown in Figure 59 is obtained. Taking a derivative of this angle with respect to the area, one gets the curve in Figure 59. Interestingly, a Gaussian curve with a confidence

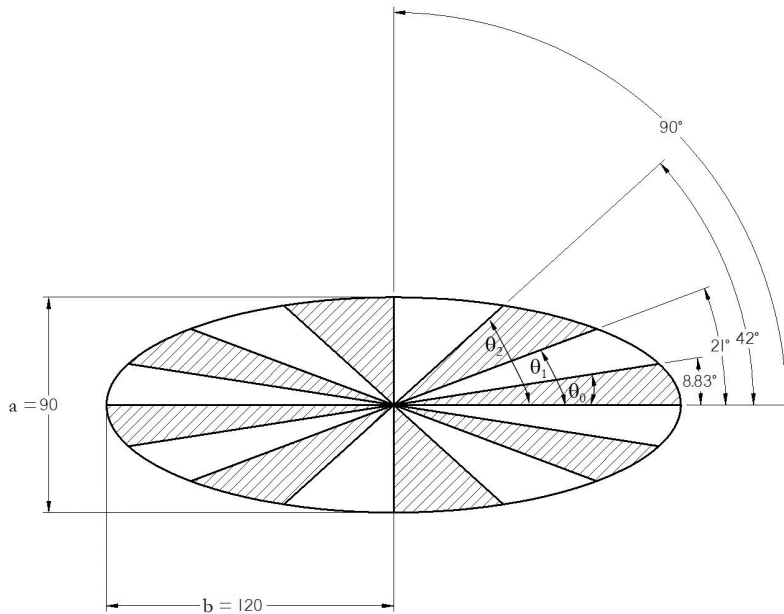


FIGURE 58 Kepler's second law of planetary motion, for an imaginary planet *DRLSM*-dimensions in AU (drawings created using Solid Edge CAD tool).

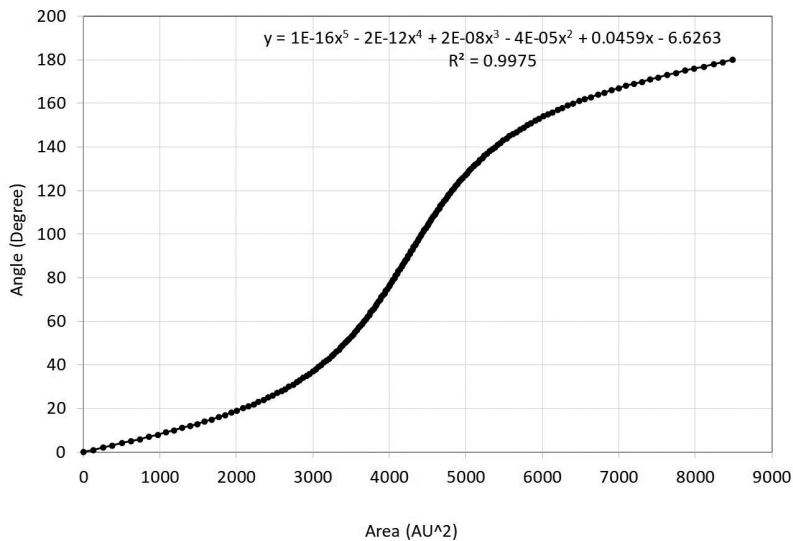


FIGURE 59 Angle versus the area for an ellipse trajectory presented to orbit *DRLSM*.

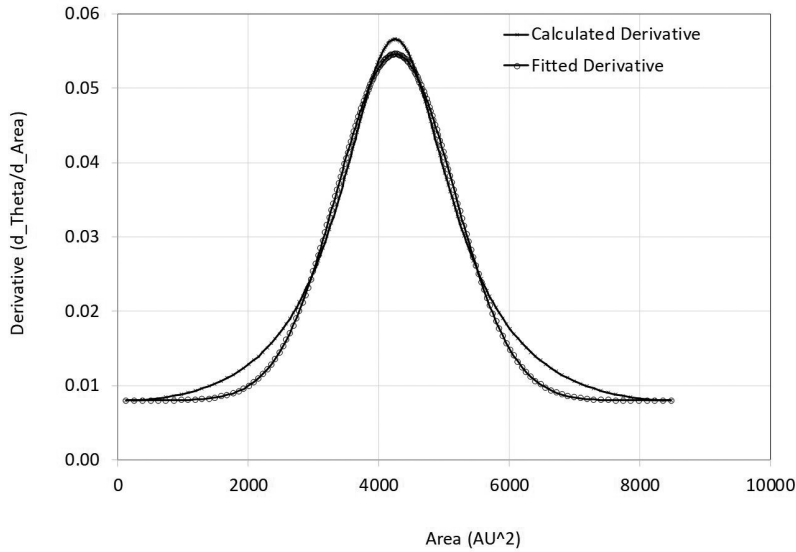


FIGURE 60 Derivative of ellipse angle with respect to the area versus the area.

level above 99 percent may be fitted to this curve, with the fit presented in Figure 60.

Equation (111) and equation (112), where $u = (A' - 4255.772)/1268.806$, are the integral of this Gaussian curve, which should yield a function similar to the one presented in the previous figure.

$$\theta = \int_0^{A'} \left[0.008 + 0.04664301 \times e^{-u^2} \right] dA \quad (111)$$

$$\theta = 0.008 A' + 52.44 \operatorname{erf}(A') \quad (112)$$

For the imaginary planet (*DRLSM*) shown in Figure 58, the relationship between the area covered (*Area*) at 1/16 of the orbit time and the associated angle (θ) is presented by equation (113). Regression analysis has been employed to model this behavior, and the model is presented with a confidence level above 93 percent.

$$\theta = -31.9 + 0.028758 \operatorname{Area} \quad (113)$$

Kepler's third law suggests that the ratio of the square of the heavenly body (planet) period (T^2) to the cube of the distance of the orbiting object from the center of the heavenly body— $(R + d)^3$ —is constant and equal for all heavenly bodies: $T^2/(R + d)^3 = cte$. Using this formula, you are able to find either the orbit distance from the surface of the planet (d), the radius of the heavenly body (R), or the period of the heavenly body (T) based on the known data for the Earth. Inserting the radius of the Earth in the previous

relation and assuming that the radius of the orbit may be approximated as the radius of the Earth, the constant universal value of 2.87723×10^{-11} is obtained—assuming 86,400 s as the period of the Earth’s rotation about its axis (also known as a civil day) and the Earth diameter of 12,756 km.

Generally speaking, if you throw a ball, you can expect it to return to the Earth. The question is whether there is a way for the ball to spin around the Earth instead of getting back to it. To overcome the attraction of the Earth—to be able to escape from it—the ball needs to reach the *escape velocity*. In this situation the conservation of energy, which states that the total energy consisting of the kinetic and potential energies remains constant, may be useful. The total initial energy consists of the kinetic ($0.5m_o v^2$) energy and potential energy ($m_o g_r r$), where (m_o) is the mass of the object, r is the distance between the centers of the heavenly body and the object, v is the launch velocity of the object, and g_r is the gravity of the planet at distance r . When the object is launched, it is to overcome the resistance, and it will eventually get to zero velocity as the minimal velocity required to escape gravity. Additionally, the gravity reduces to near zero for the extremely large distance from the Earth, since it is inversely related to the second power of the distance based on Newton’s universal law of gravity. Since the eventual kinetic energy is to approach zero, the initial kinetic energy must equal the difference in potential energy between the launch point and the final location. Note that the potential energy is calculated relative to the center of the Earth, since it is associated with gravity. In other words, the kinetic and potential energies are to be the same—the launch velocity that is predicted based on this logic (the escape velocity) is $v_e = \sqrt{2g_r r}$. Based on this formula, the approximate escape velocity of 11.17 km/s is found, which would apply to an object at the surface of the Earth at the Equator (ignoring the Earth’s angular speed). If the object were to be launched from the North Pole, the escape velocity would increase by 0.34 percent, since the gravity at the North Pole is 1.35 percent less than that at the Equator. At a distance of 100 km from the surface of the Earth, the escape velocity is 11.09 km/s. At the uppermost part of atmosphere, the base of the exosphere (400 km AGL), the escape velocity is about 10.85 km/s. The last two velocities are obtained by assuming that the launch is performed from the Equator.

Increasing the launch velocity of the flying object (or a satellite) makes it enter into the orbit—also known as the trajectory—which approaches an ellipse, based on Kepler’s first law of planetary motion, with the Earth being located at one of the foci. For a satellite to follow an orbit around any heavenly body, gravitational force provides the centripetal force required

to make the object follow the curved trajectory. Newton's law of universal gravitation states that any two bodies are attracted to each other with a force which is proportional to their masses and inversely related to the square of their distance. Based on this law, any two objects attract each other by a gravitational force existing between them. For that force to be strong enough, either the objects are to be massive, for example like the Earth; the force of gravitation is to be very high; or they are to be located in close proximity to each other. The analogy is completed by varying its parameters. If either the distance (r) between the objects is increased or the masses of the objects (m_o) or heavenly body (m_h) are decreased, the force (F) will decrease. The distance must be the only parameter with a significant effect on the force of attraction. You decrease the attraction by increasing the distance between the objects based on the universal law of gravitation. With that analogy in mind, equation (114) is presented herein, where G is the gravitational constant ($6.674 \times 10^{-11} \text{ Nkg}^{-2}\text{m}^2$).

$$F = G \frac{m_o m_h}{r^2} \quad (114)$$

It is said that Newton obtained the gravitational constant by assuming that the density of the Earth is five to six times greater than that of water—this number is very close to the value presented previously. Using the relation $G = g / (8\rho_w \pi r_{\text{Earth}})$, assuming that the density of the Earth is six times that of water (solid state— ρ_w (915 kg/m^3)) and the Earth's mass may be obtained by multiplying its density by its volume (assuming a sphere of radius r_{Earth}), the gravitational constant (G) within 0.42 percent of the currently known accurate value may be obtained.

Since the mass of a planet (like the Earth) can be assumed to be constant, it is convenient to calculate the object's weight by multiplying its mass by the gravitational acceleration ($F = m_o g$). Substituting this gravitational force in equation (114), you can express the gravitational acceleration (g) in terms of the rest of the parameters in equation (114)—mass of the heavenly body (m_h), gravitational constant (G), and the distance between the centers of the two objects (r)—equation (115). This equation is easier to apply to the common task of calculating the gravitational force on the object of a known mass.

$$g = G \frac{m_h}{r^2} \quad (115)$$

Equation (115) states that the closer you are to the COG, the higher the gravitational force is. As you get farther, like when you are flying at high altitudes, the gravity decreases. Assuming that the mass and radius of any heavenly body are known, you can calculate the gravitational acceleration on

its surface. Table 25 presents heavenly body data along with the calculated values for gravitational acceleration, the drop in an object's elevation after one second of freefall on the planet's surface, and the planet's orbital velocity and period. Note the duration of the local day (one rotation about its axis) associated with each heavenly body and expressed in Earth days. You would have to complete your daily activities in 0.24 fraction of the time available to you if you were to live on Mercury, while you would have about 248 times more time available to complete the same tasks if you lived on Pluto, followed by Neptune (165), Uranus (84), Saturn (30), and Jupiter (12)—you may decide to either get more done or rest more (see Table 25). Venus (0.62) and Mars (1.88) seem to be the least disrupting scenarios.

Note that the solar day is not to be mistaken with the sidereal day, which is the time it takes for the Earth to complete one rotation relative to the background stars (23 hrs 56 min 4 s). For the Earth orbiting around the Sun as well, an additional (3'56") is required to complete a solar day (civil day—from the sunrise to the next rise). Equation (116) presents the relationship between the orbital period (t), distance from the heavenly body (r), and mass of the parent (heavenly body or planet) (m_h) whose object (particle) is orbiting around it. Equation (117) presents the relation between the orbital velocity (v), mass of the parent (heavenly body) whose object is orbiting around (m_h), and R , the radius of the heavenly body. The period of the heavenly body is obtained from Kepler's third law, $T^2/(R + d)^3 = cte$, where ($r = R + d$) is the distance from the center of the orbiting object to the center of its parent. The circular orbital velocity is obtained from $v_{oc} = \sqrt{Gm_{hp}/(R + d)}$, where m_{hp} is the mass of the heavenly parent (e.g., the Earth is the parent of the Moon). The elliptical orbital velocity is similar to that of the circular orbital velocity that has been corrected to include the semi-major axis of the heavenly body, $v_{oe} = \sqrt{Gm_{hp}(2/r - 1/a)}$, where a is the semi-major axis.

Table 25 shows the relationship between the escape velocity (v_e) and the distance from the center of the heavenly body (r) presented by equation (118), where (ρ) is the density of the heavenly body. The alternative expression for the escape velocity presented by equation (118) is for the uniformly distributed matter (i.e., homogeneous) scenarios.

$$t = 2\pi \sqrt{\frac{r^3}{Gm_{hp}}} = \sqrt{\frac{3\pi}{G\rho}} \quad (116)$$

$$v = \sqrt{\frac{Gm_{hp}}{r}} = \sqrt{\frac{4\pi G}{3}} r \sqrt{\rho} \quad (117)$$

$$v_e = \sqrt{\frac{2Gm_{hp}}{r}} = \sqrt{\frac{8\pi G}{3}} r \sqrt{\rho} \quad (118)$$

Note that equations (117) and (118) are obtained based on the assumption that the atmosphere does not introduce resistance (i.e., drag) to the object's motion. It also assumes that the orbit is circular. If the orbit is elliptical, the vis viva (meaning *living force* in Latin) relation—also known as orbital-energy-invariance law—is employed to describe the motion of the orbiting body. To obtain this relation, energy conservation and conservation of angular momentum laws are applied, similar to the previous scenario using apogee and perigee—the farthest and nearest points from the heavenly body or planet reached by an object or particle orbiting it. The total energy consists of kinetic ($0.5m_o v_e^2$) and potential ($m_o g_r r$) energies, where m_o is the object mass, r is the distance between the centers of the planet and the object, v_e is the escape velocity of the particle, and g_r is the gravity of the planet at distance (r). The total energy is to remain the same for all locations, including those two ($0.5m_o v_e^2 + m_o g_r r = cte$), where *cte* represents a constant value. The conservation of angular momentum results in $rv_e = cte$. Therefore, by equating the two relations for the two locations (apogee and perigee), a more general form of the relationship presented by equation (119) is derived, which introduces a correction factor based on the semi-major axis into equation (118)—note that in $2a = r_a + r_p$ and $b^2 = r_a r_p$, subscripts “a” and “p” are associated with the apogee and perigee, respectively.

$$v_e = \sqrt{Gm_h \left(\frac{2}{r} - \frac{1}{a} \right)} \quad (119)$$

Based on equation (119), the specific orbital energy (i.e., the total energy of the particle per unit mass) is presented by $(-Gm_h/2a)$. Note the negative sign for the gravitational force applied to the particle is toward the planet. The escape velocities at the apogee (v_{ea}) and perigee (v_{ep}) are obtained using equation (120)—where r_a and r_p are the distances from the center of the particle to the center of the planet at the apogee and perigee, respectively. Note that variable $r_{(a,p)}$ is presented as an alternative to indicate the same relationship is valid if either subscript “a” or “p” is required and is not to be mistaken with the plane coordinates. The specific angular momentum—the cross product of a particle position and momentum vectors per unit mass of the particle— $r_{(a,p)} v_{e(a,p)} = b\sqrt{Gm_h/a}$ —is another important variable that along with the linear momentum and energy of the particle are defined in quantum mechanics—recall the translational, rotational, and vibrational

modes of energy transfer in molecules. The orbital period remains the same as that of the circular orbit, as in equation (116).

$$v_{e(a,p)} = \frac{b}{r_{(a,p)}} \sqrt{\frac{Gm_h}{a}} \quad (120)$$

Equation (120) is applied to a ballistic object (e.g., a cannonball) not aided by a propulsion system (e.g., a rocket). The equation gives the speed required by the object to be launched from the Earth or an altitude in order to overcome gravity. This parameter in addition to the chosen trajectory, the angle at which the object is erected with respect to the horizon of the launching location, are the factors determining if the object orbits the planet (e.g., satellite launched from the Earth), runs out of energy and hits a surface (e.g., missiles), or continues its journey forever (e.g., meteorites). It is to be noted that atmospheric conditions and the rotation of the Earth are not included in these calculations; therefore, as before, you are to keep in mind the direction of the rotation of the Earth (counterclockwise if you are looking down on the North Pole) and deduct or add the velocity to obtain relative velocities based on easterly or westerly launches when performing launch calculations. Because of the jet stream—winds aloft traveling to the east with the flow fluctuating to the north and south—it usually takes longer to fly west in the Northern Hemisphere—considering the Earth's counterclockwise rotation—than traveling to the east. For instance, flying west from London to Toronto, against the jet stream, causes the plane's ground speed to decrease and as a result the flight time to increase compared to flying this route to the east. Note that the Earth's rotation has a partial role in the generation of the jet stream.

Nevertheless, the orbiting objects equipped with propulsion systems are supported by the additional force available to them to reach the escape velocity required. There are circumstances in which the particles (orbiting objects) are launched first to a lower orbit as an intermediate step. Within this orbit, they may travel as many complete turns as planned before reaching the escape velocity required to move to the upper orbit belonging to another planet. An example is a Mars satellite that is launched from the Earth that first follows the Earth orbit, and then is transferred to the Mars orbit. The transfer path, which is a straight line, is also known as the Hohmann transfer orbit.

The physics and mathematics of an orbiting satellite when entering a higher orbit are complex. Nevertheless, a brief example is discussed here for enthusiasts. Let us assume that the Mars explorer satellite mission is an

orbiting DAQ system that collects information related to the Sun's Coronal Mass Ejection (CME) activities and relays this information back to the Earth. The explorer is to be placed in the Mars orbit, with Mars being located at one of its foci based on Kepler's first law of planetary motion. As the satellite travels the path, there are times when it is directly exposed to the Sun's radiation. In radiation heat transfer this is known as a shape factor of one—meaning that it is capable of receiving the Sun's uninterrupted radiation. Because of this direct radiation exposure, the satellite's structure needs to be resistant to high temperatures, and so measures need to be taken to minimize radiation impact. Thus, the aluminum alloy exterior surfaces of satellites are coated with special white paints (e.g., AZ-93) to reflect about 85 percent of the Sun's radiation as a means of thermal management [151]. Use of metamaterial Optical Solar Reflectors (OSR) as a surface coating on the exterior surfaces of the spacecraft in order to emit solar heat is suggested to thermally control heat exposure. These surfaces are made of quartz tiles, which are relatively heavy, costly to manufacture and launch, difficult to assemble, and cannot be easily implemented on surfaces with complex geometries [152]. Nevertheless, they are resistant to thermal radiation effects. The metal oxide embedded in these coatings results in a metamaterial that has very high emissivity and small absorptivity to the solar energy.

There are instances, however, when Mars comes in between the satellite and the Sun—for example, at the farthest distance from the Sun, where an eclipse is formed. In this scenario, the radiation information is recorded before entering the eclipse and right after leaving the eclipse with a time interval that is smaller than that of the regular DAQ for further processing. The Mars explorer not only records the radiation information and surface activities of the Sun but also absorbs the Sun's energy through its solar panels that are automatically adjusted for the highest energy absorption to generate the power required to operate its mechanical and electrical systems. These are examples of the engineering problems that need to be addressed before launching a satellite. There are dedicated mathematical programs which predict temperature variations as a function of the orbital locations (e.g., ESATAN-TMS—Thermal Modelling Suit), where the system exposed to the thermal radiation is modeled either using a lumped capacitance technique, a Finite Element Method (FEM), or a combination of both.

Clearly, it is desired to make the launch under the most suitable conditions possible. That includes minimizing the drag due to adverse atmospheric conditions, minimizing gravity, maximizing the initial launch

velocity to save on the initial energy requirements, and traveling over less populated areas (e.g., traveling east over the Atlantic Ocean, which is mostly unpopulated). Note that preferred launch locations are closer to the Equator (e.g., from Cape Canaveral, Florida), even though the reduction of gravity is only 1.35 percent and that of launch velocity is on the order of 0.34 percent. Before the launch takes place, launch commit criteria, the conditions that are to be met in order for the space launch countdown to take place, must be met. These conditions are not “nice to meet” criteria but are firm requirements put in place to safeguard the enormous investments made in these enterprises. Conditions such as wind, ceiling, wind on the ground, wind aloft, wind shear, cloud layers, cumulus clouds, proximity to the edge of the thunderstorms and related anvil clouds and debris, strengths of electrical fields in the atmosphere, proximity to the flight path where there is high chance for the possibility of turbulent vortices, precipitation, and freezing conditions are the factors that are taken into account when such launches for satellites, with or without propulsion systems, take place. Safety always comes first; therefore, compliance with these requirements as well as thermal and stress design requirements during all the stages, from the conception to the planning and execution of the launch, are of the utmost importance. The satellites must tolerate extreme hot and cold conditions and overcome the resultant temperature-induced stresses with thousands of hours of tests and preparation, as the smallest compromise may result in a disaster (e.g., the O-Ring seals on the space shuttle Challenger OV-99 that were not exhaustively tested for their performance in extreme cold conditions).

Figure 61 shows the gravitational acceleration versus the distance from the heavenly bodies plotted using equation (115). The plots are grouped for ease of visualization, so the planets with small diameters are presented separately from those with large diameters. The surface of the heavenly body is assumed as the reference point and is at distance zero. Note the increase of the acceleration with the decreasing distance from the center of the Earth. Note that the gravity force will actually decrease as one descends below the Earth’s surface, becoming, theoretically, close to zero at the center, where the Earth’s mass will be equally distributed around the object, with equal pull in all directions, resulting in a zero-net force. Furthermore, the Earth’s density varies along its layers, which is a contributing factor to its mass distribution along its depth and gravitational acceleration as a result. Figure 62 presents the relationship between the orbital velocity of the Earth and distance from the Sun; note that they are inversely related, as in equation (119).

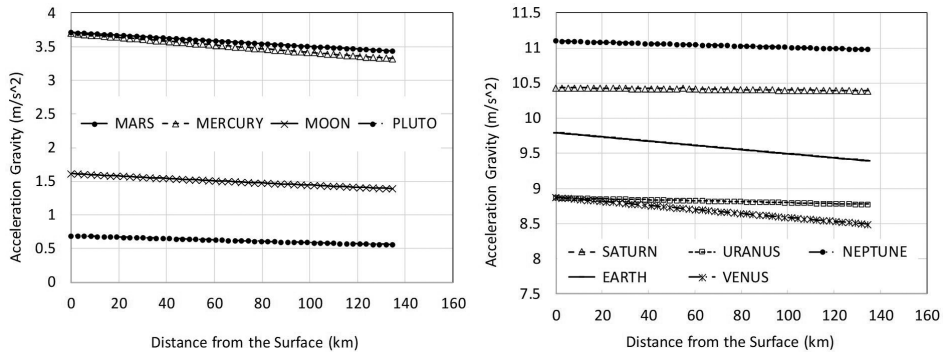


FIGURE 61 Acceleration due to gravity as a function of the distance from the surface of heavenly bodies for: (a) planets with small diameter (left), and (b) planets with large diameter (right).

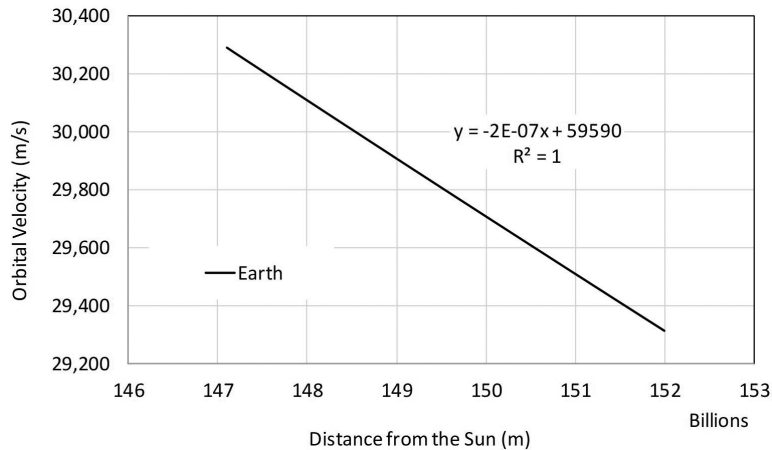


FIGURE 62 Velocity of the Earth orbiting the Sun from Perihelion ($R = 147$ million km) to Aphelion ($R = 152$ million km).

Figure 63 shows the diagram of the escape velocity (far left axis) and the gravity (far right axis) versus the distance from the Earth. It is seen that they also may be curve-fitted with a confidence level above 99 percent. For a rotating body (e.g., the Earth), the relative velocity (the resultant vector of the addition of the two velocity vectors) is to be considered in escape velocity calculations. This means that the direction of the rotation of the rotating heavenly body whose orbit is of interest is also to be considered. For example, due to its rotation, the Earth's surface moves at 465.11 m/s, meaning that if the launch is to be made at the Equator and to the west, the relative velocity is 10.71 km/s, while when launching it to the east, it is 11.64 km/s. Recall gravity increases as you get closer to the North Pole, since the diameter of the Earth decreases (the difference of approximately

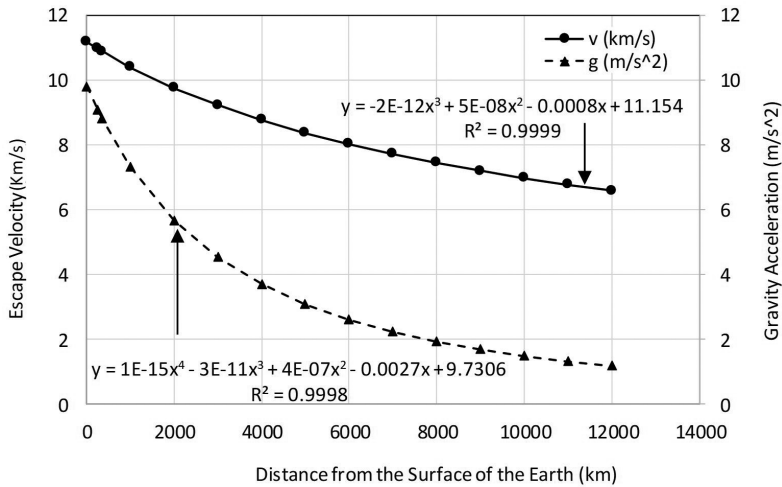


FIGURE 63 Escape velocity required for the orbital launch and gravity at the surface of the Earth versus the distance from the surface of the Earth.

42.77 km) and the Earth's rotational speed approaches zero—you may expect an escape velocity of 11.10 km/s. The 1.35 percent increase in the gravity results in additional fuel and, as a result, increased associated cost; therefore, it is preferable to launch the objects (e.g., satellites) as close as to the Equator as possible. To obtain the launch velocity at different locations, a correction factor (cosine of latitude) is to be included. For example, a launch from Kennedy Space CenterSM with 28.59°N latitude results in westerly and easterly launch velocities of 10.77 km/s and 11.59 km/s.

The curve fitted to the Earth's data in Figure 63 presents the relationship between the speed and the distance from the surface of the Earth. The surface of the Earth is assumed as the reference point of zero km. If you were to extend the diagram to include the negative distance (below the ground levels) and apply the same logic as that of AGL, the gravity acceleration would appear to increase together with the escape velocity. But as pointed out previously, this is not the case. It would be correct if you kept shrinking the Earth's size, but then you would not be going inside of it. Recall *Journey to the Center of the Earth* (1864) by Jules Verne, the French novelist, poet, and playwright known for his science fiction books and vivid imagination. If gravity did increase with depth, one can imagine the meticulous professor Otto Lidenbrock and his friends (Axel and Hans) being stuck under the ground, gravity keeping them hostage forever. If not for the increasing internal pressure of the Earth and the force of the water sliding them upward toward the mouth of the Stromboli Volcano and their eventual eruption, they would not have been alive today.

7.3 Drones

The term *drone* is very versatile. It is used in diverse fields (e.g., melittology—specialized entomology, film and television, science fiction, science, and technology) and names (e.g., music, bands, albums, and songs). Perhaps the two fields that may be related in several ways are melittology, the science of the study of bees, and science and technology. A “drone” in melittology refers to fertile male ants (not the female workers or the male soldiers) that mate with the females to expand the kingdom of ants. It is also a name for similarly functioning bees who neither sting nor gather nectar but mate with an unfertilized queen and guard hives from predators. They travel from hive to hive mating with the receptive queens, not the virgin ones [153]. The virgin ones are the most active and fly away if they are disturbed, for fear of execution, and they are harder to spot. A virgin queen is easily accepted in a new hive while a mated one is at high risk of being killed by the workers. The mating takes place in the sky as they fly. Recall the fueling of a jet plane on a mission by the Boeing 747. The hovering bee with all its flight control mechanisms is like a UAV—in fact, it is more correct to say the opposite, since the bees came into existence long before UAVs (and to answer that old riddle—it is the egg that came before the chicken, for crocodiles were the earlier inhabitants of the Earth who laid eggs).

Examples of different types of UAVs are unmanned combat aerial vehicles also known as drones (to carry airborne weapons such as missiles); multi-rotors, also known as multi-copters (to carry people or ammunitions and which are radio-controlled); quadcopters (a subset of multi-rotors), also known as quadrotor helicopters (to carry out projects in the military, photography, journalism, art, sports, and research); passenger drones (to move people); delivery drones (to transport goods and packages); and agriculture drones (to monitor and increase crop growth and production).

All of these flying objects have common flight mechanics. For them to ascend, descend, hover, and move in the desired direction, and to demonstrate the full range of capability of flying objects such as roll (moving about the longitudinal axes), pitch (moving about the lateral axis), and yaw (moving about the vertical axis), they are to satisfy the physics laws of linear and angular motions as well as balance of forces. Recall the earlier discussion on the balance of forces related to flying objects and fixed-wing aircraft. The drones are flying objects with rotors as the main mechanism to provide the thrust required to make the aforementioned motions possible. Each rotor interacts with the surrounding air by means of a propeller, which is either

single-bladed or multi-bladed. A multiple rotor drone may be designed so that the direction of the rotation of the rotors or the relative location of the axes of rotations may vary (e.g., twin-synchropter, twin-coaxial, twin-lateral, twin-tandem, triple, and quadruple helicopters).

Consider the case of a single-copter—that is, assume there is only one rotor (rotor 1) versus four as shown in Figure 64. The thrust is generated by the rotational motion of the rotor blades. The resisting forces are the weight and the drag. If the balance of the forces (equivalent components of drag, weight, and thrust) is nonzero, the drone will accelerate in the direction where the force balance vector is pointing. This can be ascent, descent, or horizontal movement. Once the new desired velocity along a linear path is established, the balance of forces will become zero, since the acceleration is zero.

Think about this as a violin player who is to deliver Sergei Rachmaninoff's Suite in D-Minor with a touch of Chopin and Liszt [154]. Although each musician is to exercise his own harmonic ingenuity, the music is open to interpretation, and it cannot be fully molded into the musical uniqueness that Rachmaninoff stands for. The violin player is free to introduce new harmonious arrangements while respecting the integrity of the original piece.

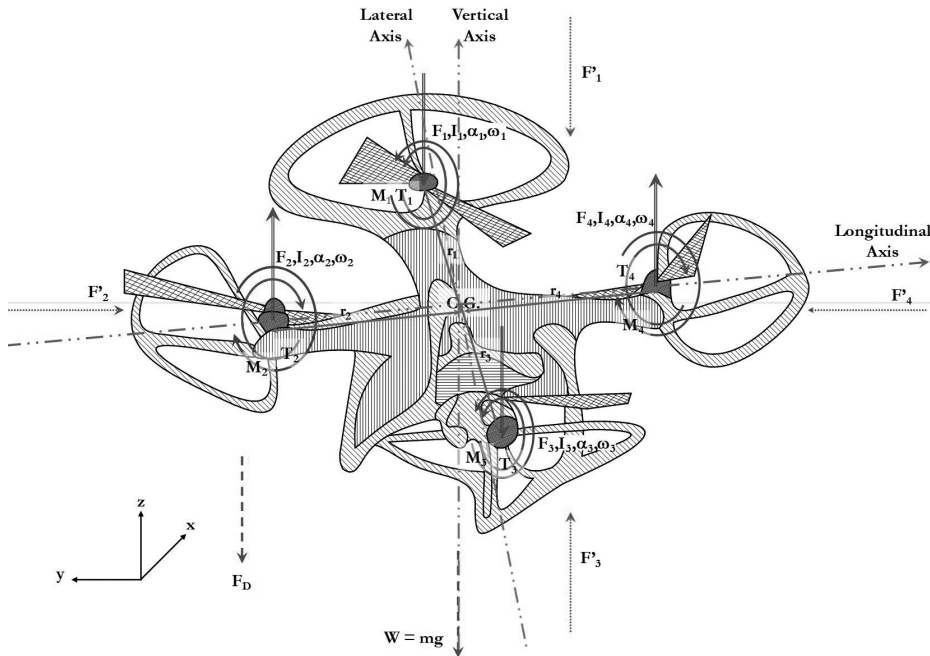


FIGURE 64 Quadcopter (drone) diagram along with the applicable forces in a horizontal orientation (drawings created using Solid Edge CAD tool).

Now assume that the number of players is increased—a string quartet. The team is the violist, cellist, and two violinists. In this scenario, the additional players are to take their cues from the lead violin player (acting also as the conductor) in order to create a harmonious piece. Each musician may play the same piece of music in different arrangements or tempo. Although one may assume that they have practiced the piece for many long hours and have the notes memorized by heart, their ensemble may not sound as pleasant as intended in the original work—some may call that a “modern” piece. For this team to work flawlessly, they must have practiced alone and together to address the technical challenges as well as the synchronization of the efforts—the single effort in addition to the collective one. That has occurred by the means of the communication and feedback mechanism between the team. They may vary the tempo to emphasize the feeling, introduce more vigor in some places. No matter what the ultimate objective is, the collaboration of the parts is a must. The same applies to the drone operation. The balance of the forces and torques must produce the desired motion; the rotors are to work with each other correctly through carefully designed control and feedback mechanisms in order to make the required changes at the right time to follow the desired path.

Among the multi-rotors, fixed-wings, single-rotor helicopters, and fixed-wing hybrid Vertical Take-off and Landing (VTOL) drones, multi-rotors are the most popular due to their small size, high maneuverability, and agility. These qualities make them well-suited for small-area mapping; however, limited endurance and speed make them unsuitable for large-area mapping, that is, one lasting over 30 min—the approximate life of their battery. The fixed-wing type is capable of flying for up to 16 hrs, carrying larger payloads, and employing gas engines. On the other hand, they are not as maneuverable, cannot hover over a point, and require more space for landing. Other considerations that affect their usage are associated cost, whether the collected data requires extensive processing, and even whether they are more prone to be attacked by birds of prey (e.g., wedge-tailed Australian Eagles also known as bunjil).

The single-rotor helicopter has one rotor with a slower spinning speed and larger blades, which makes it a more efficient option compared to a multi-rotor drone. For the same reason, a quadcopter drone is more efficient than a hexa-rotor drone. Nevertheless, the structure of a single-rotor helicopter is more complex and is associated with higher maintenance and operation costs. The VTOL aircraft combines the efficiency of a fixed-wing with the agility of a helicopter by adding a tilting rotor mechanism. There

are also drones with swiveling wings to which propellers are attached. They are vertical at takeoff and then become horizontal in level flight. A tail sitter, sitting on its tail on the ground, is another possible configuration.

Figure 64 shows a multi-rotor drone, a Micro-Aerial Vehicle (MAV), in a horizontal orientation (for hovering, ascending, or descending while stationary relative to ground), and Figure 65 shows the same drone tilted (for translating relative to ground) along with the applicable forces and moments in both cases. These drones are capable of navigating inside buildings and tight spaces without the need to use a GPS system (which may not be available due to a lack of satellite signal reception); therefore, they are suitable for surveying inside and in between tall buildings, as well as underground and in tight spaces. This makes them suitable for investigating natural disaster zones and other hazardous areas. The four thrusts shown in Figure 64 and Figure 65 are to balance the four individual rotors' weights in addition to the weight of the frame, onboard processors, communication systems, cameras or laser scanners, and batteries—the total weight—due to gravity as well as the drag force.

Assume that the weight of the drone is equally divided among the rotors. Then, the total body weight is w ($mg = g \sum_{i=1}^4 m_i$), where m_i is the mass of the

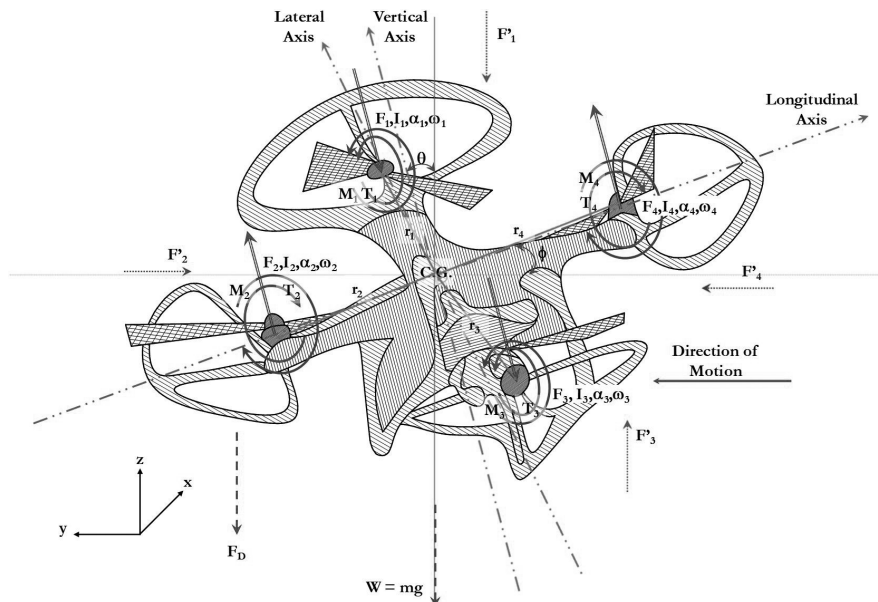


FIGURE 65 Quadcopter (drone) diagram along with the applicable forces in a tilted orientation (drawings created using Solid Edge CAD tool).

rotor “ i .” The weight is balanced by the thrust of the four rotors ($F_i = K_{fi}\omega_i^2$)—where K_{fi} is a proportionality constant that considers the mass (m_i) and ω_i is the angular speed of the rotor. One pair of opposing rotors spins clockwise and the other spins counterclockwise in order to balance the torques. The resultant force vector and torque determine the direction of acceleration and the rate of rotation. Ignoring any wind disturbance, for the drone to hover over a location, the four thrusts are to be the same to create the opposite effect to gravity so that the resultant is zero, as in equation (121), where a is acceleration. Assuming similar-weighting rotors, the thrust is a quarter of the total weight ($mg/4$). If the total thrust is more or less than the weight, the drone will accelerate in the direction of the resultant force according to Newton’s second law of motion.

$$\sum_{i=1}^4 F_i = F_1 + F_2 + F_3 + F_4 - mg - F_D = ma \quad (121)$$

Equation (121) may be written in the form of equation (122), where \ddot{y} is the second derivative of the position vector (y) along the y -coordinate—also known as acceleration along the y -coordinate. Let u_1 be the objective function representing the force (i.e., mass by the acceleration) along the y -coordinate, transverse to the hovering direction. The purpose is to obtain an objective function for linear acceleration (u_1) that minimizes the deviation from the input state variable (y) and desired location ($y_{desired}$) at a given time—in other words, you are to minimize the error, as in equation (123). The resultant moment is presented by equation (124), where I is the moment of inertia, $\ddot{\varphi}$ is angular acceleration, M_i is the reaction moment created by each rotor due to its angular rotation, and u_2 is the torque objective function.

$$\sum_{i=1}^4 K_{fi}\omega_i^2 + mg = ma = \frac{m d^2 y}{dt^2} = m\ddot{y} = u_1 \quad (122)$$

$$e(t) = y(t) - y_{desired}(t) \quad (123)$$

$$\begin{aligned} \sum_{i=1}^4 M_i &= F_1 \cdot r_1 + F_2 \cdot r_2 + F_3 \cdot r_3 + F_4 \cdot r_4 - M_1 - M_2 - M_3 - M_4 \\ &= I_{xx} \alpha = I_{xx} \ddot{\varphi} = u_2 \end{aligned} \quad (124)$$

For the object to move in the positive horizontal direction along the y -coordinate, the resultant thrust vector due to rotor 2 is to be greater than that of rotor 4. This results in the thrust vector tilting toward the direction

of motion and producing the motion due to the horizontal component of the resultant thrust vector. For the drone to move in any direction, there must be a component of the thrust in the direction of the motion, as in equation (125). Note that ψ is the rotation about the longitudinal axis (x)—also known as roll, φ is the rotation about the lateral axis (y)—also known as pitch, and θ is the rotation angle about the vertical axis (z)—also known as yaw. Equation (126) is the moment equation that is based on the angular acceleration and moment of inertia in the y-z plane.

The vector form of equations (125) and (126) is presented by equation (127), showing the linear and angular accelerations along the y , z , and φ . Note that the third matrix (3×3) from the left in equation (127) may be represented as a resultant rotation transformation matrix by φ about the y-coordinate, which is a one-rotation matrix. It is possible to rotate the frame by a sequence of ψ or φ ; these rotational angles are also known as Euler angles. A 3×3 orthogonal matrix, which is a function of time (t), can be used to define each rotation; the total rotation may be obtained from the product of the three matrices. Orthogonality means that $R(t)R^T(t) = 1$.

Equations (128), (129), and (130) represent the rotations about the x axis (x , ψ), y axis (y , φ), and z axis (z , θ). The transformation representing the sequence of these three rotations is given by equation (131). Note that for the rotation to take place about an axis, the axis does not move—in other words, there is a point within that frame (body-fixed or space-fixed) whose position does not change—equation (132)—assuming that the original frame of a body at time t from $p(t)$, which is a 3×1 vector, has rotated to $q(t)$ by means of a rotation matrix $R(t)$. It is possible to take a time derivative of this equation and therefore predict the velocity in the body frame ($R^T(t)\dot{q}(t) = R^T(t)\dot{R}(t)p(t)$) and space frame ($q(t) = R(t)p(t)$)—where $R^T(t)\dot{R}(t)$ is the angular velocity in the body-fixed frame and $\dot{R}(t)R^T(t)$ is the angular velocity in the space-fixed frame. The body-fixed frame is attached to the body; the body-fixed frame is moving with respect to a space-fixed frame (a fixed frame somewhere in the environment), as in the second part of equation (132). For a translation and rotation motion, however, the entire frame and therefore the associated points move.

$$\left(\sum_{i=1}^4 F_i \right)_y = ma_y \rightarrow m\ddot{y} = -mg + \cos\varphi (\overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4 + \overline{F}_D)_y \quad (125)$$

$$\sum_{i=1}^4 M_i = I_{xx}\alpha \rightarrow I_{xx}\ddot{\varphi} = F_1.r_1 + F_2.r_2 + F_3.r_3 + \quad (126)$$

$$F_4.r_4 - M_1 - M_2 - M_3 - M_4$$

$$\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} -\sin\varphi & 0 & 0 \\ \sin\theta & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{m} & 0 \\ \frac{1}{m} & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (127)$$

$$Rot(x, \psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix} \quad (128)$$

$$Rot(y, \varphi) = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix} \quad (129)$$

$$Rot(z, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (130)$$

$$R = Rot(x, \psi) \times Rot(y, \varphi) \times Rot(z, \theta) = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} = \begin{bmatrix} \cos\varphi\cos\theta & -\cos\varphi\sin\theta & \sin\varphi \\ \sin\psi\sin\varphi\cos\theta + \cos\psi\sin\theta & -\sin\psi\sin\varphi\sin\theta + \cos\psi\cos\theta & -\sin\psi\cos\varphi \\ -\cos\psi\sin\varphi\cos\theta + \sin\psi\sin\theta & \cos\psi\sin\varphi\sin\theta + \sin\psi\cos\theta & \cos\psi\cos\varphi \end{bmatrix} \quad (131)$$

$$\begin{aligned} q(t) &= R(t)p(t) \rightarrow \dot{q}(t) = \dot{R}(t)p(t) \rightarrow R^T(t)\dot{q}(t) \\ &= R^T(t)\dot{R}(t)p(t) \rightarrow \dot{q}(t) = R^T(t)\dot{R}(t) \end{aligned} \quad (132)$$

The drones shown in Figure 64 and Figure 65 have six degrees of freedom, three translational along the x , y , and z -coordinates and three rotational about the lateral (pitch), longitudinal (roll), and vertical (yaw) axes (ϕ , ψ , and θ angles), which along with their derivatives and second derivatives determine the drone dynamics. It can be inferred then that the rotor angular velocity, the forces due to the thrust by each rotor, and the resultant force are the factors in determining the direction of the drone translation, rotation, hovering, acceleration, and deceleration. Controlling the combination of these parameters is technically challenging, since for the

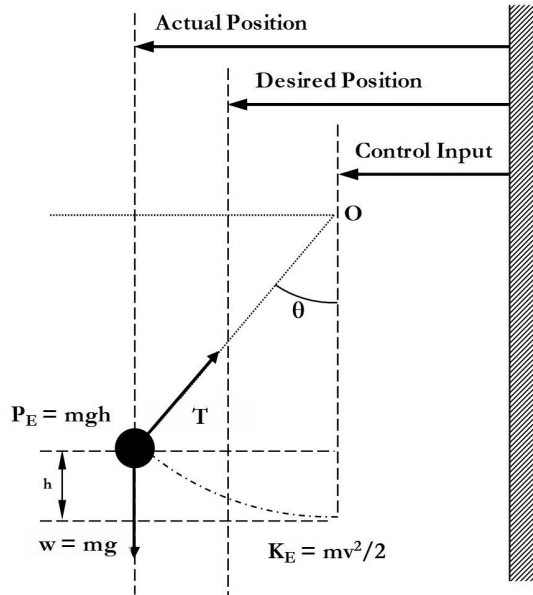


FIGURE 66 Example of a pendulum position control.

mechanical system to work as expected, the rotors are to be interconnected to adjust their reaction based on the received feedback—meaning that the combination of four rotors, which results in twenty-four outputs, needs to be adjusted simultaneously, which is not practical given the complexity of the system. A control feedback system is then to be designed for such systems.

To get a better idea of how a control system works, consider holding in your hand a string to which a small mass suspended in the air is attached, that is, a pendulum, as in Figure 66. The task of this “control system” is to move this small mass along a horizontal line, from one point to another. The desired output in this case is the mass position along the line of movement. For example, you can specify one possible desired output as a step function of time whose value changes from 0.1 to 1 at 0.3 s—in other words, it is desired that the mass moves from position 0.1 to a new location at position 1 at 0.3 s.

If you were to simply execute this motion with your hand, without any consideration of what the mass was doing, you can imagine that, when you abruptly started moving, the mass initially lagged behind, and when you stopped the movement, the mass kept swinging like a pendulum for some time. Now consider that you are carefully watching the small mass as you move it. You observe how faithfully the mass follows the desired movement

and try to adjust your hand position so as to minimize the deviation from the desired position. What you are doing is using your visual feedback system to adjust the control input (your hand position) to minimize the error between the desired and actually observed mass position.

Figure 67 illustrates the behavior of the control system described in the previous example. The figure shows a solid line with hollow circles depicting the reference signal. This is the desired output for your system (a step change in position from 0.1 to 1 at time 0.3 s). So, at 0.3 s, when this new desired position is set, if you have a feedback mechanism in place, it will immediately inform your control system—with the coolness of a robot—that the mass is off-target by the position error $e_p(t)$. You can then generate a control signal (your hand movement speed) proportional to this error, with the proportionality constant equal to K_p .

The mass responds to your movement, and as a result its position decreases or increases over time. This position change, however, reflects neither the future nor the past response. Thus, the pendulum may overshoot or undershoot the target. To account for the future response, the rate of change of the amplitude (i.e., the derivative with respect to the time) needs to be calculated. This new feedback mechanism, which is called a derivative control, with proportionality constant K_d , is added to the previously described proportional control. It acts similarly to a dashpot (or a shock absorber) to dampen oscillations in a controlled system.

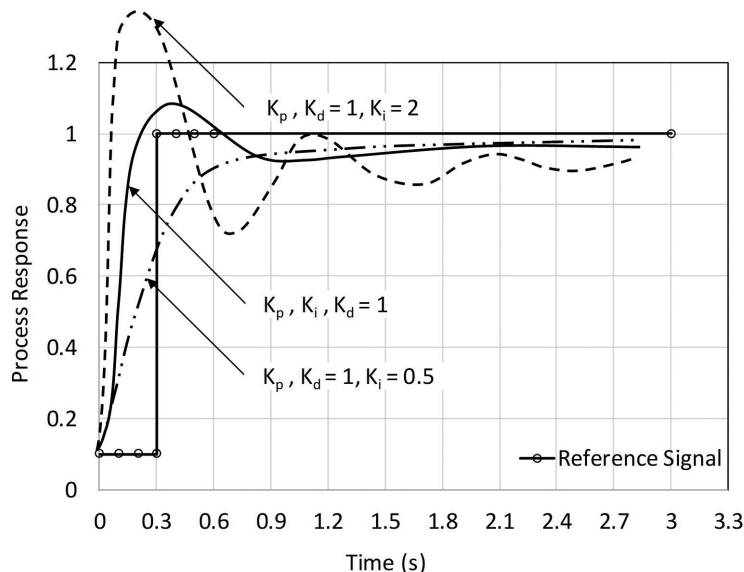


FIGURE 67 Example of response versus the time for a PID system.

This new control system design provides both instantaneous and future-time-related feedback to the amplitude response; however, it does not consider the history of the motion. For example, if there is a persistent small error between the desired and actual position, the proportional response may be too weak to correct it. However, if we keep adding up this error over time (i.e., integrating it), it will accumulate to provide sufficient corrective action. Therefore, this feedback mechanism is to consider the past states of the system, and they are to be accumulated to reflect the effect of the temporal changes of the spatial variables to modify the present results—an integral control is introduced by the proportionality constant K_i . The sum of the new integral control input and the previous two control types now determine the destiny of the mass. Depending on the values given to these three control constants, the system response can vary from having a large overshoot (dashed line), a small overshoot (solid line), or a damped response (dashed-double-dotted line), as in Figure 67.

The previous discussion demonstrates the concept of the Proportional, Integral, and Derivative (PID) control systems. A properly designed control system combines the three types of feedback so that the objective temporal-spatial function is achieved (Figure 68). Similar control mechanisms (PID) control the rotors mounted on a rigid frame and work independently from each other. They cause increase or decrease in angular velocity of the rotors, and the result is the roll or pitch motions in the direction of the acceleration. The created thrust is normal to the rotation plane. To make the drone move horizontally, the drone is to pitch forward so that the thrust vector has a component in the direction of the motion. To make the drone stop, the angle of the pitch is to be reversed until it gets to the equilibrium state where the destination is reached.

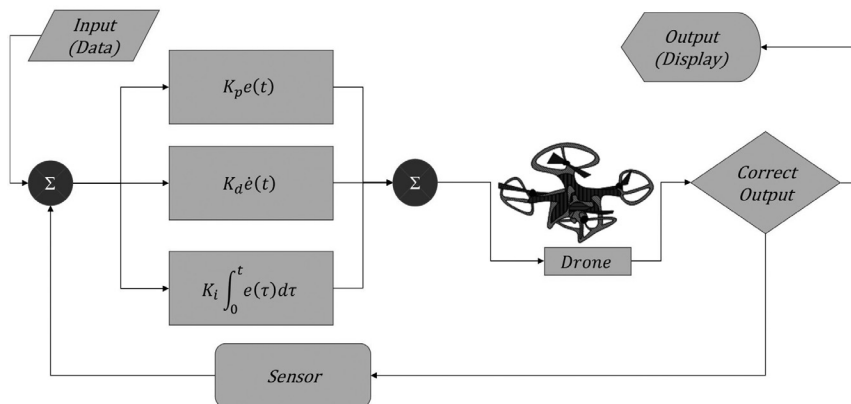


FIGURE 68 Example of a PID control diagram for a drone (drawings created using Solid Edge CAD tool).

Regardless of the types of drones being used, the main functionalities of drones are observing the process parameters (i.e., identifying accurate position, angular velocity, and rotation angle), controlling the process parameters (i.e., minimizing the deviation between the current and desired status), mapping (i.e., identifying the locations over which the drone is flying), and planning (i.e., following the desired programmed path). Recall the error objective function that is to be minimized so that the drone follows a path least deviated from the desired path—equation (133)—where K_p , K_d , and K_i are proportional, derivative, and integral constants of the employed PID controls and are positive to make a stable system. Equation (133) may be rewritten to present the time-dependent acceleration vector ($u(t)$) as a function of the rest of the parameters—equation (134)—Figure 68.

$$\ddot{e}(t) + K_d \dot{e}(t) + K_p e(t) + K_i \int_0^t e(\tau) d\tau = 0 \quad (133)$$

$$u(t) = \ddot{z}_{desired}(t) + K_d \dot{e}(t) + K_p e(t) + K_i \int_0^t e(\tau) d\tau \quad (134)$$

Given the aforementioned concept to design an efficient and effective drone, not only the mechanics and control systems are to be considered but also the applications in which the mechanism is to be employed. Therefore, the required agility, size, total weight, and component characteristics (e.g., battery capability, rotor configuration, thrust, and propeller size) as well as the existence of accessories such as laser scanners and motion cameras affect the drone range. These components require power to operate, which result in an increase of energy consumption for the entire system due to the extra payload they introduce. The intent is for the drone to both accelerate and achieve the desired position as quickly as possible. In other words, the objective is to maximize both linear acceleration ($a = u_1/m$) and angular acceleration ($\alpha = u_2/I_{xx}$). Note that, for a solid body, the mass is proportional to the third power of characteristic length (l), and the moment of inertia is proportional to the fifth power of the same length—making size an important factor when selecting components. Additionally, the thrust is related to the second power of the propeller speed and radius of the propeller ($F \sim v^2/l$) where ($v = l\omega$), and the moment is related to the distance between the COG of the propeller and the drone. It is assumed that the arm length and propeller radius are approximately equal.

7.4 Examples

7.4.1 Case Study – Distance between Two Aerodromes

For the following scenario of throwing a ball from Norman Rogers Airport (CYGK) in Kingston (Canada) to Smiths Falls (CYSH) in Smiths Falls-Montague (Canada), explain the theory of the flying the ball discussed as follows.

Alpha, who is located at CYGK, is to throw a baseball to Bravo, who is located at CYSH. Norman Rogers Airport (CYGK) in Kingston (Canada) is at $N44^{\circ}13.55'/W76^{\circ}35.80'$ and at a 303-foot elevation. Smiths Falls-Montague Airport (CYSH) in Smiths Falls (Canada) is at $N44^{\circ}56.75', W75^{\circ}56.43'$ and at a 416-foot elevation. The SkyVector® indicates a distance of 51.6 NM (95.6 km) on the great circle between the two points; connecting the departure and destination points reveals a bearing of about 35° to the northeast. Investigating the latitude and longitude from the SkyVector® data, Alpha figures out that CYSH is $43.2'$ to his north and $39.6'$ to his east. Alpha may calculate the distance from CYGK to CYSH by means of the latitude and longitude information. He has a right triangle that is horizontally away by 28.1 NM (52 km) and vertically away by 43.2 NM (80 km). He may calculate the hypotenuse as 51.5 NM (95.4 km). Alpha's calculation deviates by 0.14 percent from the 51.6 NM (95.6 km) great circle distance suggested by the SkyVector®. Thus, it was sufficiently accurate. Given the elevation of CYGK ($303'$) and that of CYSH ($416'$), the ground rise of $113'$ (34.4 m) is expected. The slope of the Earth's surface, the ratio of the rise over the run, is equal to the rise of one meter for every 1,510.3-m horizontal distance. It is an uphill walk to get to CYSH, in other words, if Alpha chooses to do so. If the ball is to be thrown in the other direction (the reciprocal, 145°), the ground surface falls one meter per every 1,510.3-m horizontal distance traveled. The gravity will make the ball come back to the Earth unless Alpha throws it fast enough to reach the escape velocity. The ball is thrown in a 45-degree trajectory by Alpha and is caught by Bravo in CYSH.

If Bravo were to throw the ball back to Alpha horizontally, how fast should the ball be thrown to reach CYGK? Since the Earth drops from CYSH to CYGK, the gravity helps Bravo's ball to travel farther without hitting a wall, since the ground surface drops. Bravo, who is interested in taking math as a hobby, calculates the travel time required for the ball if it were to match the gravity acceleration ($t = \sqrt{2h/g}$), where h is the difference between the elevations of the two aerodromes. The answer is 2.7 s. Assuming that the ball is launched horizontally, Bravo then calculates

the average speed of the ball required to match this time given the horizontal distance between them ($v = x/t$) as 9.81 km/s (35,308.1 km/hr), which equals Mach 106.6. This can only be achieved if Bravo either eats more spinach to gain Popeye's super strength or calls the gods for assistance (provided they are capable of or willing to assist). The speed of this ball would be about half of the speed of a meteoroid passing through the Earth's atmosphere. This speed is so high that it causes the meteoroid to heat up and disintegrate due to friction with the air. It just leaves a glowing trail of debris in its wake, which we see as a shooting star in the sky. On the other hand, this speed is not quite high enough to escape the Earth's gravity—the escape velocity is about 11.2 km/s.

7.4.2 Case Study – Planetary Data

Based on the mass, diameter, and distance for the heavenly bodies presented in Table 25, calculate:

(a) planet's average density, (b) gravity acceleration on the surface of the planet, (c) height drop due to the gravity for the first second, (d) the Earth's rotational period in seconds, (e) ratio of the planet's period to that of the Earth (in Earth days), (f) orbital period (in the Earth years), (g) orbital velocity, assuming the orbit is circular, and (h) orbital velocity, assuming the orbit is elliptical.

Density is obtained by dividing the mass of the heavenly body (m_h) by its volume (V) ($= m_h/V$), with a spherical shape assumed. The period of the heavenly body with respect to the Earth is the ratio of the period of the heavenly body to that of the Earth. The period of the heavenly body is obtained from Kepler's third law— $T^2/(R + d)^3 = cte$ —where R is the radius of the heavenly body and ($r = R + d$) is the distance from the center of the heavenly body to the center of the Sun. The planetary day expressed in Earth days is the ratio of the planetary day to the Earth day (24 hrs). The gravity acceleration is obtained from $g = G(m_h/r^2)$, where G is the gravitational constant ($6.674 \times 10^{-11} \text{ Nkg}^{-2}\text{m}^2$). The height drop (h) in one second is calculated from ($h = 0.5gt^2$), where t is the time and g is the gravitational acceleration. The circular orbital velocity is obtained from $v_{oc} = \sqrt{Gm_{hp}/(R + d)}$, where m_{hp} is the mass of the heavenly parent (e.g., the Earth is the parent of the Moon). The elliptical orbital velocity is similar to that of the circular orbital velocity that has been corrected to include the semi-major axis of the heavenly body— $v_{oe} = \sqrt{Gm_h(2/r - 1/a)}$ —where a is the semi-major axis.

TABLE 25 Data for the selected heavenly bodies including the accelerations due to the gravity at their surfaces [155].

Planet	MERCURY	VENUS	EARTH	MOON	MARS	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO
Mass $m_b (\times 10^{24})$ kg	0.33	4.87	5.97	0.07	0.64	1898.00	568.00	86.80	102.00	0.02
Diameter $2r$ (km)	4,879	12,104	12,756	3,475	6,792	142,984	120,536	51,118	49,528	2,370
Density calculated versus (reported) ρ (kg/m ³)	5,426.5 (5,427)	5,245.0 (5,243)	5,493.3 (5,514)	3,322.5 (3,340)	3,913.3 (3,933)	1,240.0 (1,326)	619.4 (687)	1,241.1 (1,271)	1,603.4 (1,638)	2,094.6 (2,095)
Distance from Sun $(r + d)$ (10 ⁶ km)	57.9	108.2	149.6	0.38 (Earth)	227.9	778.6	1,433.5	2,872.5	4,495.1	5,906.4
Semi-major Axis a (km)	5.7909 E+07	1.0821 E+08	1.4960 E+08	3.8440 E+05	2.2794 E+08	7.7857 E+08	1.4335 E+09	2.8708 E+09	4.5044 E+09	5.9061 E+09
Gravity Acceleration g (m/s ²)	3.70	8.87	9.80	1.61	3.72	24.78	10.44	8.87	11.10	0.69
Height drop due to gravity for the first second h (m)	1.85	4.44	4.90	0.81	1.86	12.39	5.22	4.43	5.55	0.35
Rotation Period T (in hours)	1407.6	-5832.5	23.9	655.7	24.6	9.9	10.7	-17.2	16.1	-153.3
Period T (in Earth seconds)	20,438	79,861	86,400	2,368,626	33,569	3,242,449	2,509,673	693,111	661,025	6,919
Period T (in Earth hours)	5.68	22.18	24.00	657.95	9.32	900.68	697.13	192.53	183.62	1.92
Orbital Period T (in Earth days)	88	225	365	0.05	687	4,337	10,834	30,731	60,159	90,610
Orbital Period T (in Earth years)	0.24	0.62	1.00	27.42	1.88	11.87	29.66	84.14	164.71	248.08
Orbital Velocity v_{oc} (km/s)-Circular Orbit	47.88	35.03	29.79	1.02	24.13	13.06	9.62	6.80	5.43	4.74
Orbital Velocity v_{oe} (km/s)-Elliptical Orbit	47.89	35.03	29.79	588.26	24.14	13.06	9.62	6.80	5.44	4.74

7.4.3 Case Study – Orbital Data for the Largest Moons

The seven largest moons belong to the Earth (Moon); Jupiter (Io, Europa, Ganymede, and Callisto); Saturn (Titan); and Neptune (Triton). Based on the semi-major axes and parents' data presented for the five largest moons and ISS in Table 26, calculate:

- (a) period (the Earth day), and (b) circular orbital velocity.

The period of the heavenly body is obtained from Kepler's third law— $T^2/(R + d)^3 = cte$ —where R is the radius of the heavenly body and $(r = R + d)$

is the distance from its center to the center of the Sun. The planetary day expressed in Earth days is the ratio of the planetary day to the Earth day (24 hrs). The circular orbital velocity is obtained from $v_{oc} = \sqrt{Gm_{hp}/(R+d)}$, where m_{hp} is the mass of the heavenly parent (e.g., the Earth is the parent of the Moon).

TABLE 26 Seven largest moons and ISS, periods and orbital velocities [156].

Case	Moon	Parent	Parent Mass $m_{hp} (\times 10^{24})$ kg	Semi-Major Axis a (km)	Period T (Earth day)	Orbital Velocity v_{oc} (km/s)
1	Io	Jupiter	1,898	421,800	1.77	17.33
2	Europa	Jupiter	1,898	671,100	3.55	13.74
3	Ganymede	Jupiter	1,898	1,070,400	7.16	10.88
4	Callisto	Jupiter	1,898	1,882,700	16.69	8.20
5	Moon	Earth	6	384,000	27.42	1.02
6	ISS	Earth	6	353 (circular orbit above Earth)	91.62 (min)	7.69

7.4.4 Case Study – Single-Rotation Angular Velocity

Obtain the angular velocity in the space-fixed frame for a single rotation about the z-axis given by $Rot(z, \theta)$ in equation (130).

$$R_z = Rot(z, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \alpha = \dot{R}R^T =$$

$$\begin{bmatrix} -\sin \theta & -\cos \theta & 0 \\ \cos \theta & -\sin \theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\theta} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

7.4.5 Case Study – Double-Rotation Angular Velocity

Obtain the angular velocity for a double rotation about the x-axis and y-axis given by $Rot(x, \psi)$ and $Rot(y, \phi)$ in equations (128) and (129) for ψ and ϕ values of ninety and zero degrees, respectively.

$$R_{xy} = Rot(x, \psi)Rot(y, \phi) = R_x R_y \rightarrow \alpha = \dot{R}R^T = (\overset{y}{R_x R_y})^T (R_x R_y)^T$$

$$= (\dot{R}_x R_y + R_x \dot{R}_y)(R_y^T R_x^T) = \dot{R}_x R_y R_y^T R_x^T + R_x \dot{R}_y R_y^T R_x^T$$

$$= \dot{R}_x R_x^T + R_x \dot{R}_y R_y^T R_x^T =$$

$$R(x, \psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix}$$

$$R^T(x, \psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{bmatrix}$$

$$\dot{R}(x, \psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\sin \psi & -\cos \psi \\ 0 & \cos \psi & -\sin \psi \end{bmatrix} \dot{\psi}$$

$$R(y, \varphi) = \begin{bmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ -\sin \varphi & 0 & \cos \varphi \end{bmatrix}$$

$$R^T(y, \varphi) = \begin{bmatrix} \cos \varphi & 0 & -\sin \varphi \\ 0 & 1 & 0 \\ \sin \varphi & 0 & \cos \varphi \end{bmatrix}$$

$$\dot{R}(y, \varphi) = \begin{bmatrix} -\sin \varphi & 0 & \cos \varphi \\ 0 & 1 & 0 \\ -\cos \varphi & 0 & -\sin \varphi \end{bmatrix} \dot{\varphi}$$

$$\dot{R}_x R_x^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\sin \psi & -\cos \psi \\ 0 & \cos \psi & -\sin \psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{bmatrix} \dot{\psi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \dot{\psi}$$

$$R_x \dot{R}_y R_y^T R_x^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} -\sin \varphi & 0 & \cos \varphi \\ 0 & 1 & 0 \\ -\cos \varphi & 0 & -\sin \varphi \end{bmatrix}$$

$$\begin{bmatrix} \cos \varphi & 0 & -\sin \varphi \\ 0 & 1 & 0 \\ \sin \varphi & 0 & \cos \varphi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{bmatrix}$$

$$\dot{\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{bmatrix}$$

$$\dot{\phi} = \begin{bmatrix} 0 & 0 & 1 \\ \sin \psi & \cos \psi & 0 \\ -\cos \psi & \sin \psi & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{bmatrix}$$

$$\dot{\phi} = \begin{bmatrix} 0 & -\sin \psi & \cos \psi \\ \sin \psi & \cos^2 \psi & \cos \psi \sin \psi \\ -\cos \psi & \cos \psi \sin \psi & \sin^2 \psi \end{bmatrix} \dot{\phi}$$

$$\dot{R}_x^y R_x^T + R_x^y \dot{R}_y^y R_y^T R_x^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \dot{\psi} + \begin{bmatrix} 0 & -\sin \psi & \cos \psi \\ \sin \psi & \cos^2 \psi & \cos \psi \sin \psi \\ -\cos \psi & \cos \psi \sin \psi & \sin^2 \psi \end{bmatrix} \dot{\phi}$$

$$f = \psi = 90^\circ \text{ and } \varphi = 0^\circ \rightarrow R_{xy} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \dot{\psi} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\phi}$$

PERFORMANCE CHARTS

This chapter targets those aviation enthusiasts with an engineering background and some flare for physics and mathematics who would like to learn about flight mechanics as well as the available methods of analyzing aircraft performance data. You may be working as a test supervisor, high-level analyst, research and development scientist, or university professor. To make the experience of following this section beneficial to the greatest number of readers, the author has selected for this comprehensive case study the Cessna 172M, which is one of the numerous variants of the Cessna 172, a small single-piston-engine aircraft extensively employed for flight training as well as private and commercial rentals and ownership. The more advanced readers may apply this methodology to other aircraft. The author invites readers to take on an active role when reading this chapter and treat it as a mini-research project, applying the concepts to the aircraft of their choice and sharing their experiences with the author.

8.1 Statistical Analysis Through the Ages

Regression analysis is carried out to find the relationship between variables by fitting a mathematical model which will then predict the effect of independent variables, also known as predictors, on a dependent variable, also known as the response. For example, if you were to compress a helical spring by a certain distance (x) and to record the corresponding force (F), you would have a set of data points (x, F). You could then apply linear regression analysis to this set of data and find that $F = kx$, which will

inform you that F is linearly dependent on x . A regression analysis method helps to find the best estimate for the value of k .

The earliest method of regression analysis reported is the least squares technique presented in 1805 by Johann Carl Friedrich Gauss, a German mathematician and physicist, and in 1809, by Adrien-Marie Legendre, a French mathematician. The method was developed to determine the orbits of bodies about the Sun, especially comets and minor planets. The first official use of the term *regression* was recorded in the 1870s and attributed to Francis Galton, a British statistician, polymath, and sociologist. He applied the concept in a biological field, stating that the descendants of ancestors with tall genes tend to become shorter and eventually regress down toward a normal average. British statisticians Karl Pearson (1893) and Udny Yule (1897) expanded the application of Galton's theory from bio-species to more general statistical subjects. They assumed a Gaussian distribution for the samples. Sir Ronald Aylmer Fisher, a British statistician and geneticist known for his 1930 publication *The Genetical Theory of Natural Selection*, assumed a Gaussian distribution for the conditional distribution of response variables but not the joint distribution. Therefore, he closely followed the Gauss theory.

Use of calculator for regression analysis started in the 1950s, and it is reported that the solutions sometimes took up to 24 hrs to obtain. With development of the field of statistical analysis, regression analysis has also progressed. New methods to tackle robust regression (a method to overcome some problems related to earlier traditional methods); time series (a series of data in time order); and growth curves (a specific form of the multivariate linear model) have been presented. In any of these methods, the predictors (independent variables) or response variables (dependent variables) may be extracted from curves, images, graphs, or other complex data objects. Regression methods may also accommodate different types of missing data; these methods often include nonparametric regression models made based on available data and not the data trend, Bayesian regression (defining the true state of affairs based on the degree of belief), regressions with more predictors than observations, and regressions including causal inference.

Statistical tools are used in many fields as well as in the atmospheric sciences for weather forecasts. Recall using TAFs to learn about weather data when making a flight plan. Recall also that weather stations are not found at all aerodromes or their vicinities, meaning that the TAF data are not directly obtainable at specific locations where the flight is to commence or terminate. Adopting the weather reports from neighboring aerodromes may be possible, but it does not always result in making accurate decisions

when making flight plans, especially when weather conditions are close to the minima. For example, it is possible that the TAF reports the visibility at an aerodrome as marginally VFR while in fact it is an IFR condition. Weather may vary significantly even at distances as close as twenty miles. Therefore, Model Output Statistics (MOS) are used, which employ the raw weather data to process them statistically, considering the history of the specific location for which the weather data are to be reported, and produce a report that combines both predicted and variable data. The data may have been obtained by interpolating the predicted data between the two locations, similar to what is done when the wind data for different elevations are calculated based on the data at the adjoining altitudes. The statistical model is a numerical model over a certain period of time. It has been possible to extend MOS predictions to about three days, while the weather predictions are limited to a period of 24 hrs for TAFs. It is possible to predict the base ceiling and surface winds using MOS; however, the multiple ceiling predictions that TAFs can provide are not possible.

8.2 Statistical Analysis of Performance Data

Statistical analyses of aircraft performance data are important to both understand the current product trends and performance and also the future of the platform and industry trends—where the product is headed, what the objective of the product is, and if the direction it is headed to is aligned with its mission both technically and economically, on local and global levels. Therefore, in addition to a technical individual who will take advantage of this section and the presented methodology, a technical individual with business acumen will also learn the concepts, and so will the Black Belt Six Sigma certified champion whose mission is to decrease product defects and irregularities, and eventually enhance the company's bottom line.

Aircraft performance data are usually given in section 5 of the POH. Aircraft performance chart data are used to calculate climb performance (e.g., required fuel, time, and distance to climb), cruise performance (e.g., brake horsepower, TAS, and rate of fuel usage), and takeoff and landing performance. These charts also provide data for converting CAS to the IAS and identifying the stall speed under various conditions. These data are paramount when you are making flight plans, especially when identifying if the runway is long enough for landing or takeoff. The latter is of particular importance under extreme atmospheric conditions, when aircraft performance decreases due to high temperature, humidity, or pressure altitude.

Performance charts given in Section 5 of the *Cessna Skyhawk 1976 Model 172M Pilot's Operating Handbook* are used herein as examples. This

aircraft, introduced in 1955, has remained in production since then, and it holds the distinction of having the highest number of manufactured units (over 44,000) of any aircraft in history. It is a high wing, four-seat, single-piston-engine aircraft with fixed tricycle landing gear and a fixed-pitch, two-blade propeller. Its performance characteristics are summarized in Table 27.

Since statistical analysis methods are extensively employed in this section, the reader is encouraged to take advantage of available software tools such as Microsoft EXCEL, Minitab® Statistical Software, JMP Statistical Discovery™ Software, MathWorks® Statistics, and Machine Learning Toolbox™. Regression analysis has been heavily employed for the majority of the investigations presented herein. The reader may come across expressions such as fitted curves, significant variables, analyses summaries, and normal probability plots. The purpose of these regression analyses is to predict the relationship between a performance parameter of interest (response or dependent variable) and a few selected parameters (predictors or independent variables). To some extent, the models developed here may be extended to similar weight or category aircraft to gain a general feel for the relationship between these variables. However, the reader is reminded that only the aircraft's POH may be used for any flight-related calculations.

TABLE 27 Performance data for Cessna 172M [157,158,159,160].

Row	Aircraft	Gross Mass m [kg (lb)]	Empty Mass m [kg (lb)]	Maximum Structural Speed v_{NO} [m/s (kt)]	Maximum Flap Extended Speed v_{FE} [m/s (kt)]	Power P [kW (hp)]	Wing Surface Area A [m ² (ft ²)]	Wing Span m (ft)	Fuel Capacity $Fuel$ [gal (lit)]	Propeller
1	Cessna 172M	1,043.3 (2,300)	629.1 (1,387)	65.8 (128)	43.7 (85)	111.9 (150)	16.2 (174)	11 (36)	42 (159) 52 (197)	2-bladed fixed-pitch 1C160/DT M7553
2	Carburetor Air Temperature T [°C (°F)]	Takeoff (Ground Roll at MSL, 20 °C), m (ft)	Takeoff (Clear Over 50-ft obstacle at MSL, 20 °C), m (ft)	Landing (Ground Roll at MSL, 20 °C), m (ft)	Landing (Clear Over 50-ft obstacle at MSL, 20 °C), m (ft)	Service Ceiling km (ft)	Rate of Climb Rate MSL m/s (fpm)	Flaps Down Stall Speed m/s (kt)	Maximum Engine Speed (rpm)	Airfoil
	-15 (5) 5 (41)	272.8 (895)	480.1 (1,575)	161.5 (530)	385.6 (1,265)	3.4 (13,100)	3.3 (645)	22.6 (44)	2,700	NACA 2412

As mentioned earlier, in the majority of cases when carrying out a regression analysis, the scientist is to select a measurable parameter that identifies the performance of the system as a whole and is a function of a number of other parameters to be identified. For example, when investigating landing performance data, the parameter that describes the outcome (response) is the landing distance. One can say that this parameter is “perhaps” a function of (dependent upon) a number of other parameters (predictors). Note the use of the term *perhaps*—often the scientist does not really know which parameters are important factors in determining the landing distance. Fortunately, prejudices have little to do with performing statistical analyses and the results obtained. The previous considerations normally apply to experimental research work, where one is often not sure what effect different factors have on the outcome. Thus, one does the analysis to assess whether this factor is important or not. In a POH, the reader does not deal with mysterious unknown factors. The reason the performance tables have been created is to inform the reader of the relationship between the well-known important factors (critical few) which affect the landing distance, as in this example. If the factors were not important (i.e., trivial many), they would surely not include them.

Assume that you select a number of parameters, believing that they are important. The parameters that are proved to be important are among critical variables and the ones that are not so important are among the trivial variables. Ideally, based on your experiments and judgment, you would select the critical few variables. If you are taking this flight to deliver a bride to her groom and analysis is part of this auspicious occasion, you may categorize the tailwind and headwind as influential variables that affect your landing while the importance of the wedding dress may only be due to its weight, not its designer. By the same token, while transporting a bride to her wedding venue, the number of the bride’s eyeliner shade is not important at all, unless you experience cases where wearing a certain shade has resulted in a significant difference in average landing distance. Even so, you are to collect enough evidence (over 100 samples) for better accuracy of the statistical analysis to make clear conclusions. You may then conclude that since brides with a certain eyeliner shade may cause poor landing performances, you either refuse to deliver them to their grooms or they must apply their makeup after being delivered to their grooms. This playful example is to illustrate a methodology for treating dependent and independent variables and identifying the few that are critical to making sound decisions.

The variables that are important, also known as the critical few variables, are further categorized based on their effect on the dependent

or response variable (e.g., landing distance). If their significance is within the chosen confidence level, they are statistically significant. It is possible to have few critical variables whose significance is above 99.9 percent. A linear regression assumes a linear relationship between the response parameter and the vital few predictors, while the nonlinear regression assumes a nonlinear relationship among them. Note that *an error* (or residual) in regression analysis is defined for a particular combination of independent variables as the difference between the actual observed value and the value predicted by the regression. Ideally, though normally unlikely for experimental data, the residual would be zero and that would make the regression model (linear or nonlinear) function a perfect fit. A summary of regression analysis is usually given in the form of a table that lists the variance (mean of the square of deviation), square of variance, sum-squared (summation of the errors squared), and the root-mean-squared of the data. Obviously, the smaller the sum-squared, the less deviation from the mean value, and as a result tighter data distribution and less error. In addition, a probability plot shows the probability distribution of the residual, and it should follow a straight line in case of a perfect fit.

There are sometimes unexpected variables that come into play that you had not thought about before. You may assign a percentage to them (e.g., 5 percent) and announce that the criteria for validity of your statistical analysis—or at least what you hope to achieve—is presented within a confidence level of 95 percent. It is paramount to critically evaluate the results of a regression analysis. There are goodness-of-fit tests that may be performed in order to evaluate the applicability of the results using different techniques and confidence levels. The data may be correlated (similar trends) or cointegrated (similar trend and stationary spread). They may or may not be interrelated in any of these scenarios. For example, although you may find a similar trend in the increasing population of gibbons and plush aircraft, there is not really any real relationship between the two. The increase of the bottom line and reduction of product waste—Transportation, Inventory, Motion, Waiting, Over-production, Over-processing, Defects, and Skills (TIMWOODS)—however, are interrelated, correlated, and often cointegrated.

In the following section, the relationship between an independent variable—for example, a propeller's angular speed (*RPM*), pressure altitude (*PA*), and temperature (*T*)—for a fixed-pitch propeller scenario and dependent variables—for example, horsepower (*BHP*), true airspeed (*KTAS*), and fuel usage (*GPH*)—is described by fitted equations. Regression analysis is employed to model these characteristics for the

Cessna 172M, and the model is presented with a confidence level above 95 percent. Tables and figures—for example, Figure 70 and Table 28—are given that present the summary of the analyses and also the probability plot for the model fit. When the effect of a few selected critical-to-quality performance variables is not statistically significant with a confidence level above 95 percent, new variables are introduced, or higher-level interactions are included in the model. Except for those scenarios, selected critical variables are statistically significant. As mentioned previously, the data may be correlated, cointegrated, interrelated, or any combination of these. The examples presented as follows are to represent this relationship and what it would mean if you were to make an educated and non-educated combination of the variables or a grouping of the responses as one of the predictors. The most common-sense scenario (e.g., rate of fuel usage versus the engine angular speed, temperature, and pressure altitude) is the first scenario whose regression analysis is presented in the following sections.

The first step to make these analyses is to collect the related data from the POH; for example, one can use the data for cruise performance. The next step is to input the data into the statistical tool in its preferred method of communication. These data are usually imported in separate columns with identifying headings for each. The statistical tools are called in as the next step, where the response and predictors are identified as well as the analysis method, dominant mathematical correlation techniques, and confidence level in which the results are to be entered. You may incorporate a linear or nonlinear relationship between the dependent and independent variables. You may also define a correlation method between two, three, or all the independent variables. For example, if $F = f(x, y, z)$, F may take any of the forms $ax + by + cz$, $ax^2 + by^{1/2} + cz$, $ax + bxy + cxyz$, and so on—which present linear, nonlinear, and interrelated scenarios with cross-predictor terms. You can find thirty-four combinations of the three terms (x , y , and z) that include the interactions through the third order, cross predictors, and terms in the model. Assuming that $F = f(x, y)$, you are able to obtain nine relations and interrelations based on the aforementioned configuration. Assuming that $F = f(x, y, z, t)$, sixty-nine combinations may be derived. Figure 69 provides a relationship between the number of critical few variables and possible combinations. As you see, a cubic polynomial is a very good model for this relation, with a coefficient of determination (R^2 value) over 99.9 percent. The R^2 value describes the proportion of the variance in the dependent variable predictable by the model from the independent variables. A curve is fitted to the data so that in the future, one can predict the fitted parameter for any values between those used to estimate the model (known as interpolation). Predicting

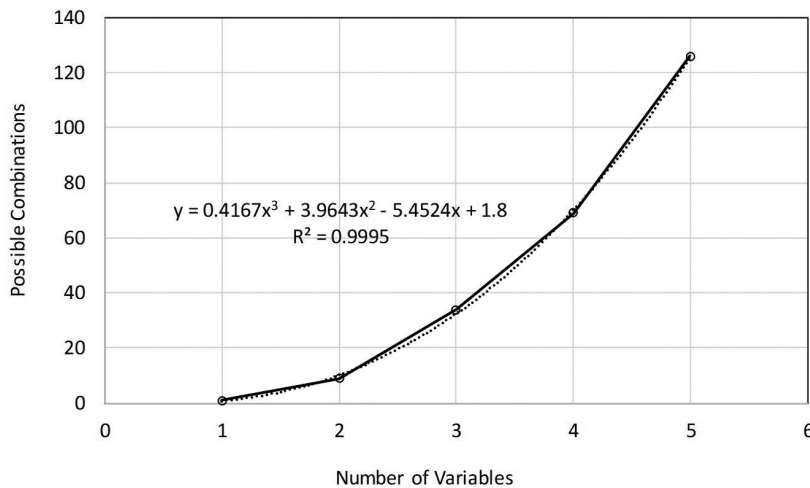


FIGURE 69 Possible numbers of the combinations versus the number of variables for the regression analysis.

outside the range of input data (extrapolation) is also possible, but there is greater likelihood that the model will deviate from the actual data. A diagram may then be presented to compare the results of the regression analysis to the raw data for specific sets of input variables.

For a higher number of independent variables (predictors), it is recommended that you pick the critical few variables judiciously as well as the order of the cross predictors. Note you cannot form a trend with only two data points—at least three are required. To investigate the influence of the second or higher order variables and their interactions, the second or higher order analysis may be performed. Although this may increase the confidence level, it makes the process, the interpretation of its results, and its implementation more complex, which discourages the use of higher order equations. Recall the truncation and roundoff errors, decreasing one far away from its optimum value increases the other one significantly. The author recommends not to engage in complex relationships unless they are worth it, which is not the case in the majority of scenarios. The analyst may end up paying much more than they gain with no substantial added benefits. In this scenario, for instance, you are to tackle additional second order or higher order terms, and this requires additional resources (e.g., computer capability, human knowledge, and associated costs in the form of money and time) for dubious gain. Perfection is to know when to stop being perfect. Keep It Simple, Stupid (KISS), “less is more” [161].

8.2.1 Pressure Altitude and Temperature

Let us investigate a relatively simple example for a regression analysis of the temperature variation with altitude. Recall that the gradient of temperature change with altitude presents the atmospheric lapse rate. This investigation is independent of the aircraft type—it only depends on the atmospheric lapse rate. The raw data uses a standard-adiabatic lapse rate of $1.98\text{ }^{\circ}\text{C}$ ($\approx 2\text{ }^{\circ}\text{C}$) per 1,000 ft ($6.5\text{ }^{\circ}\text{C}/\text{km}$) altitude with the standard sea level temperature of $15\text{ }^{\circ}\text{C}$ (288 K, 59°F). Depending on the lapse rate (i.e., reduction of temperature with altitude), which is directly related to the atmospheric conditions (i.e., dry, standard, saturated, or a combination of three in different layers), different rates are estimated. As an example, the relationship between pressure altitude in feet above the MSL (PA) and temperature in degrees Celsius (T) is presented by equation (135). Model fit is presented with a confidence level of 100 percent. In this scenario, the temperature is the response variable, and the pressure altitude is the predictor. The analysis summary (Table 28) and the probability plot (Figure 70) are presented as well. The confidence level is 100 percent.

$$T(^{\circ}\text{C}) = 15 - 0.002\text{ } PA(ft) \quad (135)$$

TABLE 28 Summary of the regression analysis model for the temperature as a function of the pressure altitude presented by equation (135).

Response	S	R-sq	R-sq(adj)	R-sq(pred)
$T(^{\circ}\text{C})$, equation (135)	0	100.00%	100.00%	100.00%

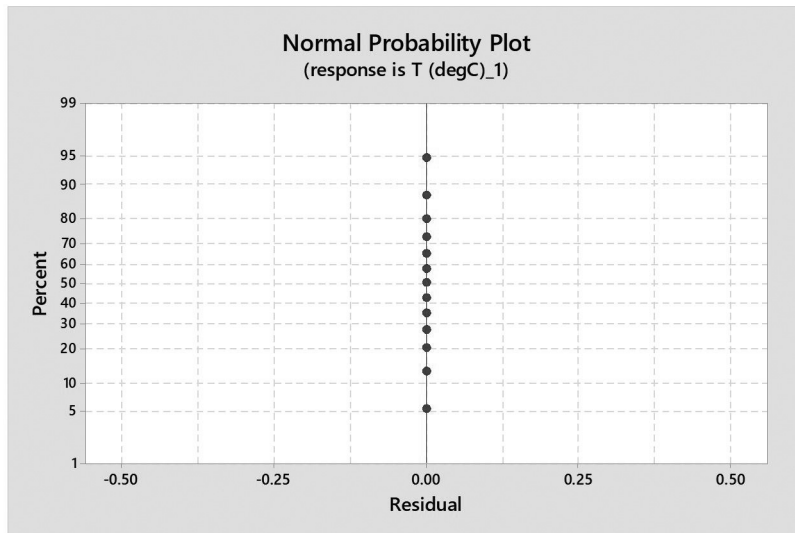


FIGURE 70 Normal probability plot for the regression analysis showing the relation between the temperature and pressure altitude presented by equation (135).

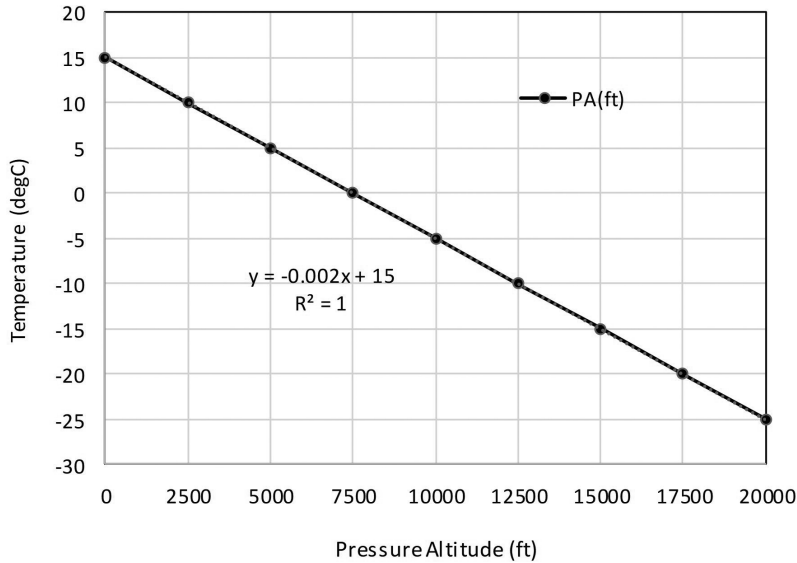


FIGURE 71 Temperature versus the pressure altitude.

Temperature (T) versus the pressure altitude (PA) is shown in Figure 71. It is seen that the fitted diagram closely follows the regression analysis results presented by equation (135). This method validates the regression analysis and also provides a visualization aid for understanding this relationship.

8.3 Cruise Performance

The cruise performance charts for the Cessna 172M describe the aircraft's performance for a range of values of the pressure altitude (PA), propeller angular speed (RPM), and air temperature (T). As output, the charts list the brake horsepower (BHP) as a percentage of full power, true airspeed ($KTAS$), and rate of fuel usage in GPH . Performances at 20 °C below the standard temperature (ISA standard temperature) and 20 °C above the standard temperature are listed. As the air temperature decreases, the power increases since the colder air is denser. The brake horsepower range reported is from 38 to 80 percent, and the altitudes range from mean sea level to 12,000 ft.

8.3.1 Rate of Fuel Usage

Rate of fuel usage in GPH (or another preferred unit) is one of the most important variables and is dependent upon predictors such as pressure altitude (PA), propeller angular speed (RPM) (for a fixed-pitch propeller),

and air temperature (T). The relationship between the rate of fuel usage (GPH) as a function of the aforementioned independent variables is presented by equation (136), where confidence level is above 94 percent. Table 29 presents the analysis summary.

$$GPH = -3.648 - 0.000132 PA(ft) + 0.004698 RPM - 0.01390 T(^{\circ}C) \quad (136)$$

Figure 72 displays the plots of the cruise fuel (GPH) versus the angular speed (RPM) for the pressure altitudes of 6,000 ft (at $-17^{\circ}C$) and 12,000 ft (at $-29^{\circ}C$). The raw data are extracted from the aircraft's POH. Error is under 6.5 percent. The regression model output are plotted with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

TABLE 29 Summary of the regression analysis model for the rate of fuel usage as a function of the pressure altitude, angular velocity, and temperature presented by equation (136) for the Cessna 172M.

Response	S	R-sq	R-sq(adj)	R-sq(pred)
<i>GPH</i> , equations (136)	0.203983	94.34%	94.16%	93.83%

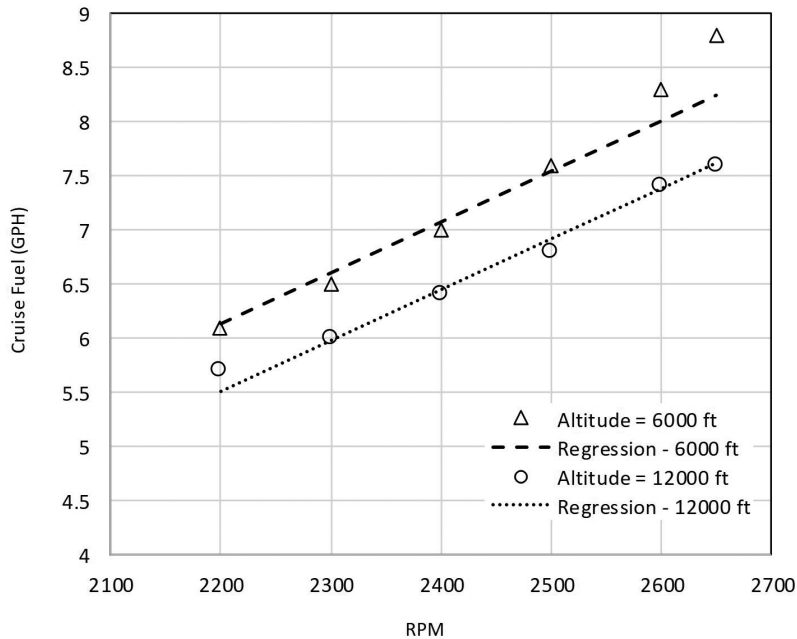


FIGURE 72 Cruise fuel versus the RPM at 6,000 ft ($T = -17^{\circ}C$) and 12,000 ft ($T = -29^{\circ}C$) pressure altitudes and indicated temperatures for the Cessna 172M.

8.3.2 Brake Horsepower

Brake horsepower (BHP), measured in percent of full power, is represented here as a function of the pressure altitude (PA), angular speed (RPM), and temperature (T). The obtained relationship is shown by equation (137), where the confidence level is above 99 percent. Table 30 presents the analysis summary.

$$BHP(\%) = -73.63 - 0.001668 PA(ft) + 0.059794 RPM - 0.16618 T(^{\circ}C) \quad (137)$$

Figure 73 shows the plots of the brake horsepower (BHP) versus the angular speed (RPM) for the pressure altitudes of 6,000 ft (at $-17^{\circ}C$) and 12,000 ft (at $-29^{\circ}C$). The raw data are extracted from the aircraft's POH. Error is under 3 percent. The regression model output is plotted with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

TABLE 30 Summary of the regression analysis model for the brake horsepower as a function of the pressure altitude, angular velocity, and temperature presented by equation (137) for the Cessna 172M.

Response	S	R-sq	R-sq(adj)	R-sq(pred)
$BHP(\%)$, equation (137)	0.953019	99.19%	99.17%	99.11%

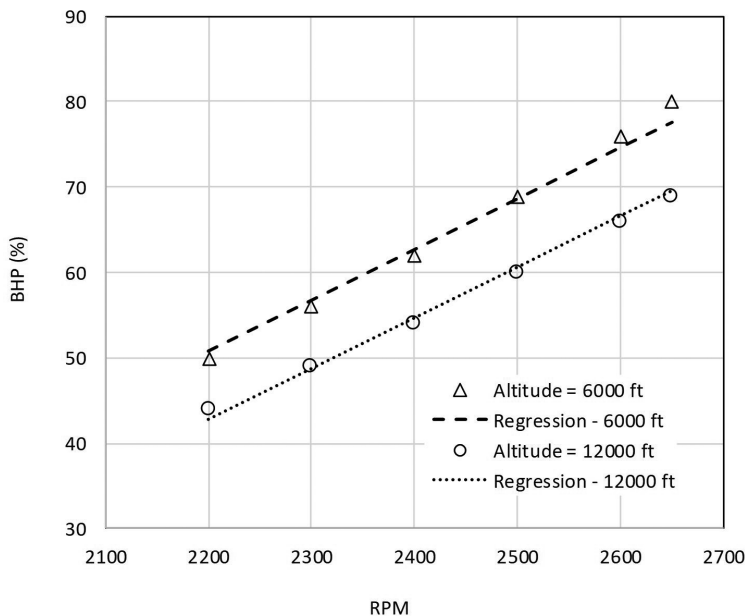


FIGURE 73 Brake horsepower versus the RPM for 6,000-ft ($T = -17^{\circ}C$) and 12,000-ft ($T = -29^{\circ}C$) pressure altitudes and indicated temperatures for the Cessna 172M.

8.3.3 True Airspeed (TAS)

True airspeed ($KTAS$) is examined here as a function of the pressure altitude (PA), propeller angular speed (RPM), and temperature (T). The relationship is shown in equation (138) with a confidence level above 99 percent. Table 31 presents the analysis summary.

$$KTAS(kt) = -30.14 - 0.000618 PA(ft) + 0.057508 RPM - 0.05809 T(^{\circ}C) \quad (138)$$

Figure 74 shows the plots of the true airspeed ($KTAS$) versus the angular speed (RPM) for the pressure altitudes of 6,000 ft (at $-17^{\circ}C$) and 12,000 ft (at $-29^{\circ}C$). The raw data are extracted from the aircraft's POH. It is seen that the two diagrams are in good agreement with an error under 1.3 percent. The regression model output is plotted with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

TABLE 31 Summary of the regression analysis model for the true airspeed as a function of the pressure altitude, angular velocity, and temperature presented by equation (138) for the Cessna 172M.

Response	S	R-sq	R-sq(adj)	R-sq(pred)
$KTAS(kt)$, equation (138)	0.682199	99.47%	99.45%	99.42%

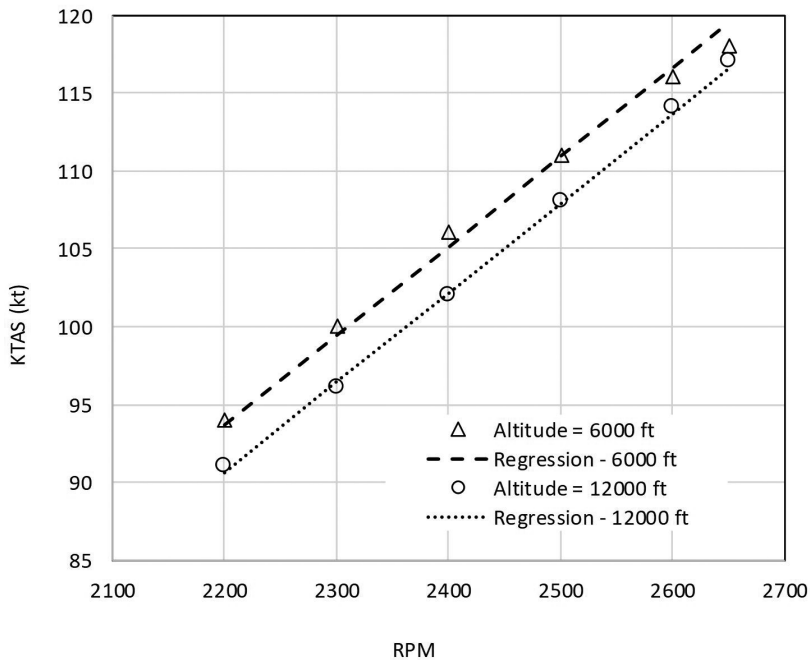


FIGURE 74 True airspeed versus the angular speed for 6,000-ft ($T = -17^{\circ}C$) and 12,000-ft ($T = -29^{\circ}C$) pressure altitudes and the indicated temperatures for the Cessna 172M.

8.3.4 Visualization of Cruise Performance Regression Analysis for a Single Critical Variable

To visualize the influence of the *RPM* on the *KTAS* and the *GPH* on the *RPM*, equations (139) and (140) are presented using regression analysis by including only one independent variable—*RPM* and *GPH* in the former and latter cases—whose results are compared with the fitted diagrams presented in Table 32, Figure 75, and Figure 76. Note the consistency between the fitted curves as well as the R-squared values, which validate the regression analysis results presented herein. The regression model output is plotted with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

$$KTAS(kt) = -31.50 + 0.05622RPM \quad (139)$$

$$RPM = 1356.4 + 157.8GPH \quad (140)$$

TABLE 32 Summary of the regression analysis model for the true airspeed and angular velocity as a function of the angular velocity and rate of fuel consumption presented by equations (139) and (140) for the Cessna 172M.

Response	S	R-sq	R-sq(adj)	R-sq(pred)
<i>KTAS</i> (<i>kt</i>), equations (139)	2.05632	95.04%	94.99%	94.86%
<i>RPM</i> , equation (140)	87.8761	69.90%	69.60%	68.62%

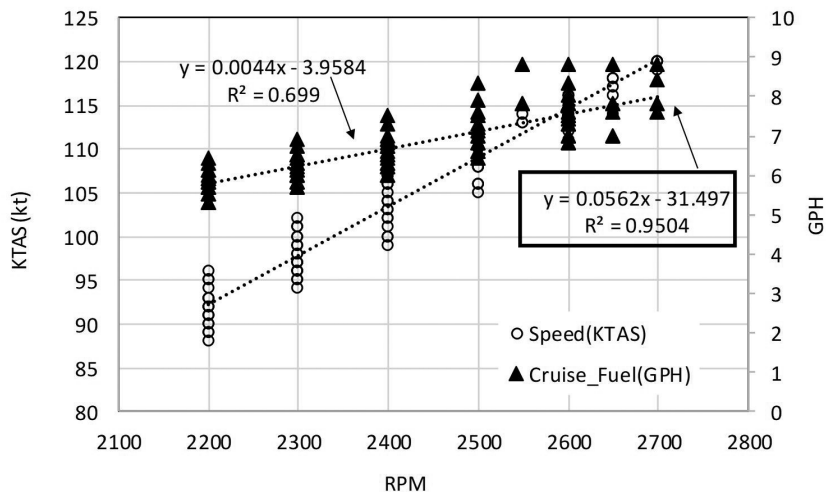


FIGURE 75 Cruise fuel and true airspeed versus the angular speed for the Cessna 172M.

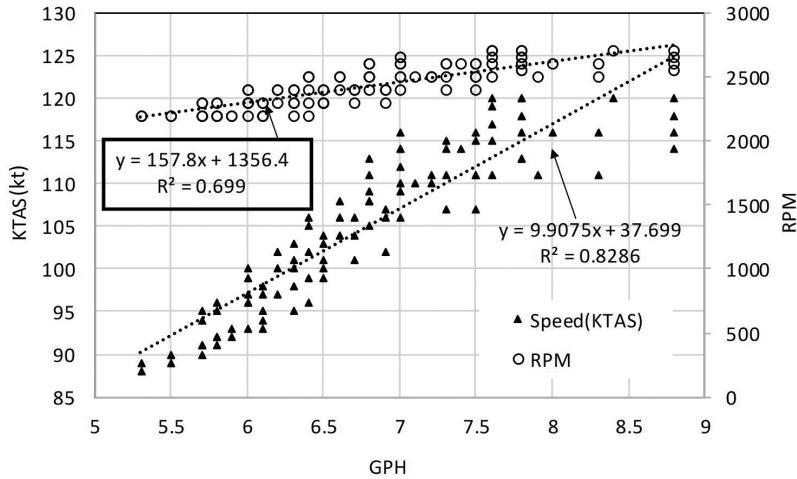


FIGURE 76 True airspeed and angular speed versus the rate of fuel usage for the Cessna 172M.

8.3.5 Indicated (IAS) and Calibrated (CAS) Airspeeds

In a majority of scenarios, the indicated airspeed is available and the data are to be calibrated to correct for the internal errors of the measurement devices. The discrepancy between the indicated and calibrated airspeeds is significant mostly at lower speeds; it is negligible at higher speeds. The difference is also more pronounced with extended flaps.

The relationship between calibrated airspeed ($KCAS$) as a function of the indicated airspeed ($KIAS$) and flap setting (FP), where they may be zero degrees, 10° , and 40° , is shown by equation (141)—where the confidence level is above 99 percent. Table 33 presents the analysis summary for the model. These fits are similar to those of the regression analysis with an accuracy of about 1.3 percent.

$$KCAS(kt) = 9.83 + 0.8960 KIAS(kt) - 0.0134 FP(^{\circ}) \quad (141)$$

Figure 77 shows the plots of the calibrated airspeed ($KCAS$) versus the indicated airspeed ($KIAS$) for 0-degree and 40-degree flap angles. The raw data are extracted from the aircraft's POH. The regression model output is plotted with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

TABLE 33 Summary of the regression analysis model for the calibrated airspeed as a function of the indicated airspeed and flap setting presented by equation (141) for the Cessna 172M.

Response	S	R-sq	R-sq(adj)	R-sq(pred)
$KCAS(kt)$, equation (141)	1.74211	99.58%	99.54%	99.41%

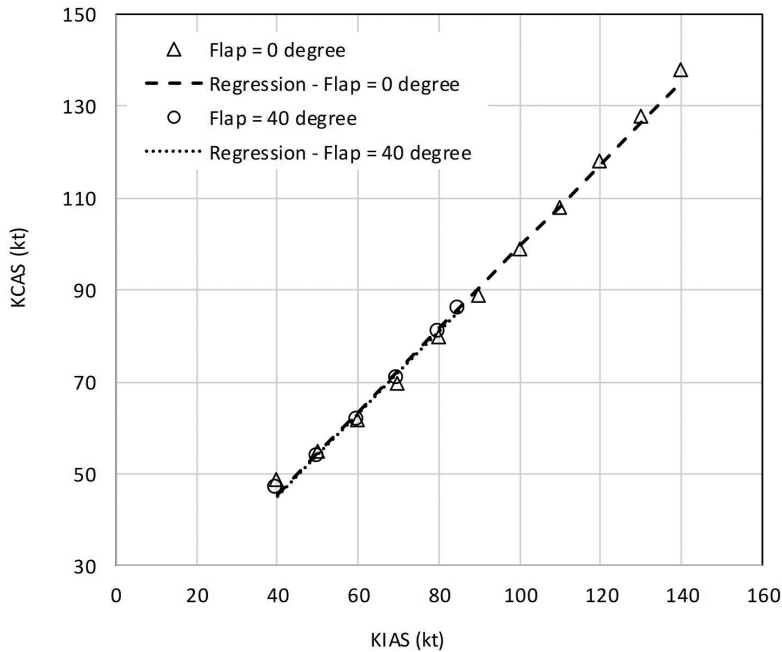


FIGURE 77 Calibrated airspeeds versus the indicated airspeed for 0-degree and 40-degree flap angles for the Cessna 172M.

8.4 Climb Performance

The three important response variables for climb performance are time (t), fuel used (GPH), and distance (d). These responses depend upon predictors such as aircraft weight (W), pressure altitude (PA), and air temperature (T)—keeping in mind that temperature and pressure altitude are interdependent. There are also other variables such as climb speed and rate of climb in feet per minute (ROC) that may be treated as response variables. Table 34 presents the analysis summary for the climb performance regression. Note that the weight affects the performance parameters. Climb performance data obtained from the POH are based on the MTOM of 2,300 lb.

Climb speed (CS) as a function of the pressure altitude (PA) is presented by equation (142), where the confidence level is 100 percent.

$$CS(KIAS(kt)) = 78.00 - 0.001 PA(ft) \quad (142)$$

Rate of climb (ROC) from the sea level as a function of the pressure altitude (PA) is presented by equation (143), where the confidence level is above 99 percent.

$$ROC(fpm) = 645.275 - 0.041648 PA(ft) \quad (143)$$

Time required to climb (t) from the sea level as a function of the pressure altitude (PA) is presented by equation (144), where the confidence level is above 96 percent.

$$t(min) = -3.07 + 0.002857 PA(ft) \quad (144)$$

Fuel required to climb ($FUEL$) from the sea level as a function of the pressure altitude (PA) is presented by equation (145), where the confidence level is above 98 percent.

$$FUEL(gal) = -0.356 + 0.000490 PA(ft) \quad (145)$$

Distance required to climb (d) from the sea level as a function of the pressure altitude (PA) is presented by equation (146), where the confidence level is above 95 percent.

$$d(NM) = -4.27 + 0.003802 PA(ft) \quad (146)$$

TABLE 34 Summary of the regression analysis model for the climb speed, rate of climb, time to climb, fuel required to climb, and distance to climb as a function of the pressure altitude presented by equations (142) to (146) for the Cessna 172M.

Response	S	R-sq	R-sq(adj)	R-sq(pred)
$CS(KIAS(kt))$, equation (142)	0	100.00%	100.00%	100.00%
$ROC(fpm)$, equation (143)	1.61165	99.99%	99.99%	99.99%
$t(min)$, equation (144)	2.35891	96.04%	95.68%	93.56%
$FUEL(gal)$, equation (145)	0.272884	98.16%	97.99%	97.01%
$d(NM)$, equation (146)	3.27678	95.70%	95.31%	93.04%

Figure 78 shows the relationship between the rate of climb and fuel required to climb versus the pressure altitude for the Cessna 172M. The fuel required for climb suggests a nonlinear function with the pressure altitude, while the rate of climb demonstrates a reverse linear relationship with the pressure altitude. The behaviors are curve-fitted with a confidence level above 98 percent. Figure 79 shows the relationship between the time to climb and climb speed versus the pressure altitude for the Cessna 172M. The time to climb also indicates a nonlinear association with the pressure altitude, while the climb speed shows a reverse linear relationship with the pressure altitude. The relationships are curve-fitted with a confidence level above 96 percent.

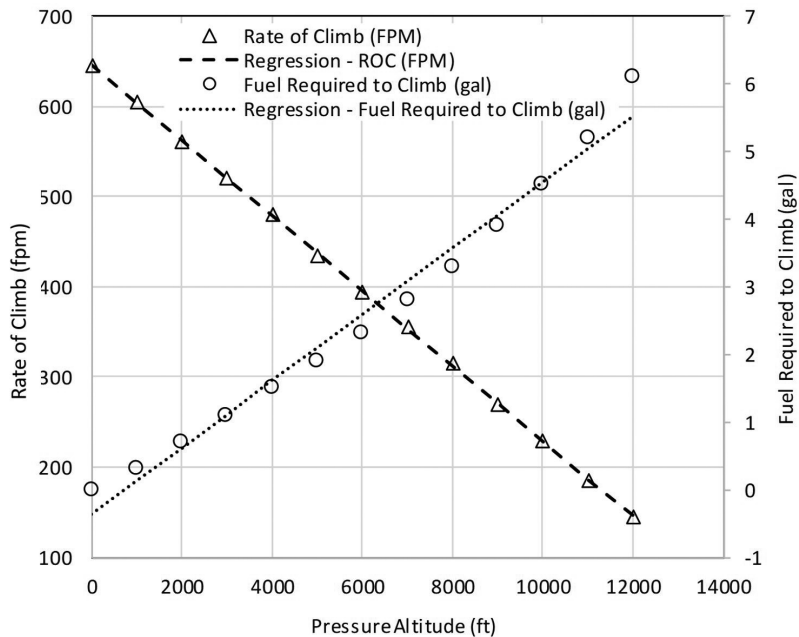


FIGURE 78 Rate of climb and fuel required to climb versus the pressure altitude for the Cessna 172M.

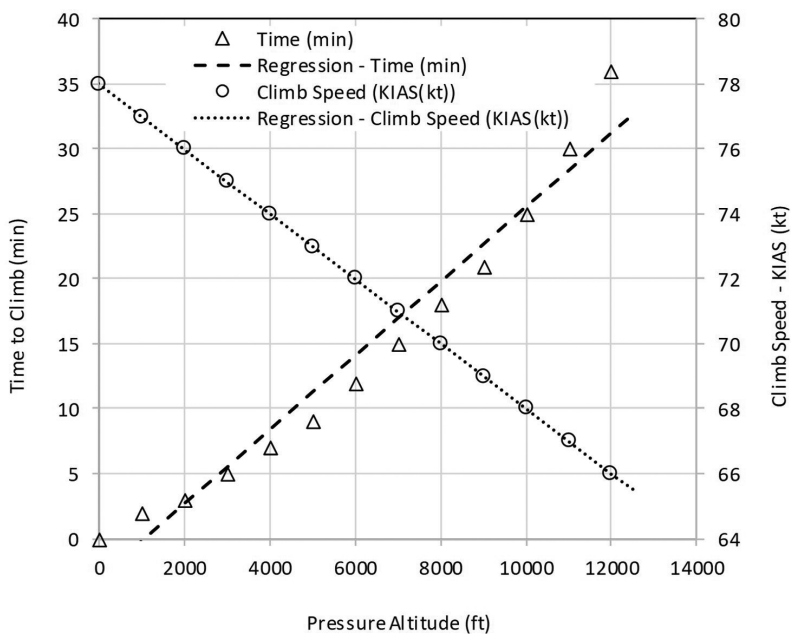


FIGURE 79 Time to climb and climb speed versus the pressure altitude for the Cessna 172M.

Figure 80 shows the relationship between the time to climb and distance to climb versus the pressure altitude for the Cessna 172M. Both diagrams suggest that the dependent variables are nonlinear functions of the pressure altitude. The models are presented with a confidence level above 95 percent. The regression model output is plotted with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

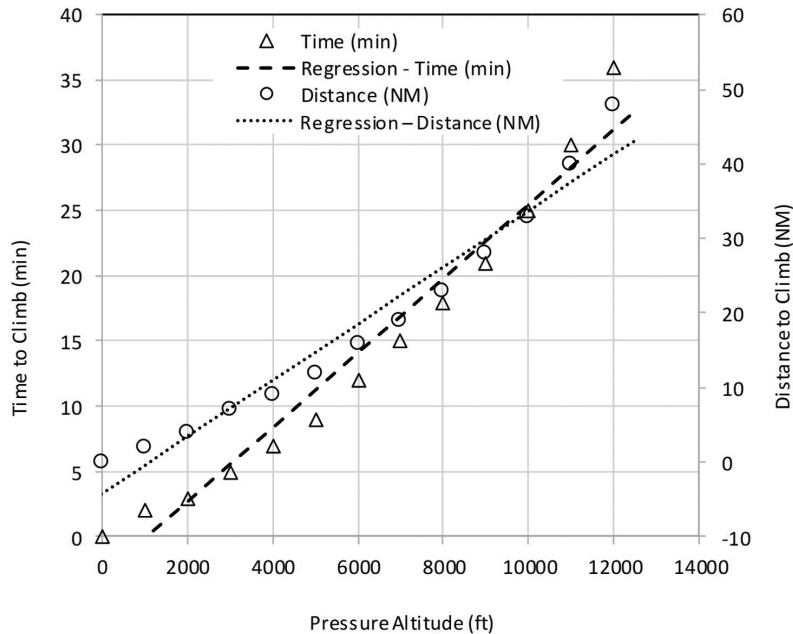


FIGURE 80 Time to climb and distance to climb versus the pressure altitude for the Cessna 172M.

8.5 Stall Conditions

Indicated and calibrated stall airspeeds are related to the flap setting and angle of bank. These two airspeeds diverge with an increasing angle of bank. The *KCAS* becomes greater than the *KIAS* as the angle of bank increases. Weight also affects this relationship, though the weight-dependent data are not presented in the Cessna 172M POH. Thus, the following analyses are performed for the MTOM of 2,300 lb. Table 35 presents the summary of the analyses.

Indicated stall speed (*KIAS*) versus the flap setting (*FP*), angle of bank (*AOB*), and center of gravity (*COG*) (the most rearward at 47.3 in and the most forward at 38.5 in) is presented by equation (147). Model fit is

TABLE 35 Summary of the regression analysis model for the indicated and calibrated stall speeds as a function of the angle of bank, flap setting, and the COG presented by equations (147) to (150) for the Cessna 172M.

Response	S	R-sq	R-sq(adj)	R-sq(pred)
$KIAS(kt)$, Multi Predictor, equation (147)	3.19059	85.94%	83.84%	79.94%
$KCAS(kt)$, Multi Predictor, equation (148)	3.36255	86.46%	84.43%	80.48%
$KIAS(kt)$, Single Predictor, equation (149)	5.27643	57.72%	55.79%	49.79%
$KIAS(kt)$, Single Predictor, equation (150)	7.6285	11.61%	7.60%	0.00%

presented with a confidence level above 85 percent. Table 35 presents the analysis summary.

$$KIAS(kt) = 72.74 - 0.1558 FP(^{\circ}) + 0.266 AOB(^{\circ}) - 0.72 COG(inch) \quad (147)$$

Calibrated stall speed ($KCAS$) versus the flap setting (FP), angle of bank (AOB), and most rearward (47.3 in) and forward (38.5 in) center of gravity (COG) is presented by equation (148). Model fit is presented with a confidence level above 86 percent.

$$KCAS(kt) = 68.72 - 0.1697 FP(^{\circ}) + 0.3117 AOB(^{\circ}) - 0.455 COG(inch) \quad (148)$$

Figure 81 and Figure 82 show the calibrated stall speed ($KCAS$) versus the angle of bank for two different flap settings for the Cessna 172M; the

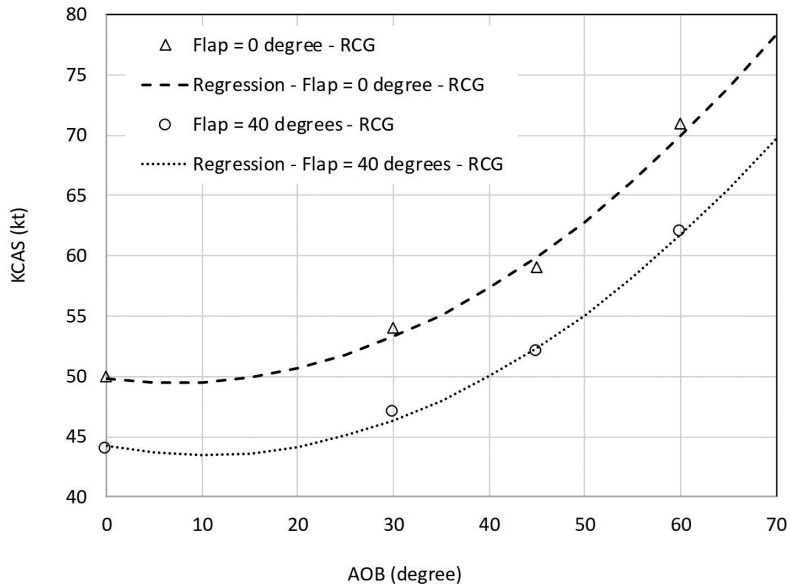


FIGURE 81 Calibrated stall speed versus the angle of bank for the most rearward COG location and 0-degree and 40-degree flap angles for the Cessna 172M.

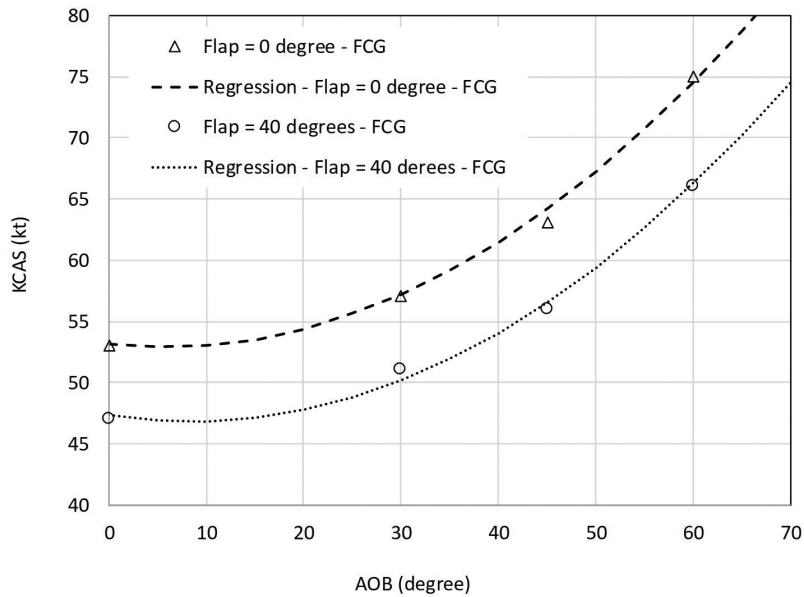


FIGURE 82 Calibrated stall speed versus the angle of bank for the most forward COG location and 0-degree and 40-degree flap angles for the Cessna 172M.

former figure for the most rearward and the latter figure for the most forward COG locations. The relationship is curve-fitted with a confidence level above 85 percent, presented by equations (147) and (148). Equations (149) and (150) present indicated airspeed (*KIAS*) as a function of the angle of bank (*AOB*) and flap setting (*FP*) as single predictors.

$$KIAS(kt) = 39.27 + 0.266 AOB(^{\circ}) \quad (149)$$

$$KIAS(kt) = 50.85 - 0.1558 FP(^{\circ}) \quad (150)$$

Figure 81, Figure 82, and Figure 83 show that with an increasing angle of bank, the stall speed increases. Increased angle of bank while turning causes higher wing loading, which requires a higher angle of attack to generate more lift and thus earlier onset of the stall (higher stall speed). The load factor is the secant of the angle of bank. For example, for a 30° and 60° angle of bank, the load factor increases by 15 and 200 percent. This is why one should be careful about steep banking as one slows down on approach. It is seen that increasing the flap extension (angle) decreases the stall speed. Flaps allow the plane to slow down to landing speed while minimizing the danger of stalling by lowering the stall speed. Extending flaps increases the wing's airfoil camber, which translates into a higher

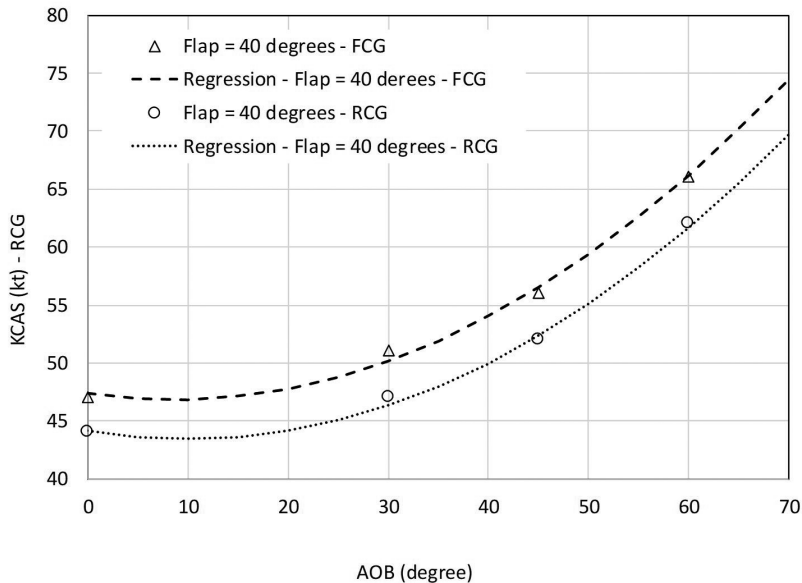


FIGURE 83 Calibrated stall speed versus the angle of bank for the forward and rearward COG locations and 40-degree flap angle for the Cessna 172M.

critical angle of attack and greater lift coefficient before stalling, while on the other hand also increasing the lift-induced drag. The regression model output is plotted in these figures with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

The stall speed increases by about 3 kt for all AOB values when the COG is forward compared to the case where the COG is rearward. The forward COG suggests higher stall speeds and harder recovery from stall conditions if it occurs while flying since the nose drops more readily, and it also suggests the possibility of wheel-barrowing. The rearward COG when landing suggests the possibility of a tail strike and higher true airspeed but also harder recovery from a stall when flying.

8.6 Fuel Usage for the Climb-Cruise Performance

Total fuel usage consists of fuel usage during taxi, climb, and takeoff. The following analysis presents a model that excludes the fuel usage for groundwork activities. Table 36 presents the summary of the analyses. The relationship between total fuel usage (*Fuel*) as a function of the pressure altitude (*PA*), angular speed (*RPM*), outside air temperature (*T*), and flight time (*t*) is presented by equation (151), where the confidence level is above 99 percent. Note that the fuel used for the groundwork is to be added to the results to obtain the total fuel used. For this aircraft, 1.1 gallons of fuel

may be added for 15-min-groundwork. Note that the selected independent variables are significant in this analysis. For the comparisons between the data presented in the POH and that of the regression analysis to be meaningful, POH temperature-dependent information should be for the same temperature.

$$FUEL(gal) = -57.348 - 0.000119 PA(ft) + 0.02535 RPM + 6.8127t(hr) - 0.05988T(^{\circ}C) \quad (151)$$

TABLE 36 Summary of the regression analysis model for the fuel as a function of the pressure altitude, angular speed, flight time, and outside air temperature using equation (151), 1.2-hour flight time for the Cessna 172M.

Fuel Components	S	R-sq	R-sq(adj)	R-sq(pred)
<i>FUEL(gal)</i> , Multi Predictor, minus Groundwork Fuel, equation (151)	1.27328	99.67%	99.67%	99.67%

Figure 84 shows the total fuel versus the angular speed for 6,000 ft (at 23 °C, which is 20 °C above the standard temperature for the altitude) and 12,000 ft (at -29 °C, which is 20 °C below the standard temperature for the altitude) pressure altitudes for the Cessna 172M. The regression analysis results are compared with the POH data for the cases shown. The regression model output is plotted in the figure with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles.

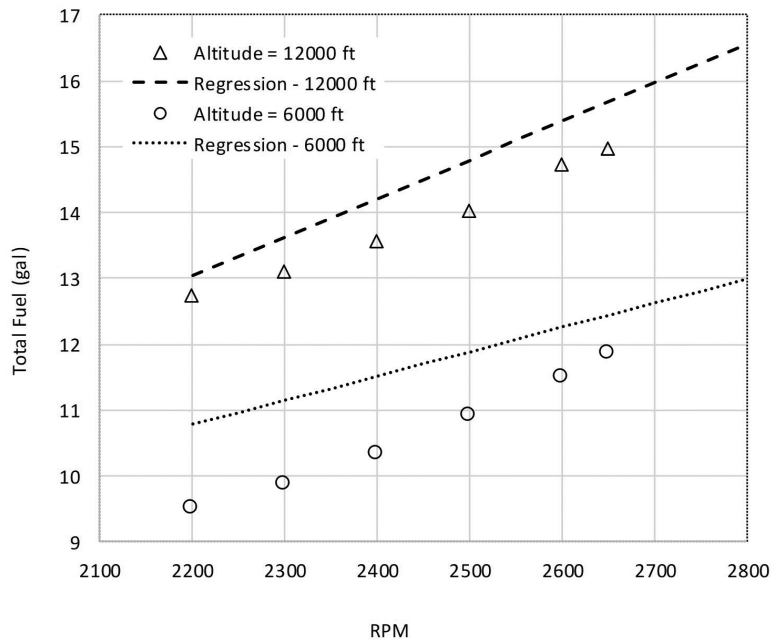


FIGURE 84 Total fuel excluding the groundwork fuel versus the angular speed for 6,000-ft ($T = 23^{\circ}C$) and 12,000-ft ($T = -29^{\circ}C$) pressure altitudes for the Cessna 172M (1.2-hour flight time).

8.7 Takeoff and Landing

The purpose of this section is to provide the information required as to what runway to choose to take off or land given its length and surface conditions along with other performance considerations that affect related decisions. The fundamentals discussed herein apply to all types and categories of aircraft, since they share similar technical language when describing takeoff and landing performances—from a very light single-engine, single-seat homebuilt aircraft such as the Bede BD-5J [162] with an MTOM of 162.7 kg (358.8 lb) and its first flight in 1971, to the Antonov An-225 Mriya [163] with an MTOM of 640 tons (1,433,600 lb) and its first flight in 1988 [164,165].

Environmental conditions such as temperature, wind, and aerodrome altitude affect landing or takeoff distances since these conditions influence aircraft performance and, as a result, the choice of aerodrome or runway to land on or take off from. Local weather will also affect the runway surface index and vertical and horizontal visibility. Another factor affecting the required takeoff or landing distance is the runway slope.

8.7.1 Takeoff Distance Requirements

Takeoff ground roll distance (GR) as a function of the temperature (T) and pressure altitude (PA) is presented by equation (152), where the confidence level is above 98 percent. Table 37 presents the summary of the analysis model fit.

For these analyses, it is assumed that flaps are up, full throttle is applied before releasing the brakes (short takeoff), the runway is paved, level, and dry, and there is zero wind.

$$GR(ft) = 695.4 + 8.205T(^{\circ}C) + 0.11798PA(ft) \quad (152)$$

Takeoff distance to clear a 50-ft obstacle ($TC50$) as a function of the temperature (T), and pressure altitude (PA) is presented by equation (153), where the confidence level is above 97 percent.

$$TC50(ft) = 1211.1 + 14.089T(^{\circ}C) + 0.22183PA(ft) \quad (153)$$

TABLE 37 Summary of the regression analysis model for the takeoff ground roll and takeoff distance to clear a 50-ft obstacle as a function of the temperature and pressure altitude presented by equations (152) and (153) for the Cessna 172M.

Takeoff Distance	S	R-sq	R-sq(adj)	R-sq(pred)
$GR(ft)$, Multiple Predictors, equation (152)	35.5837	98.50%	98.42%	98.17%
$TC50(ft)$, Multiple Predictors, equation (153)	80.6038	97.82%	97.69%	97.33%

Figure 85 and Figure 86 show the takeoff ground roll and takeoff distance to clear a 50-ft obstacle versus the pressure altitude and temperature for specific conditions (temperature is fixed in Figure 85 at 20 °C and pressure altitude is fixed in Figure 86 at 6,000 ft). The behavior is modeled with a confidence level above 97 percent. The purpose of this presentation is to visually compare the regression analysis result with the POH data in Figure 85 as a validation method and also to visualize the takeoff distances versus the pressure altitude. The regression model output is plotted in these figures with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles. One can observe that the takeoff distances increase linearly with increasing pressure altitude and temperature. From the regression coefficients, one can also say; for example, that the ground roll increases by about 118 ft for every 1,000-ft increase in pressure altitude and by about 82 ft for every 10 °C rise in air temperature.

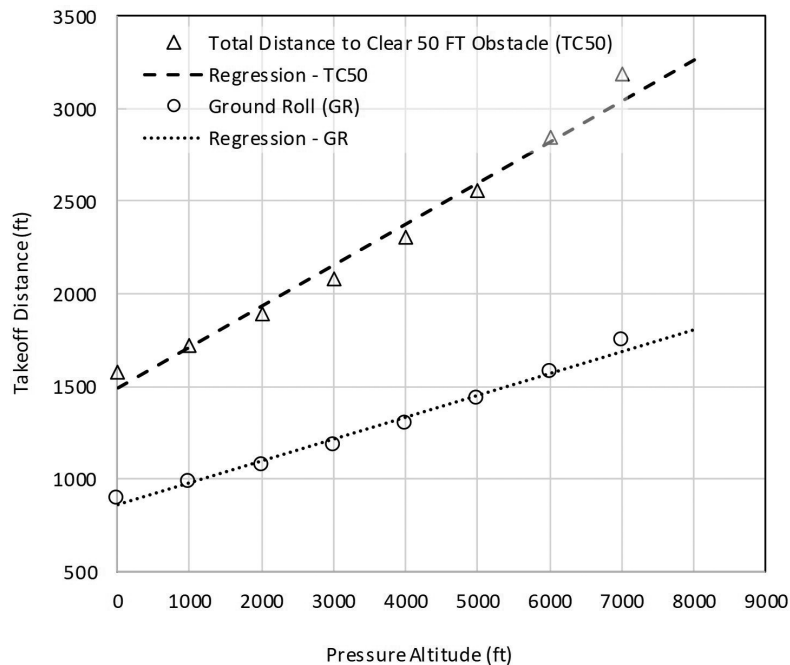


FIGURE 85 Ground roll takeoff distance and total takeoff distance to clear a 50-ft obstacle versus the pressure altitude for the Cessna 172M ($T = 20\text{ }^{\circ}\text{C}$).

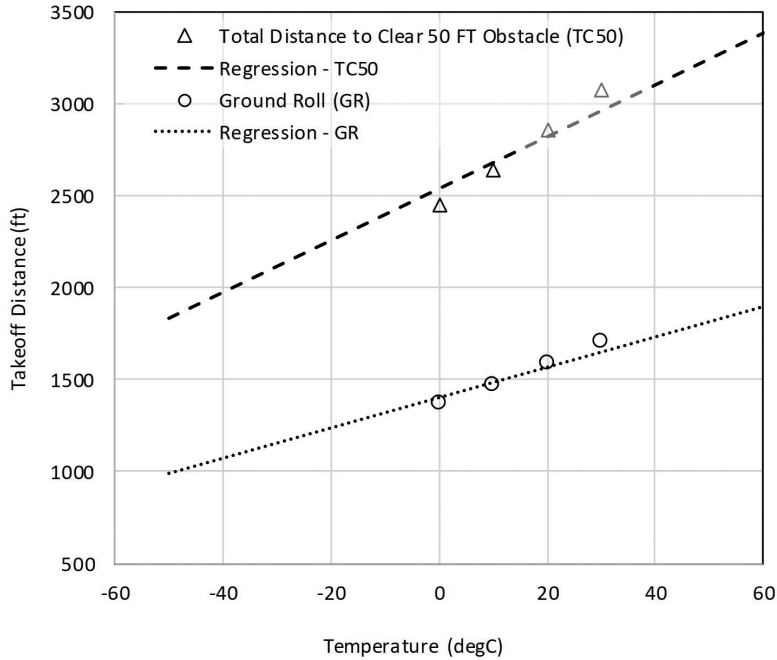


FIGURE 86 Ground roll takeoff distance and total takeoff distance to clear a 50-ft obstacle versus the temperature for the Cessna 172M ($PA = 6,000$ ft).

8.7.2 Landing Distance Requirements

Landing ground roll distance (GR) as a function of the air temperature (T) and pressure altitude (PA) is presented by equation (154), where the confidence level is above 99 percent. For these analyses, it is assumed that the flaps are set to 40° (short landing), the throttle is idle, maximum braking is applied, the runway is paved, level, and dry, and there is zero wind. The extra quadratic terms are included in the fitted model as a result of several trials to represent the relationship more accurately.

$$GR(ft) = 493.03 + 1.814T(^{\circ}C) + 0.0175PA(ft) - 0.0004T(^{\circ}C)^2 + 0.00008T(^{\circ}C)PA(ft) \quad (154)$$

Landing distance to clear a 50-ft obstacle ($TC50$) as a function of the air temperature (T) and pressure altitude (PA) is presented by equation (155), where the confidence level is above 99 percent.

$$TC50(ft) = 1186.20 + 3.6444T(^{\circ}C) + 0.039117PA(ft) \quad (155)$$

Figure 87 and Figure 88 present landing distance for the case of a ground roll and to clear a 50-ft obstacle as functions of the pressure altitude

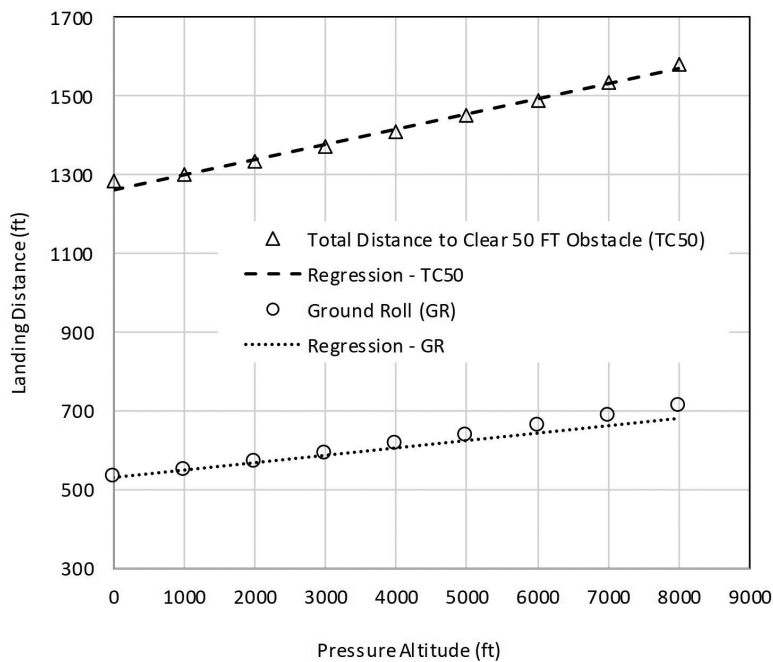


FIGURE 87 Ground roll landing distance and total landing distance to clear a 50-ft obstacle versus the pressure altitude for the Cessna 172M (T = 20 °C).

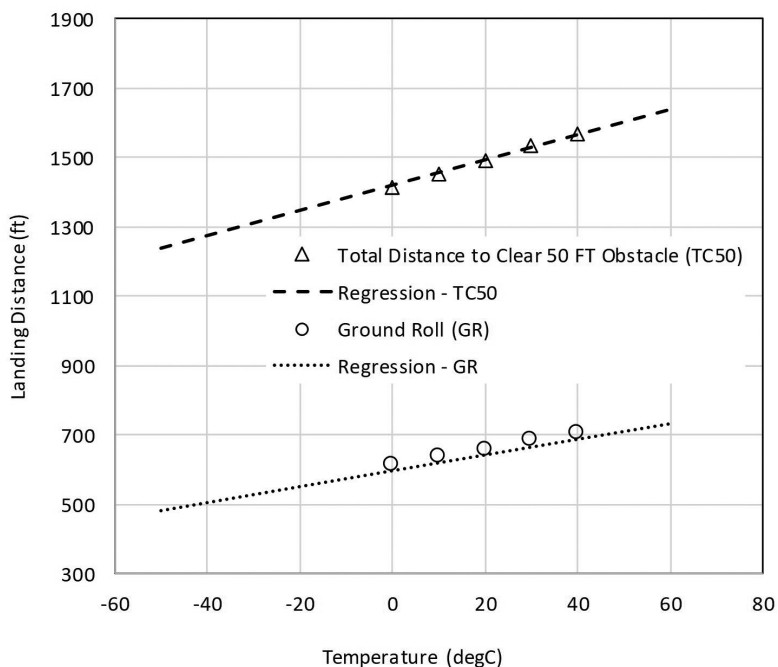


FIGURE 88 Ground roll landing distance and total landing distance to clear a 50-ft obstacle versus the temperature for the Cessna 172M (PA = 6,000 ft).

and temperature for specific scenarios (temperature is fixed in Figure 87 at 20 °C and pressure altitude is fixed in Figure 88 at 6,000 ft). The behavior may be modeled with a confidence level above 79 percent. The purpose of these plots is to compare visually the regression analysis results with the POH data in Figure 87 and Figure 88 as a validation method and also to visualize the landing distances versus the pressure altitude. The regression model output is plotted in these figures with either dashed or dotted lines, while the raw data are shown by hollow circles or triangles. Table 38 presents the summary of the analysis model fit.

A comparison is made between the suggested regression models (dashed lines) and POH data (triangle and circle hollow dots), and the errors are about 0.25 percent for the total distance to clear a 50-ft obstacle and about 2.5 percent for the ground roll distance. The plots show that the landing distances increase linearly with increasing pressure altitude and temperature. From the regression coefficients one can also say; for example, that the landing ground roll increases by about 18 ft for every 1,000-ft increase in pressure altitude, ignoring the small effect of the temperature and PA product term.

TABLE 38 Summary of the regression analysis model for the landing ground roll and landing distance to clear a 50-ft obstacle as a function of the temperature and pressure altitude presented by equations (154) and (155) for the Cessna 172M.

Landing Distance	S	R-sq	R-sq(adj)	R-sq(pred)
<i>GR(ft)</i> , Multiple Predictors, equation (154)	1.49184	99.96%	99.95%	99.94%
<i>TC50(ft)</i> , Multiple Predictors, equation (155)	8.43639	99.49%	99.46%	99.39%

CHAPTER 9

NAVIGATION

Navigation involves steps taken to determine precisely one's location, to plan a path toward the desired destination, and to successfully follow that path. It is a word rooted in the early sixteenth century, from the Latin word *navigatio* from the verb *navigare* (i.e., *navis* for *ship* and *agere* for *drive*). The action may be done in the sky, land, sea, and space [166]. It is also the field in which navigators put their art to good use, which is to find their location relative to the heavenly bodies and known locations—also known as fixes. The object to navigate is a ship, aircraft, or any other form of transport, and it is to be performed using instruments or maps. For navigation to be completed (i.e., to complete a route) in a proper fashion, you need to know your current location (i.e., start point) and destination (i.e., end point)—also known as the line of position (LOP). The LOP may be in the form of an arc, circle, hyperbola, or straight line. The duration of your trip (time) from the start to the end point depends on how far you are from the destination and with what speed you are moving toward it. This relates to the concepts of length and direction that, combined with time, lead to their derivatives such as velocity and resources required to complete the desired route. Depending on your means of transportation, the desired route may be affected by experiences along the way—fuel stops, food breaks, the times you need to answer the call of nature, or the roses you may wish to smell.

Thus, the art of navigation is to be able to identify vectors by their magnitudes and directions and create geometrical shapes in order to

achieve what is desired—getting to the destination safely and on time. This art, which is a branch of applied geometry, found its way among the seven articles of the liberal arts known—until the ninth century—as weaving and tailoring, agriculture, architecture and masonry, hunting and martial arts, trade, cooking, and metallurgy. The modern interpretation of the liberal arts, since the start of the twelfth century, also known as mechanical arts, has added medicine, theatrical arts, and navigation, replacing the agriculture, cooking, and trade. In the beginning of the nineteenth century, the concept of the mechanical arts was separated from the fields associated with groups such as art performers, fine artists, and intelligentsia, and became a practical field now commonly known as engineering.

9.1 Navigation Through the Ages

The earliest form of long-range navigation is associated with the Polynesians, who used celestial locations, natural variations of wildlife species, and the shape of water waves in order to travel and find dwellings on the oceanic islands. The concept of scientific instruments, such as early astrolabes, is often associated with Hipparchus—a Greek astronomer, mathematician, and geographer from about 120 BC—who showed that the circles on the sphere that do not pass through the center of the Earth (i.e., the projection) can conform to the circles on the plane. The early astrolabe was invented around 150 BC in the Hellenistic civilization by Apollonius of Perga—a Greek geometer and astronomer with known theories on conic sections. Muhammad al-Fazari—a Persian mathematician, philosopher, and astronomer, the son of Ibrahim-al-Fazari, also an astronomer and mathematician in the eighth century—introduced angular scales to the original design to identify the azimuth on the horizon and the direction of Mecca and the time for sunrise and fixed stars. Abd-al-Rahman Sufi—the tenth century Persian astronomer and the well-known author of *Book of Fixed Stars* after whom the lunar crater Azophi is named—introduced more than 1,000 uses of astrolabes in fields varying from astronomy to navigation, surveying, timekeeping, and astrology. For the next period, more developments of the design and use of astrolabes were made that included the armillary sphere (Abu Abbas Al-Nayrizi, a Persian mathematician and astronomer in the ninth century known for compiling astronomical tables and atmospheric phenomena) and the linear astrolabe (Sharaf-al-Din Tūsī, a Persian mathematician and astronomer in the twelfth century after whom the main-belt asteroid 7058 Al-Tūsī, discovered by Henry E. Holt, an American astronomer, in 1990, was named).

The simplest version of the astrolabe is the *balesilha*, used by sailors to measure latitude for sea voyages. It was employed by the Portuguese sailors sponsored by Prince Henry in 1418 during early Atlantic coast explorations reaching the Indian ocean in 1488, sailing to the Indies, leading to the discovery of America in 1492 by Christopher Columbus, an Italian explorer and navigator, as well as sailing to India in 1498 and China in 1512. In the fifteenth century, Jean Fusoris, a French instrument maker, made astrolabes in his shop in Paris, thirteen of which still survive today. In the 1500s, Georg Hartmann—a German engineer, instrument maker, and astronomer—was the first to fabricate this ancient instrument in large quantities.

The pinnacle of marine navigation was the circumnavigation of the Earth by Ferdinand Magellan—a Portuguese explorer who was in charge of the 1510–1522 expedition later named after him. After Magellan's 1521 death in a battle with native tribes in the Philippines, it was Juan Sebastián Elcano, a Spanish navigator, who took command of the remaining ships (2 out of 7), crossed the Indian Ocean, rounded the Cape of Good Hope, and returned to Spain with only one ship (aptly named *Victoria*). While the westward crossing of the Pacific was known and used by Magellan, the opposite, eastward path from the Philippines back to the Americas—the path also known as *Tornaviaje*—was only discovered forty years later, when Andrés de Urdaneta, a Spanish cosmographer, sailed from the Philippines, north to parallel 39°, and hit the eastward Kuroshio Current, which guided his galleon across the Pacific, arriving in Acapulco in 1565.

The start, destination, and via points are identified on the maps and connected by means of lines. The navigator picks two or more locations with geographic coordinates to draw the LOP. The navigator is always located at the intersection of the two lines of position, or for three lines of position, somewhere within the triangle bounded by them, given the difficulty that having three lines meet at one point introduces. The intersection of the LOPs is a fix. There are different ways to navigate using a single LOP: (a) celestial observation—which is identifying the position of the heavenly bodies, represented by points of equal altitudes, by LOPs obtained from the almanac and spherical trigonometry; (b) terrestrial range—in which two desired points are aligned; (c) compass bearing to a point of interest; (d) radar range to a desired point; (e) a depth sounding generated from an echo sounder on some coastlines or a hand lead line; and (f) frequent identification of current position with respect to geographical features—also known as pilotage.

One traditional technique known as rising and dipping lights is not widely used anymore. In this method the concept of Line of Sight (LOS) is employed, where a light source (e.g., a sea lantern) visible on the horizon may be employed as a guide to identify the bearing to establish an educated guess for the fix. Given the spherical shape of the Earth and that the light intensity may vary depending on the LOS (i.e., the distance from the source and the height of eye level compared to that of the source), the dimming light increases in intensity until it is bright enough to be identified by the navigator and compared using an almanac table in order to calibrate for the distance away from the light source. The position of the fix is then finalized.

Modern navigation has become far more accessible with the introduction of satellite-based navigation to augment radio-based and radar-based navigation. Satellite-based navigation includes the Global Navigation Satellite System (GNSS), GPS, Global Navigation Satellite System (GLONASS) used in Russia, BeiDou navigation Satellite system (BDS) used in China, Global Navigation Satellite System (GNSS Galileo) used in Europe, Indian Regional Navigation Satellite System (IRNSS), and Quasi-Zenith Satellite System (QZSS) used in Japan. There are older satellites that are not active anymore, such as the Soviet MKRC Legenda targeting system connected to specialized missiles (e.g., SS-N-19 missile), which consisted of the Signals Intelligence (SIGINT) satellites and nuclear-powered Radar Ocean Reconnaissance Satellites (RORSAT), which collected intelligence by intercepting signals in the former case and performed reconnaissance missions in the latter case.

Radio-based navigation includes measurements such as distances, directions, and velocities to and from electronic beacons, which are achieved by means of interferometry, travel times, and radio doppler shift, respectively. In the majority of these cases there is a sensor that senses the location of antenna broadcasting information along with the conventional techniques to obtain a radio fix. Radio-based methods include: (a) systems to measure bearings (e.g., Radio Direction Finding—RDF, Reverse RDF—RRDF, and Non-Directional Beacon—NDB, consisting of an Automatic Direction Finder (ADF) instrument and an NDB transmitter, and VHF Omnidirectional Range—VOR); (b) transponder systems (e.g., radar and transponders, bombing systems, beacons, and Distance Measuring Equipment—DME); (c) beam systems (e.g., lorenz, Low Frequency Radio range—LFR, and glide path and localizer of Instrument Landing Systems—ILS); and (d) hyperbolic systems (e.g., GEE, LORAN, and Decca, LORAN-C). There are also systems employed by the military, such

as Tactical Air Navigation systems (TACAN) and a combination of VOR and TACAN (VORTAC). Radar-based navigation uses radar to identify the bearing and distance from the point of interest and is not to be confused with the use of a radar as a means for situational awareness.

Automatic Dependent Surveillance Broadcast (ADS-B) allows an aircraft to identify its position with respect to a satellite and to report it to other receivers capable of receiving the information, either on the ground or in the air, to facilitate situational awareness and maintain safe separation. This is part of the United States next generation air transport system (NexGen), which is also connected to ICAO global plan initiatives, Aviation System Block Upgrades (ASBU), and Single European Sky ATM Research (SESAR). This system is already mandatory in some parts of Australian airspace and will be required for some United States aircraft by 2020, while the implementation has already started in Europe since 2017. Canada uses this system for traffic control in the Hudson Bay region, which lacks radar coverage [167].

For short-distance navigation, GPS systems with receiver capability, such as the Wide Area Augmentation Systems (WAAS) and GLONASS, are currently being employed. Implementation of the augmentation systems allows for more accurate positioning, especially around the aerodromes, to minimize reliance on the localizers and at the same time maximize the accuracy of the GPS-based guidance for the approach, both horizontally and vertically [168].

9.2 Land Survey and Mapping Through the Ages

Land survey is an ancient practice dating back to 2000 BC, or possibly even prior to that. Groma, a surveying instrument, was invented in Mesopotamia in the first millennium BC. Stonehenge in Wiltshire, England, was built about 2500 BC by means of pegs and ropes. Researchers have investigated the accuracy of using pegs and ropes in land survey and specifically during construction of the Neolithic 56 Aubrey Hole circuit at Stonehenge. Studies carried out with computer-aided tools found that this simple yet very accurate technique has resulted in a regular 56-sided polygon, which may be employed to accurately approximate a regular heptagon. The comparison of this method with earlier methods suggests that it is both simplest and simultaneously the most accurate method of its caliber [169]. It is speculated that the Great Pyramid of Giza in Egypt, built about 2700 BC, used this method. One of the publications associated with this field is *The Seal Island Mathematical Manual* by Liu Hui, a

Chinese mathematician in Three Kingdom (era 220–280); this book itself was an extension of an earlier work, *The Mathematics Art*, compiled by the generations of scholars between the tenth and second centuries BCE. Romans treated the field (*Gromatici*) as a profession for tax and land registry purposes starting in 300 AD. William, the first Norman king of England, also known as the Conqueror, was one of the pioneers in land survey use; he commissioned a book to keep a record of the land locations and areas along with their owners and contents in 1086 [170,171,172,173].

Abel Foullon, a French descriptor in 1551, presented what is reportedly the earliest description of a plane table. Leonard Digges, a British mathematician and surveyor in 1571, described a theodolite—an instrument to present angles in the horizontal and vertical planes that is still used for land surveying, meteorology, and even rocket launches. A compass was first incorporated into a theodolite in 1576 by Erasmus Habermel, a Czech watchmaker and instrument maker. In 1615, Willebrord Snellius, a Dutch astronomer and mathematician, introduced the modern systematic use of triangulation. Edmund Gunter, a British mathematician and astronomer, introduced in 1620 a Gunter's chain—a distance measuring device used for land survey for legal and commercial purposes. The first telescope was incorporated into a theodolite in 1725 by Johnathon Sission, a British instrument maker [174,175,176,177,178,179].

The method of triangulation was adopted by Jacques Cassini, a French astronomer, and his son César-François Cassini de Thury, also a French astronomer and cartographer, in the first triangulation of France between 1733 and 1740, resulting in the resurveying of the meridian arc and the first map of France in 1745 that spread to the rest of the Europe by 1784. In 1787, Jesse Ramsden, a British mathematician and instrument maker, introduced the first precision theodolite. Major-General William Roy, a Scottish engineer and surveyor, started in 1784 the first high-precision trigonometric survey of the whole of Great Britain, also known as the *Principal Triangulation of Britain*, which was completed in 1853. The Great Trigonometric Survey of India, measuring the entire Indian subcontinent with scientific precision, began in 1802 under the supervision of General William Lambton, a British surveyor and geographer. By 1830, his successor in this monumental endeavor was George Everest, a British surveyor and geographer whom Mount Everest is named after. The survey continued in 1843 under Major-General Andrew Scott Waught, a British surveyor, was completed under General James Walker, an Anglo-Indian Surveyor between 1860 and 1871. Mount Everest and the other Himalayan peaks were mapped during this grand survey [180,181,182,183,184].

The pioneering efforts of William the Conqueror in the eleventh century were followed by Napoleon Bonaparte, who in 1808 initiated continental Europe's first cadastre, a comprehensive recording of real estate, including owners, land size, and its usage, which quickly spread to the rest of Europe. During the nineteenth century, surveying became more accessible and modernized, and the detailed geographical features, such as city and road maps, were further developed. In 1858, Sir Robert Richard Torrens, the third premiere of the south of Australia, introduced the Torrens system, which was built on the initiatives of William the Conqueror and Napoleon and simplified land transactions through a centralized land registry. Torrens's system was adopted in some other countries, especially with arrival of railroads in the 1800s. Examples include Canada (1861), New Zealand (1870), Ireland (1892), Thailand (1901), the Dominican Republic (1920), the Philippines (1922), Palestine and Israel (1948), Singapore (1960), Malaysia (1965), and (limited implementation) the United States [185,186,187,188,189].

In 1957, Trevor Lloyd Wadley, a South African engineer known for the development of the Wadley loop circuit, invented the tellurometer to measure long distances using microwave transmitters and receivers. Electronic Distance Measurement (EDM) equipment, or the range finder, was introduced in the 1950s and has been used in virtual reality applications since the 1990s. It employs a multi-frequency phase shift of light waves and saves considerable time of chain measurement requiring measuring between points kilometers apart. EDM equipment decreased in size with further development. In the 1970s, equipment was developed integrating electronic theodolite with EDM in order to read the slope between the instrument and a reference point; the unit was equipped with a DAQ system connected to a computer and a program that could make advanced coordinate-based calculations. The Navy Navigation Satellite System (NNSS), also known as the Transit system, was the first satellite navigation system, and it was launched in 1960 for the purpose of providing coordinate positions to Polaris missile submarines. It was later adopted by surveyors to find the location of benchmark points at remote locations. The Global Positioning System (GPS) was launched by the United States Air Force in 1978; it improved accuracy of the Navy's system by using a larger constellation of satellites and better signal transmission. With the improvement of the GPS systems, the original accuracy that was achieved only by recording and averaging data by a static source over hours was improved to become the efficient and effective method of Real Time Kinematic (RTK) surveying. RTK made collection of accurate data possible

by means of a fixed base station and a second roving antenna whose position may be tracked [190,191].

Nowadays, theodolite, total station, and RTK GPS survey methods are the main surveying techniques. Remote distances are being sensed and imaged topographically by satellites by means of three-dimensional scanning and lidar—a distance-measuring method illuminating the target with pulsed lasers and measuring the reflected pulses with a sensor. UAV technology, or Remoted Pilot Vehicles (RPV)—also known as drones, aircraft without a human pilot aboard—along with photogrammetric image processing, the science of using photographs to make measurements by recovering the exact locations of the surface points, are employed as well [185].

9.3 Latitude, Longitude, and Great Circle

Since the Earth is not a perfect sphere, but a spheroid, also known as an ellipsoid of revolution, the spherical relationships need to be corrected to make accurate navigation possible. Earlier in history, there was a period of scholarly argument on the exact nature of the shape of the Earth: whether it is oblate or prolate. An oblate Earth is a spheroid stretched over its Equator (like a tomato); a prolate Earth is a spheroid stretched over its Poles (like a lemon). In 1687, Sir Isaac Newton determined that the Earth was oblate (flattening of $1/230$), which was later confirmed by the measurements made in the eighteenth century. In 1669, Jean Picard, a French astronomer, measured the size of the Earth with reportedly high accuracy. In 1672, Giovanni Domenico Cassini, an Italian naturalized French engineer, mathematician, and astronomer, and later in 1713 his son Jacques Cassini, a French astronomer, measured the arc of the meridian and concluded that the Earth was prolate. The French Academy of Science then sponsored an expedition—also known as the French Geodesic Mission—in 1735 to investigate this finding, which resulted in confirming Newton's findings. In 1743, Alexis Claude Clairaut, a French mathematician, astronomer, and geophysicist, presented Clairaut's theorem, which stated the relationship between the gravity at the surface of the Earth to the position of the point based on the assumption that the Earth was oblate—a rotational ellipsoid. In the eighteenth century, different astronomers and geodesists studied the Earth's curvature, and as a result several ellipsoid models for the Earth were introduced—by Plessis in 1817, Airy in 1830, Bessel in 1830, Everest in 1830, and Clarke in 1866 [192,193].

Latitudes along with longitudes identify the distance of a point with respect to the reference meridian, also known as the prime meridian and

occasionally Greenwich meridian, and with respect to the Poles. Longitudes vary from zero degrees at the reference meridian to -180° westward and $+180^\circ$ eastward, which is the dipole of the reference meridian. Latitudes vary from zero at the Equator to $+90^\circ$ at the North Pole and -90° at the South Pole. Lines of equal longitude are called meridians; they are semicircles joining the North and South Poles. Lines of equal latitude are called parallels; they form circles around the Earth parallel to the Equatorial plane.

As a result of the Earth measurement efforts, three different kinds of latitudes have been introduced for a point “P” on the Earth’s surface: (a) geocentric (L_G)—angle between the Equator plane and the line passing through the center of the Earth; (b) astronomical (L_A)—angle between the Equator plane and a plumb line due to gravity; and (c) geodetic (L_N)—angle between the Equator plane and a normal to the tangent plane at “P” (Figure 89). Since the Earth is a spheroid, the normal to the local tangent at “P” does not pass through the center of the Earth, except at the Poles and the Equator. If someone uses the term *latitude* without specifying its type, the convention is to assume that it is a geodetic latitude.

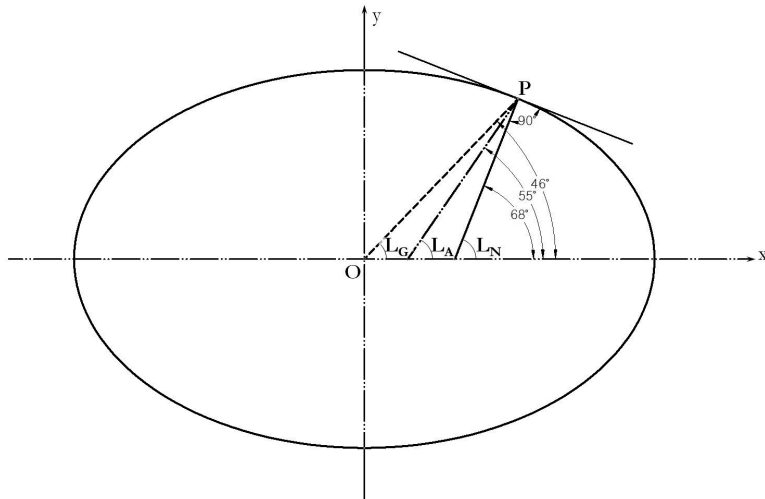


FIGURE 89 Geocentric (L_G), astronomical (L_A), and geodetic (L_N) latitudes (drawings created using Solid Edge CAD tool).

9.3.1 Latitude

Latitude corresponds to the distance from south to north on the Earth’s surface and varies from zero degrees on the Equator to $+90^\circ$ at the North Pole and negative -90° at the South Pole (Figure 90). Parallels are lines

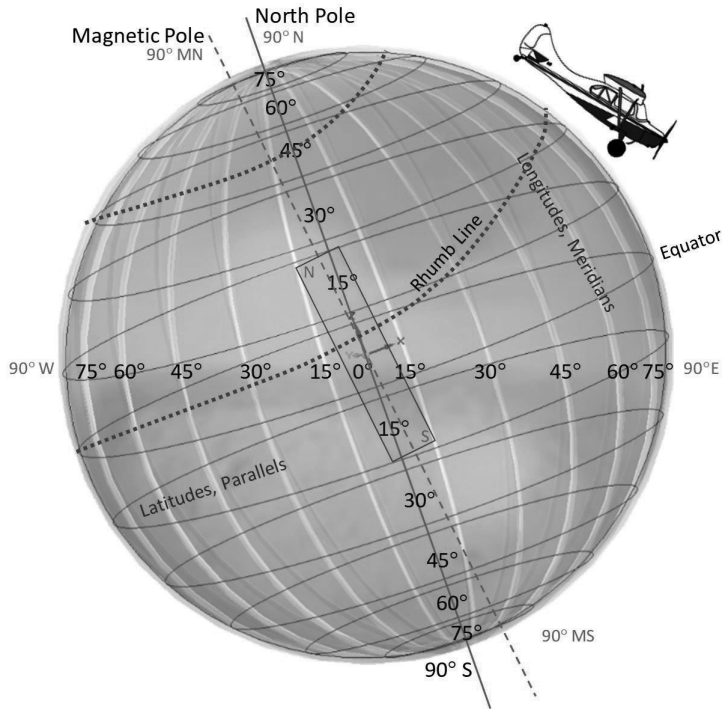


FIGURE 90 True and magnetic Poles, rhumb line, latitudes, and longitudes (drawings created using Solid Edge CAD tool).

(circles) of constant latitude on the Earth's surface. When navigating in the Northern Hemisphere by observing Polaris, also known as the North Star, with a sextant and using tables to correct for the height of eye and atmospheric refraction, the latitude of the observer is the height of Polaris (in degrees) above the horizon.

9.3.2 Longitude

Longitudes correspond to the distances to the east or west of the prime meridian, also known as the Greenwich meridian (Figure 90). They vary from zero degrees at the Greenwich meridian to -180° to the west and $+180^\circ$ to the east. Meridians are lines (semicircles) of constant longitude and run from the South to North Pole. Without access to GPS or other modern navigation tools, longitude can be calculated based on the time of sighting. If not possible, you may employ a sextant to measure lunar distance, also known as lunar observation, which, along with a nautical almanac, can be used to calculate the time at zero-degree longitude. Marine chronometers only became available in the late eighteenth century and were not widely used until the nineteenth century. A mariner with a chronometer could check its reading using a lunar determination of Greenwich time.

9.3.3 Azimuth

Azimuth is from the Arabic term *al-samt* for *the direction*. The vector from a point of origin to a point of interest is projected to a reference plane. The angle between the projected vector and a reference vector on the reference plane is called the azimuth. It is the horizontal direction of a star, or another heavenly body in the sky, in celestial coordinates. In this scenario, the heavenly body is the point of interest, the location of the observer is the center of a reference plane with a 5-km radius at sea level, and the reference vector point located on this reference plate points to true north.

9.3.4 Zenith

The term *Zenith* is from Persian astronomers, from the Arabic¹ [194] word *samt-al-raas* meaning the *direction of the head*. The zenith is a point directly above a location on the sphere. This is a vertical direction opposite to the Earth's gravity, orthogonal to the flat surface of the Earth. It is also the highest point on the heavenly body.

9.3.5 Rhumb Line

A rhumb line, also known as a loxodrome, crosses all latitudes and longitudes at the same chosen angle (Figure 90). At every point along this line, one is moving along a constant bearing relative either to true or magnetic north direction.

9.3.6 Variation

A variation or magnetic declination is the angle in degrees between the direction indicated by the compass needle (which aligns itself to the local magnetic field lines) and the direction of the meridian passing through the same point and going toward the geographic North Pole. This angle depends on the geographical location and also changes slowly over time (2° to 3° per hundred years) as the North Magnetic Pole shifts its location. The following terminology of isopleths is used when displaying variations on charts (Figure 90): (a) isogons or isogonics (lines of constant variation—an agonic line is associated with zero variation); (b) isoporics (lines of constant annual change in variation); (c) isoclinics (lines of constant magnetic dip, also known as magnetic inclination—an acclinic line is associated with zero magnetic dip); and (d) isodynamics (lines of constant intensity of magnetic force).

¹. The reader is encouraged to review the related endnote for clarification that although the word is not a Persian (Farsi) word, after the Arab conquest, some words were borrowed from Arabic by Persian scholars.

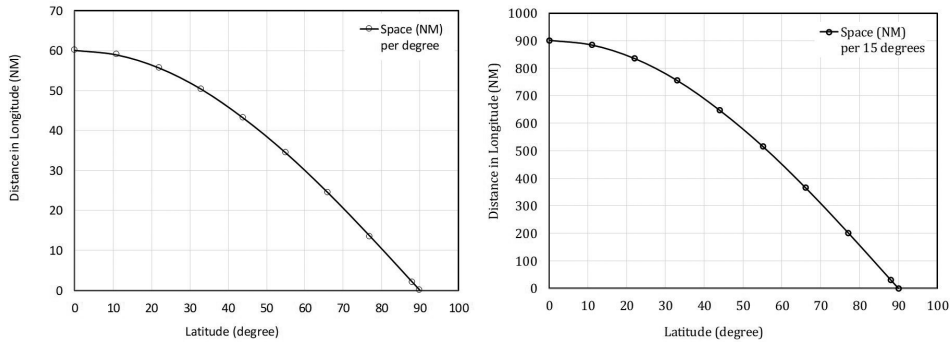


FIGURE 91 Longitude distance: (a) per one degree (left), and

Figure 91 shows the distance for one and 15° of longitude versus the latitude— $Distance = 60 \times Longitude\ Difference \times \cos(latitude)$. It is seen that the distance per degree of longitude decreases with increasing latitude—as one moves closer to the Poles, the meridians converge and thus the distance between them decreases. Note that at the Equator (zero-degree latitude), the distance is exactly 60 nm. This is not accidental—a nautical mile was originally defined as one minute (or 1/60) of one degree of latitude. At the Equator, a distance of one degree of latitude is equal to that of one degree of longitude.

9.4 Coordinate Systems

To understand navigational charts better, you should know the coordinate systems used on these maps. Figure 92 shows the azimuth α , elevation ψ , and zenith ζ angles.

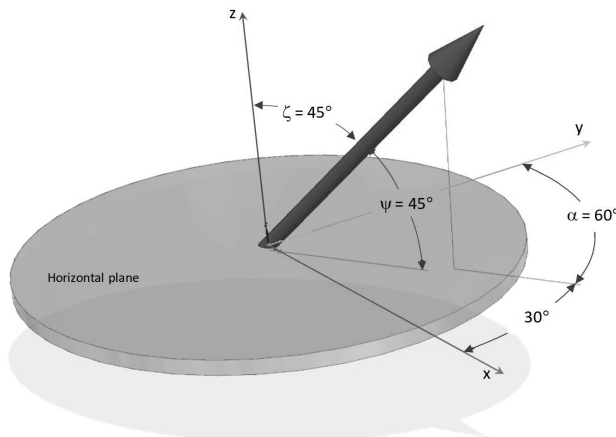


FIGURE 92 Azimuth, elevation, and zenith angles (drawings created using Solid Edge CAD tool).

Based on the projection method—cylindrical, conical, or azimuthal-planar—the main coordinate systems are presented: (a) polar—equations (156) to (159); (b) cylindrical—equations (160) to (164); and (c) spherical—equations (165) to (170). In these equations, x , y , z are the Cartesian coordinates of the point located at the distance r from the axis of the cylinder or the center of the sphere. θ and ϕ are the planar angles. For example, in polar coordinates, which is a two-dimensional coordinate system, the location is identified by $(x, y) = (r \cos \theta, r \sin \theta)$. In cylindrical coordinates, the same point is identified by $(x, y, z) = (r \cos \theta, r \sin \theta, z)$. In spherical coordinates, the same point is identified by $(x, y, z) = (r \sin \phi \cos \theta, r \sin \phi \sin \theta, r \cos \phi)$. The previous coordinates may be simplified to $r(\cos \theta, \sin \theta)$ —polar— $r(\cos \theta, \sin \theta, z/r)$ —cylindrical—and $r(\sin \phi \cos \theta, \sin \phi \sin \theta, \cos \phi)$ —spherical—coordinates, respectively. Knowing the latitude and longitude associated with the point, you may then calculate the identified coordinates and multiply them by the radius according to whatever standard you are following (e.g., WGS-84).

9.4.1 Polar Coordinates

$$x = r \cos \theta \quad (156)$$

$$y = r \sin \theta \quad (157)$$

$$r = \sqrt{x^2 + y^2} \quad (158)$$

$$\theta = \arctan\left(\frac{y}{x}\right) \quad (159)$$

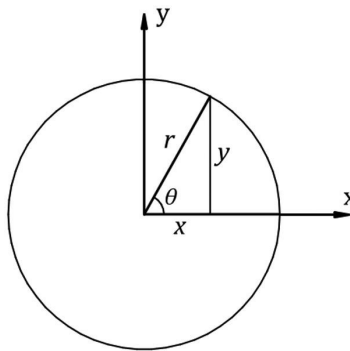


FIGURE 93 Polar coordinates (drawings created using Solid Edge CAD tool).

9.4.2 Cylindrical Coordinates

$$x = r \cos \theta \quad (160)$$

$$y = r \sin \theta \quad (161)$$

$$z = z \quad (162)$$

$$r = \sqrt{x^2 + y^2} \quad (163)$$

$$\theta = \arctan\left(\frac{y}{x}\right) \quad (164)$$

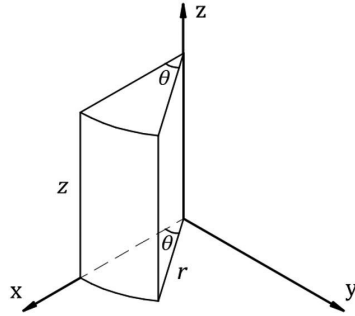


FIGURE 94 Cylindrical coordinates (drawing created using Solid Edge CAD tool).

9.4.3 Spherical Coordinates

$$x = r \sin \phi \cos \theta \quad (165)$$

$$y = r \sin \phi \sin \theta \quad (166)$$

$$z = r \cos \phi \quad (167)$$

$$r = \sqrt{x^2 + y^2 + z^2} \quad (168)$$

$$\theta = \arctan\left(\frac{y}{x}\right) \quad (169)$$

$$\phi = \arctan\left(\frac{x}{z} \sec \theta\right) = \arctan\left(\frac{y}{z} \csc \theta\right) \quad (170)$$

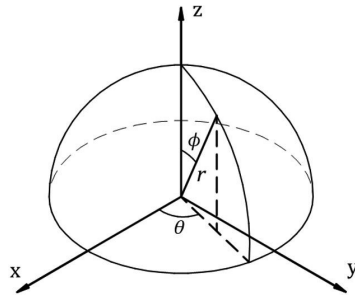


FIGURE 95 Spherical coordinates (drawing created using Solid Edge CAD tool).

9.5 Vector Algebra

A vector has two characteristics: a magnitude and a direction in space. A scalar, by contrast, only has a magnitude and a sign. For example, an airplane's movement through the air can be described by a TAS magnitude (e.g., 100 kt) and a heading (e.g., 60°), making it a vector quantity represented by two scalars. Temperature, on the other hand, is a scalar, since it is identified only by its magnitude and sign. Any group of scalars (more than one) may be treated as a vector. For a three-dimensional (3D) space, a group of three independent dimensions describes a vector in this 3D space. Depending on the coordinate system used, these three numbers can be the three Cartesian coordinates (X, Y, Z) or a distance and two angles in spherical coordinates (r, φ, θ), and so on. If the vectors are confined to a plane or any two-dimensional (2D) surface, a set of two scalars describes a vector on that surface. Thus, the aircraft velocity relative to air (\vec{v}_T) in the aforementioned example can be described as a set of two scalars $\vec{v}_T = (100 \text{ kt}, 60^\circ) = (\text{TAS}, \beta)$, where β is the aircraft heading (Figure 96).

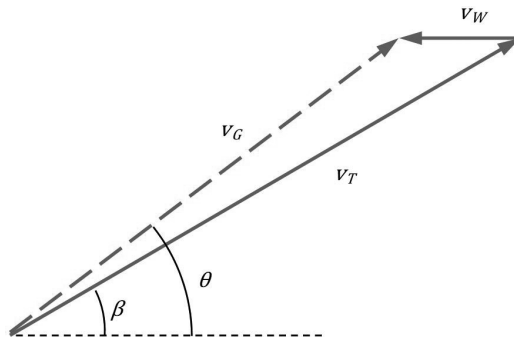


FIGURE 96 Example of vector addition for aircraft air velocity and wind vectors.

In the previous example, polar coordinates were used to define the aircraft velocity vector. The same vector may be expressed using Cartesian coordinates by—Note that the expressions are different from those in equations (156) and (157)— $\vec{v}_T = (100 \sin(60), 100 \cos(60)) = (86.6, 50.0)$. In the last expression, it is assumed that the y-coordinate is aligned with the north and the positive angle rotation is clockwise and starts at the positive y-coordinate (matching the compass angles).

Vectors can be added and subtracted. This can be shown graphically. In Figure 97, vector \overline{AB} is the sum of vectors \overline{AC} and \overline{CB} . This is written in vector notation as $\overline{AB} = \overline{AC} + \overline{CB}$. Any of the three vectors in this expression may be defined in terms of the other two. For example, $\overline{CB} = \overline{AB} - \overline{AC}$.

This can be deduced from rearranging the previous equation by moving vector \overrightarrow{AC} from the right side to the left side. The visualization may be made knowing that $-\overrightarrow{AC} = \overrightarrow{CA}$. If the vectors \overrightarrow{AB} and \overrightarrow{CA} (opposite direction to \overrightarrow{AC}) are added, vector \overrightarrow{CB} is obtained, of the same magnitude and direction as \overrightarrow{CB} shown in Figure 97.

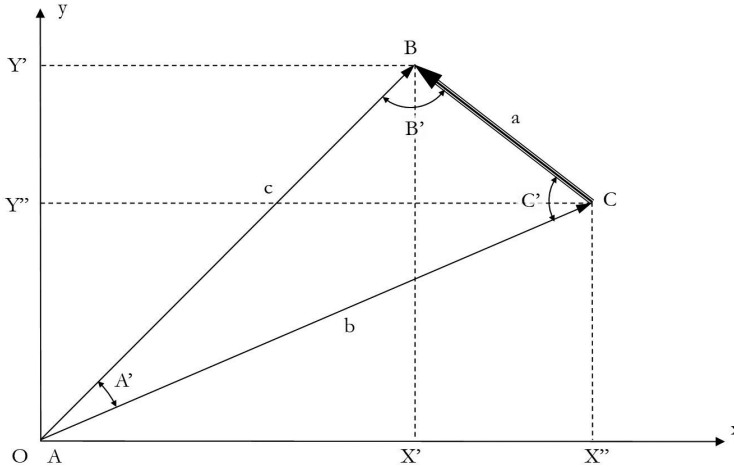


FIGURE 97 Adding vectors.

Vectors can also be added and subtracted numerically. You can express the vectors in Figure 97 in terms of their Cartesian components: $\overrightarrow{AC} = (X'', Y'') = X''i + Y''j$, $\overrightarrow{AB} = (X', Y') = X'i + Y'j$. In this relation, “ i ” and “ j ” represent unit vectors along the x and y coordinates, respectively. Thus, when multiplied by the scalars representing the vector’s Cartesian components, they result in vectors equivalent to those written as a set of numbers in brackets. The difference of the two vectors may be calculated as: $\overrightarrow{AB} - \overrightarrow{AC} = (X' - X'', Y' - Y'')$. An inner (or dot) product of two vectors gives a scalar. For the given example, it would be $\overrightarrow{AB} \cdot \overrightarrow{AC} = (X'X'', Y'Y'')$.

A cross product is a vector itself and can only be calculated in three dimensions. This vector points in the direction normal to the plane in which the two input vectors lie. To find this direction, use the *right-hand rule*, where the right hand’s four fingers curl in the direction from the first to the second vector and the thumb points in the direction of their cross product. Given two vectors \overrightarrow{AB} and \overrightarrow{AC} in xy -coordinates with the z component equal to zero, the cross product is $\overrightarrow{AB} \times \overrightarrow{AC} = (X', Y', 0) \times (X'', Y'', 0) = (0, 0, X'Y'' - X''Y')$. Observe that the cross product has zero x and y components (first two numbers in the set of three). Since the two input vectors are in the xy -plane, the cross product points along the z axis, is normal to the xy -plane, and so only has the nonzero z component (the third number).

Continuing with the example of the airplane traveling with a known airspeed and heading, let us now assume there is a wind coming from 90° at 20 kt. As a vector in Cartesian coordinates, this can be written as $\vec{v}_w = (20\sin(270), 20\cos(270)) = (-20, 0)$. In this relation, the wind direction is calculated as $90 + 180 = 270$ (since the wind is coming *FROM* 90° , the air is moving in the 270° direction). The vector addition can be now used to calculate the ground speed and course by $\vec{v}_G = (86.6, 5) + (-20, 0) = (86.6 - 20, 50 - 0) = (66.5, 50)$. Hence, the ground velocity (\vec{v}_G) has a 66.6 kt x component and 50.0 kt y component. This vector may now be converted back to the polar coordinates to obtain the ground speed and course using equations similar to equations (156) and (157), $|\vec{v}_G| = \sqrt{66.6^2 + 50.0^2} = 83.3 \text{ kt}$ and $\theta = \arctan(X/Y) = \arctan(66.5/50.0) = 53.1^\circ$. Thus, it is calculated that the ground speed is reduced from 100 to 83.3 kt and the true course is 53.1° due to the wind.

The following diagrams demonstrate the principles of vector algebra required for aviation navigation along with the calculations related to the velocity and wind predictions. Table 39 summarizes the relationships between vectors and scalars as well as the vector inner product (also known as dot product) and cross product presented in Figure 97 and Figure 98. In the table, vector \overline{AC} is the summation of vectors \overline{AB} and \overline{BC} . Each vector is identified by its components normal to its three main coordinates (defined in a three-dimensional space)—the coordinates are assumed the same for this example. Vectors may be multiplied by scalars (e.g., C) by multiplying each component by the scalar. Addition or subtraction of the vectors is achieved by adding or subtracting the corresponding components. The vector multiplications, either in the form of dot or cross products, follow the rules presented in Table 39.

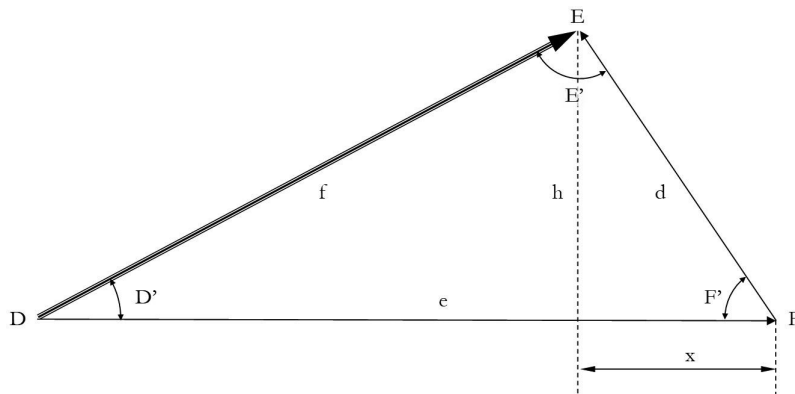


FIGURE 98 Law of cosines.

TABLE 39 Vector operations used in aviation.

Case	Vector	Scalar	Scalar Product	Inner Product	Cross Product	Angle (°)
1	$\vec{AC} = X''i + Y''j + Z''k$	C	$C(\vec{AC})$	$\vec{AC} \cdot \vec{AB} = \vec{AC} \vec{AB} \cos \theta =$ $X''X' + Y''Y' + Z''Z' =$	$\vec{AC} \times \vec{AB} = \vec{AC} \vec{AB} \sin \theta =$ $\begin{vmatrix} i & j & k \\ X' & Y' & Z' \\ X'' & Y'' & Z'' \end{vmatrix} =$ $i(Y'Z'' - Z'Y'') + j(Z'X'' - X'Z'') + k(X'Y'' - Y'X'')$	$A' =$ $\arctan\left(\frac{X'}{Y'}\right) -$ $\arctan\left(\frac{X''}{Y''}\right)$
2	$\vec{AB} = X'i + Y'j + Z'k$	C'	$C'(\vec{AB})$	$C(\vec{AC}) \cdot C'(\vec{AB}) =$ $CC' \vec{AC} \vec{AB} \cos \theta =$ $CC' (X''X' + Y''Y' + Z''Z') =$ $CC' (X''X' + Y''Y' + Z''Z')$	$\vec{AC} \times C' \vec{AB} =$ $CC' \vec{AC} \vec{AB} \sin \theta =$ $CC' \begin{vmatrix} i & j & k \\ X' & Y' & Z' \\ X'' & Y'' & Z'' \end{vmatrix} =$ $CC' \begin{bmatrix} i(Y'Z'' - Z'Y'') + \\ j(Z'X'' - X'Z'') + \\ k(X'Y'' - Y'X'') \end{bmatrix}$	$B' =$ $\arctan\left(\frac{X'' - X'}{Y' - Y''}\right) +$ $\arctan\left(\frac{X'}{Y'}\right)$
3	$\vec{R} = \vec{BC} = \vec{AC} - \vec{AB} =$ $Xi + Yj + Zk =$ $(X'' - X')i +$ $(Y'' - Y')j +$ $(Z'' - Z')k$ $a =$ $\sqrt{b^2 + c^2 - 2bc \cos(A')}$	C''	$C''(\vec{BC})$	$C''(\vec{AC} \cdot \vec{AB}) =$ $C''\vec{AC} - C''\vec{AB}$	$C'' \times (\vec{AC} - \vec{AB}) =$ $C''\vec{AC} - C''\vec{AB}$	$C' =$ $\arctan\left(\frac{Y' - Y''}{X'' - X'}\right) +$ $\arctan\left(\frac{Y''}{X''}\right) =$ $180 - A' - B'$

Case 3 in Table 39 presents the law of cosines from which you are able to find any angle of a triangle knowing dimensions of its three sides. Note that this law applies in many areas of aviation; therefore, it is worth investing some time in learning it and perhaps learning how to derive the relationships—equations (171), (172), and (173). Equation (174) demonstrates the inner product of the two vectors related to the location that is situated on the Earth's surface (at radius R). Relations for right-angled triangles along with the Pythagorean theorem are employed to derive these relationships. The Pythagorean theorem states that in a right triangle, the square of the hypotenuse (the side opposite to the right angle) equals the sum of the squares of the other two sides.

$$h^2 = f^2 - (e - x)^2 = (d \sin F')^2 \quad (171)$$

$$x = d \cos F' \quad (172)$$

$$f^2 = d^2 + e^2 - 2ed \cos F' \quad (173)$$

$$\vec{FE} \cdot \vec{FD} = |\vec{FE}| |\vec{FD}| \cos(F') \quad (174)$$

To calculate the shortest distance between any two points on the Earth's surface (i.e., the great circle distance), one method is to use the inner product of the two vectors corresponding to these points based on their geographical locations (spherical coordinates). Assume that θ and ϕ are longitudes (λ_a, λ_b) and latitudes (L_a, L_b) belonging to the two points A and B. The Cartesian vectors that connect the two points to the center of the Earth (geocentric latitude) are given by $(r \cos L_a \cos \lambda_a, r \cos L_a \sin \lambda_a, r \sin L_a)$ and $(r \cos L_b \cos \lambda_b, r \cos L_b \sin \lambda_b, r \sin L_b)$. The angle between these two vectors can be obtained from the arc(cosine) of the inner product of these vectors divided by the product of the magnitudes of the two vectors (r^2), given by $\cos F' = \cos L_a \cos L_b \cos \lambda_a \cos \lambda_b + \cos L_a \cos L_b \sin \lambda_a \sin \lambda_b + \sin L_a \sin L_b$. Converting this angle to degrees, and multiplying it by 60 NM, gives the distance. Note that 60 NM is the length of each degree along the great circle.

Another method to calculate the great circle distance is to use the stars, also known as celestial navigation, a technique extensively used in the old times by navigators. In this case, it is assumed that the longitudes (λ_a, λ_b) and latitudes (L_a, L_b) of two points are known. The longitude of one of them is assumed as the reference longitude, meaning that the second longitude is the difference longitude between the two ($t = \lambda_a - \lambda_b$). Expressing spherical coordinates as vector components in the cartesian coordinates results in: $(r \cos L_a, 0, r \sin L_a)$, $(r \cos L_b \cos t, r \cos L_b \sin t, r \sin L_b)$. The inner product is obtained ($r^2 \cos L_a \cos L_b \cos t + r^2 \sin L_a \sin L_b$), with the angle in between defined by $H_c = \arccos(\cos L_a \cos L_b \cos t + \sin L_a \sin L_b)$.

9.6 Nautical Charts

Looking up *map* in a dictionary, you will find that it is defined as an illustration that depicts: (a) a land area with bodies of water, roads, and cities; (b) the location of stars or other heavenly bodies in the sky; (c) the distribution of a quantity over specific geometrical boundaries; (d) the arrangement of genes on a chromosome; and (e) the interconnection between spaces in mathematics [195]. Simplify that and you get the representation of features of some sort within their associated space.

Maps are either static (e.g., paper) or dynamic (e.g., interactive ones on your smart device screen). Whatever they are, either real or fantasy, what you can or cannot see, in three or two dimensions, they are the figurative representation of the napkin of the world—*mappa mundi*—its medieval Latin root. The form that we commonly use in this section is a

two-dimensional map, the projection of the three-dimensional object over a surface. The most common ones are road maps. They guide you so you can get to your best-friend's wedding on time. An Appalachian trail map is another form of the road map and so is the bike map on the Campbell River. Navigational charts and their specialized form, also known as aeronautical charts, are usually drawn by local surveys made by navigators and pilots such as C. A. Lindbergh—in collaboration with his wife, Anne Morrow Spencer Lindbergh—as part of their exploratory flights. The isopleths are the collections of certain data of equal values over certain areas, showing the visual representation of a certain population similar to a contour plot; examples include elevation, temperature, and variation. Maps also have an orientation, the direction along which its sides are aligned [196].

Nowadays, there is a small guiding compass in the bottom corner of the map that you use to orient yourself, with north directed upward. However, in the old times, the east direction upward was the normal orientation for a majority of maps—the term *orient* as a matter of fact derives from *oriens* in Latin, meaning *east*. Old maps such as T and O (medieval European maps centered on Jerusalem), Edo (Japanese maps for the imperial palace), sea towns (sea), channel maps (waterway), polar maps of the Arctic and Antarctic (the Poles), reversed maps (south), Buckminster Fuller's Dymaxion maps (any order of the resultant triangular pieces from an icosahedron), and digital Geographic Information System (GIS) maps (usually north with zero degrees at the east increasing counterclockwise), are all types of maps centered on what they represent or revere (shown within the parentheses).

Another characteristic of a map is its scale, or representative fraction, which is expressed in the form of a ratio. The linear unit of measurement on the map equals this ratio multiplied by the corresponding real-world measurement. In other words, the representative fraction is the ratio of the chart length to the Earth's (or any real-world) distance—equation (175). An area map with a scale 1: 10,000 means that all its features have been resized—scaled down—by 10,000. For example, scaling down the area (10,000 × 10,000) occupied by Lake Ontario (19,000 km², 7,336 mi²) by a factor of 100,000,000 would result in an area of (190 m², 2,045 ft²) that is 6.6 times smaller in area than an Olympic-size pool—which may easily fit in your backyard; imagine having Lake Ontario in your backyard. Scale determines the accuracy with which features can be represented. A small-scale map is a zoomed-out representation and accommodates more features, a larger area, with less resolution, while a large-scale map is a zoomed-in representation and accommodates fewer features but with higher resolution. Think of you

flying at a 390-m (1,300-ft) elevation looking down at a field seeing cows gathered under the tree shadows—looking lazy and small. Standing on top of a 100-m (333-ft) high building, you may also see them swishing their tails to keep the mosquitoes away. But even the most accurate maps may not represent the accuracy you seek. For example, some minor roads may be missing, the curl on the southeast of Loughborough Lake that you like to focus on to establish your Set Heading Point (SHP) may not be at its correct location, or the bend in the road to the town of Tamworth may not follow the exact curvature. The reality sometimes ends up distorted or hidden for the purpose of simplification and facilitation of understanding; in this case, the “alternate facts” work better.

Examples of common maps and typical scales are the Mercator Conformal (MC) Projection Chart (1: 250,000), Lambert Conformal Conic (LCC) Projection Chart (1: 500,000), and World Aeronautical Chart (WAC) (1: 1,000,000).

$$\text{Scale} = \frac{\text{Chart Length}}{\text{Earth(actual) Distance}} \quad (175)$$

Looking at a WAC chart, you will see the borders separating countries as well as geographical features such as rivers, lakes, oceans, and mountains shown by contour lines. These comprehensive representations make them simultaneously functional as political, physical, and topographical maps. There are a number of charts and diagrams that facilitate navigation by identifying the direction and distances traveled. You may select the map type depending on your intention and based on the function of the map and the scale it represents—orientation is a secondary variable, although most modern maps are oriented toward north, or you may choose your orientation as you desire if you are using an electronic map. For example, on an electronic map display, you can have *North Up* or *Track Up* orientations. With the latter, the aircraft is always pointing up on the map.

Modern navigational maps share similar features such as geographical information (e.g., cities, roads, bodies of water, hydro lines, rail tracks, and antennas), land and sea aerodrome information (e.g., frequencies, distances, and airspaces), and map data (e.g., distances in degrees along with the x- and y-coordinates in the form of longitudes and latitudes, and variations). The following sections discuss the maps commonly used in aviation. They are to share a number of features, and most important of all is that they are to be orthomorphic, meaning that the small areas between the graticules (i.e., a grid of lines representing the parallels and meridians)

are presented by the correct shape and bearings are correct—this property is also known as conformal. After this condition is met, they are to project the three-dimensional space to a two-dimensional plane, such as a sheet of paper or a screen of an electronic device. Depending on the projection method (cylindrical, conical, or azimuthal-planar), the accuracy of the map and suitability to be used in certain areas of the globe are determined. Other factors, such as the shape of the graticules or shapes in general, convergence, rhumb lines, and great circles, also affect the choice of a map.

Nevertheless, maps are to be user-friendly, something that you can easily communicate with, connect to, a guide you may count on during dark hours of the day (and light hours of the night). It is said that C. A. Lindbergh used his MC Projection Chart for the majority of his famous transatlantic solo flight from New York to Paris; however, as he was preparing to land at Paris aerodrome, he got the strength he needed to complete the last few miles of the flight from St. Christopher—the patron saint of travelers. A similar medal was sewn into Geraldine Fredritz Mock's coat lining by her relative.

Visual Flight Rules Navigation Charts (VNC) is a common name for the sectional charts used in a majority of English-speaking countries (United States, Canada, and Australia) for navigation under VFRs. They are usually in 1: 500,000 scale and are named after a member city (e.g., Toronto AIR 5000 WGS84). Terminal Area Charts (TAC) are the zoomed-in versions of the VNC charts at a larger scale (1: 250,000). WAC charts are the zoomed-out versions of the VNC charts representing a smaller scale (1: 1,000,000). These compatible maps are published by FAA, NAV CANADA, Jeppesen, and other similar agencies. Some of the charts are updated every six months; NAV CANADA issues annual updates for its Visual Flight Rules Terminal Area (VTA) and VNC charts (e.g., Toronto AIR 5000 WGS84 Edition 36 April 2018).

To identify any location on the Earth, especially for air navigation purposes, a grid-based World Geographic Reference System (GEOREF) has been selected as a standard method. It is technically identifying the location by means of its latitude and longitude with a simpler notation. This technique may be employed with any map with latitude and longitude identifiers. This system divides the Earth into 15-degree equal grids in the horizontal and vertical directions (quadrangles)—resulting in twenty-four zones of longitude and twelve of latitude. The identification is done horizontally from the positive 180° moving toward the east and vertically from the -90° at the South Pole toward the North Pole. Depending on

the desired precision (i.e., geocode), one may select how the location specification is presented. A two-letter reference may be assigned to each quadrangle, starting from A to Z for the longitude (excluding letters I and O) and from A to M for the latitude (excluding letter I). Each 15-degree quadrangle is further subdivided into one-degree regions, with two extra identifiers associated with the longitude and latitude from A to Q (excluding I and O)—the directions are from the west to the east and from the south to the north. There is a further division of the one-degree quadrangles by 60 to result in minutes, with the similar direction to the previous two codes but assigned with digits (00 to 59). Each minute may be further divided into 10 or 100 and that will introduce the tenth or hundredth of the minute for more accuracy. Therefore, you will have four letters followed by four, six, or eight digits to identify your location of interest.

Note that convergency is the rate at which the meridians come together (converging); on the Earth this is zero at the Equator since the meridians are parallel, and it is at its highest at the Poles—convergency changes with the sine of the latitude angle. The charts need to represent the curved surface of the Earth on a two-dimensional paper; therefore, depending on the map in use and projection type, it is defined by various methods. When using any of the projection charts, there are multiple rules to consider, either on the ground station or in the aircraft if applicable. The measurements that are to be plotted from the ground station are as follows: (a) variation, (b) conversion angle, and (c) rhumb line. The measurements that are to be applied at the aircraft are as follows: (a) variation and heading, (b) conversion angle, and (c) reciprocal.

On some occasions, a combination of the projection maps is used. For example, C. A. Lindbergh, to prepare for his transatlantic flight from New York to Paris, plotted the great circle route as a straight line on LCC Projection Charts between departure and destination points, marked points along this route every 100 NM, and then transferred these points to his MC Projection Charts to determine the bearing at each point during his flight. He then recorded these bearings at one hour of flight intervals and kept adjusting them at these points in order to follow the great circle.

9.6.1 Mercator Conformal Projection Chart

The Mercator Conformal (MC) Projection Chart is named after a German-Flemish geographer, cartographer, and cosmographer Gerardus Mercator, who presented it in 1569. This map made it possible to sail around the globe on a constant bearing (the angle with respect to meridians), also known as rhumb lines (loxodromes), along a straight line on this map. In

other words, if you were to draw a straight line from Bern (Switzerland) to Stockholm (Sweden) on this map, the line will be at a fixed bearing (angle) relative to the meridian lines. The shortest route between the two points on the sphere is the great circle. Although selecting a route on the great circle is the shortest path you may take, using the MC Projection Chart for this purpose is not easy due to the variable bearing it introduces that needs to be adjusted periodically. Since the 1960s, with the expansion of computer use, calculating the bearing on a regular basis has become less challenging, while prior to that, compasses were to be employed to keep on track. Another disadvantage of this chart is its scale that changes rapidly with latitude as you move away from the Equator toward either of the Poles (Figure 99). Figure 100 is the MC Projection World map, generated in MathWorks® Mapping Toolbox™.

Imagine the spherical Earth that has been wrapped by a cylinder, in contact with the globe at the Equator (waist). The diameter of this cylinder is the same as of the Earth. Project the latitudes and longitudes onto this cylinder—cylindrical projection. The latitudes will form parallel lines that are not equally spaced, if you started with latitudes separated by equal angles. In the next step (not shown in Figure 99), to create the MC Projection Chart, the parallel spacing is adjusted to produce a constant bearing along a straight line. This is achieved by progressive “shrinking” of the spacing between the parallels as one goes farther north from the Equator. The longitudes also create parallel equally spaced lines, assuming

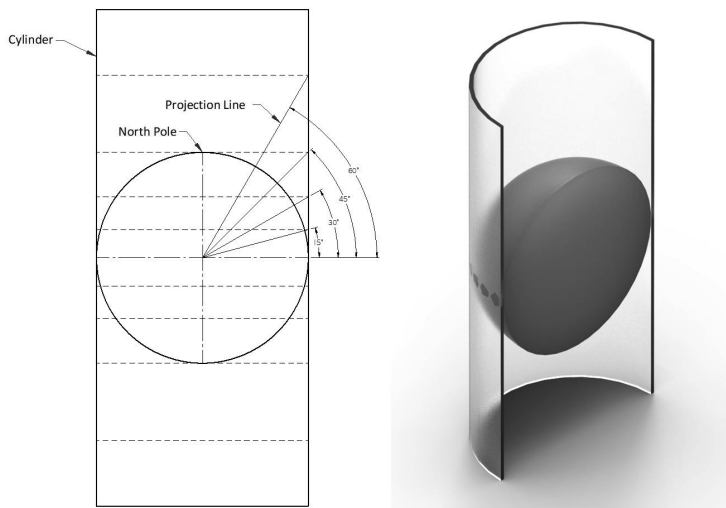


FIGURE 99 The MC Projection: (a) schematic (left), and (b) 3D visualization of the sectional view (right) (drawings created using Solid Edge CAD tool).

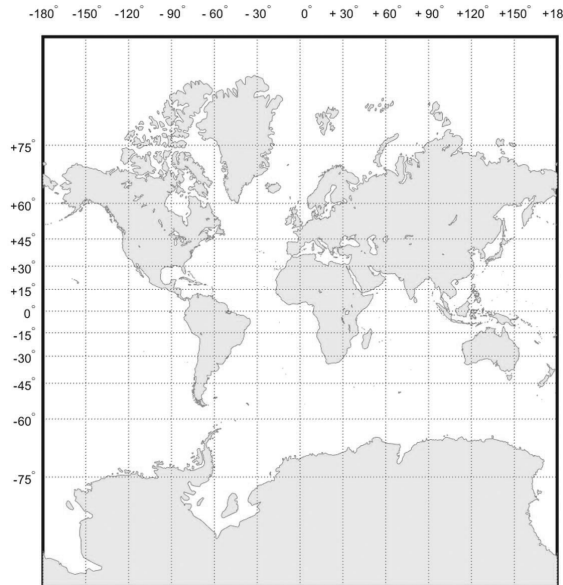


FIGURE 100 The MC Projection world map generated in MathWorks® Mapping Toolbox™.

you picked equally spaced longitudes at the Equator. The map is obtained by unrolling (flattening) the cylinder. Along the latitude coordinate near zero degrees, the Equator, you get relatively accurate representation, since you are actually presenting the real-size dimension of the Earth's diameter—the same-sized waists are in touch and create tangents. This changes though as you move farther from the Equator, creating belts of latitudes which are increasing in scale. For example, at the farthest point from the Equator (e.g., Antarctica), the land areas appear much larger than their true sizes.

The linear scale remains the same in both directions of latitudes and longitudes around any point. In other words, this map is conformal, also known as orthomorphic, since this mapping preserves the local shape of features. Nevertheless, the longitudes are projected as they are within the scale of your operations. Therefore, you will end up with a grid of equal spaces, that is, a projection chart for small regions. There are 60 NM for every one-degree longitude in this region; however, as you move away from the Equator, the meridians get closer to one another—meaning that the scale changes depending on the latitude. *Departure* (the difference between the current longitude and that of the destination for the given latitude) is then defined based on equation (176) as a function of the change of longitude in minutes (*ChLong*) multiplied by the Cosine of latitude (*Cos(Lat)*):

$$\text{Departure} = \text{ChLong}(\text{in minutes}) \times \text{Cos}(\text{Lat}) \quad (176)$$

Incorporating equation (175) into equation (176), equation (177) is obtained, where scale at any latitude ($\text{Scale}_{\text{Lat}}$) may be obtained from the scale at the Equator ($\text{Scale}_{\text{Equator}}$) by the secant of the latitude ($\text{Sec}(\text{Lat})$). For cases where two latitudes are to be directly compared, the ratios of the secants are directly associated with those of the scales—equation (178).

$$\text{Scale}_{\text{Lat}} = \text{Scale}_{\text{Equator}} \times \text{Sec}(\text{Lat}) \quad (177)$$

$$\frac{\text{Scale}_{\text{Lat A}}}{\text{Scale}_{\text{Lat B}}} = \frac{\text{Sec}(\text{Lat A})}{\text{Sec}(\text{Lat B})} \quad (178)$$

Scale is one unit at the Equator. Scale at $\pm 6^\circ$ from the Equator is within 0.5 percent, while for $\pm 8^\circ$ from the Equator it is within one percent. Scale increases with the secant of the latitude as you get farther from the Equator (Figure 101). The graticules (latitudes and longitudes) are straight and parallel lines and evenly-spaced—similar to trapezoid mesh nettings. While scale may be used to show variations for latitude, as in equation (178), the Earth convergence is used to show variations of longitude, as in equation (179). It is the angle between the straight line that connects the two points on the MC Projection Chart (rhumb line) and the tangent to the great circle the two connecting points make—equation (179). It is the angle of inclination between two meridians at a given latitude. Convergence is true only at the Equator. Note the difference between the Earth convergence and the conversion angle—equation (180). Since meridians are parallel lines at the Equator, they are not converging at the Equator (i.e., zero convergence) and are therefore correct only at the Equator and constant over the entire chart. Equations (181) and (182) present the coordinates in the MC Projection Chart in terms of latitude and longitude angles, where R is the Earth's radius. This projection is best used for the Equatorial region,

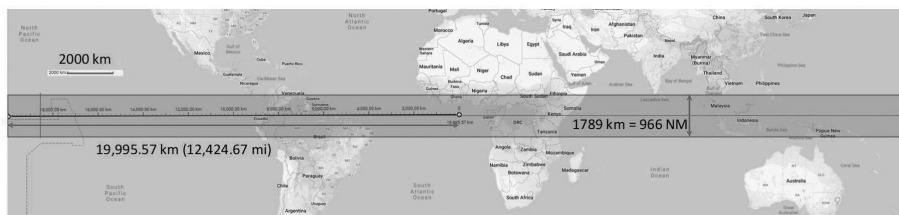


FIGURE 101 The band in which the MC Projection Chart is applicable within one-percent error (the arrow presents the 1,789.032 km—966 NM—distance with the east-west distance of 19,995.57 km—10,796.74 NM); the map is courtesy of Google Maps.

where it has the highest accuracy. Most Jeppesen charts (e.g., ED6) use the Mercator projection for this region.

$$\text{Earth Convergency} = 2 \times \text{Conversion Angle} \quad (179)$$

$$\text{Conversion Angle} = 0.5 \times \text{ChLong} \times \sin(\text{Mean Lat}) \quad (180)$$

$$x = R \times (\text{Long}_a - \text{Long}_b) = R \times (\lambda_a - \lambda_b) \quad (181)$$

$$y = R \times \text{Ln} \left[\tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \right] \quad (182)$$

9.6.2 Lambert Conformal Conic Projection Chart

The Lambert Conformal Conic (LCC) Projection Chart is named after a Swiss polymath, Johann Heinrich Lambert, who created it in 1772. This map made it possible to identify the great circle route by connecting the departure and destination points by means of a straight line. This simplified determination of the shortest navigational route. The scale in this chart is nearly constant. Therefore, it addresses the two inadequacies of the MC Projection Chart. The rhumb line, however, is not a straight line anymore.

Imagine the spherical Earth on top of which sits a cone with its apex located on the Earth's axis at some distance (h) from the Equator plane. Project the latitudes and longitudes onto this cone surface. The circles of latitude will form concentric circles centered at the cone apex; the meridians will form radial lines emanating from the apex (Figure 102). The apex angle, also known as the angle of the cone, equals 90° in this example but can be any other angle. Figure 103 is the LCC Projection World map generated in MathWorks® Mapping Toolbox™.

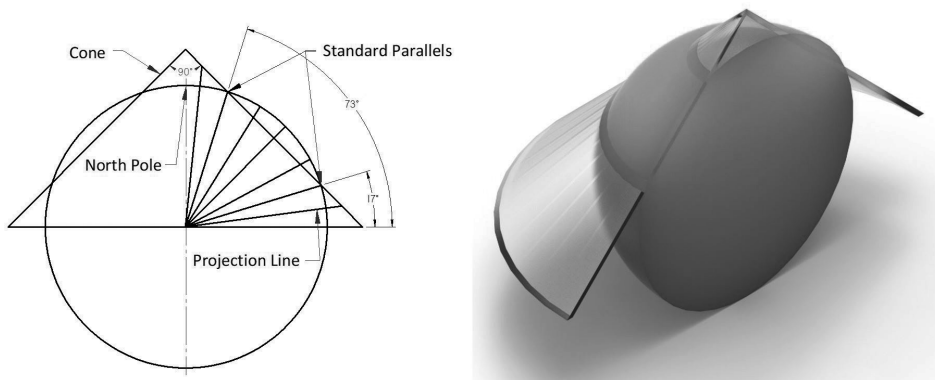


FIGURE 102 The LCC Projection: (a) schematic with the standard parallels at 17° and 73° (left), and (b) 3D visualization of the sectional view (right) (drawings created using Solid Edge CAD tool).

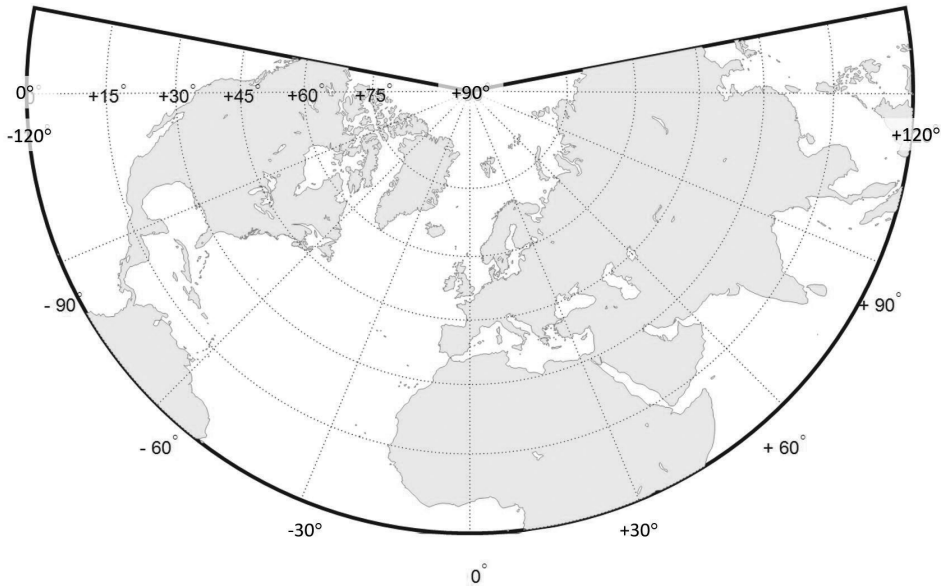


FIGURE 103 The LCC Projection world map generated in MathWorks® Mapping Toolbox™.

The cone may either touch the sphere at one parallel (which would be the 45th parallel for a 90-degree cone) or it can intersect the sphere at two parallels (as does the cone in Figure 102 at the 17th and the 73rd parallels). The two latitudes where the cone and the sphere intersect are called the standard parallels. In the case where the sphere is in contact at one circle only, both standard parallels are considered the same. The reference parallel is the average of the two standard parallels.

At the latitudes at which the cone and the sphere come in contact, you get the least distortion of the map, since the local dimensions for the cone and the sphere are the same where they are in contact. Scale is reduced between the standard parallels and increased outside of them. When creating a map using this projection, one would choose the standard parallels to pass through the region of the map in order to minimize the local scale variation. To maintain this projection's conformal (also called orthomorphic) properties, meaning the local shapes at any point on the map are not distorted, the north-south scaling away from the standard parallels must be adjusted appropriately to match the east-west scaling variation introduced by the conic projection.

The center of the map may be at coordinates (0°N, 0°E)—Gulf of Guinea. However, if your region of interest is someplace else, your new center of interest becomes the reference latitude and longitude at the new

reference coordinate (x, y) . In this case, your meridian is the only meridian parallel to the y -coordinate of your cone (h)—being the center of the developed plane—meaning that you would shift and rotate your coordinate system every time you change the (x, y) vector.

Generally, the United States systems for VFR sectional area charts and TACs are based on standard parallels at 33°N and 45°N (12° difference). This projection, also known as ETRS89-LCC, is also recommended by the European Environment Agency and Infrastructure for Spatial Information in European Community (INSPIRE) for scales smaller than or equal to 1: 500,000. Note that there are different LCC Projection Charts used around the world (e.g., RGF93 in Metropolitan France, WGS84 in India, and CCS83 in the United States). Choosing a suitable projection for a point of interest (the origin of the chart) helps with accurate representation of the area [197].

Chart convergence is the angle of inclination between two meridians at a given latitude and is calculated from equation (183). In the LCC Projection Chart, the sine of the latitude of origin is the convergence factor, also known as the constant of the cone, and is shown by n . The chart convergence for the LCC Projection Chart is constant across the chart.

$$\text{Convergence} = Ch\text{Long} \times \sin(\text{parallel of origin}) \quad (183)$$

If you were to open the cone, the arc of the sector equals the change of the longitude (in this case the entire 360° belt around the globe) by the sine of the parallel (i.e., latitude), as in equation (184).

$$\text{Arc of Sector} = Ch\text{Long} \times \sin(\text{parallel of origin}) \quad (184)$$

This is not to be confused with the Earth convergence, which is the angle of inclination between the great circle and rhumb line, and is the change of longitude multiplied by the sine of the mean latitudes, which is twice the convergence angle—equations (179) and (180).

There are large-scale LCC Projection Charts—divided into groups—which cover an entire country (e.g., the United Kingdom has three 1: 500,000 scale maps). Only one of these maps may include the standard parallels and the parallel of origin (e.g., N52 among the three 1: 500,000 scale maps used for the United Kingdom). Nevertheless, these maps share the same convergence factor n .

9.6.3 Polar Stereographic Projection Chart

This map type is well-suited for navigation in the polar regions because it is both conformal and perspective there. These regions are important

for aviation because political rivalry is almost nonexistent in these areas, meaning there are minimum limitations on flights, and because aircraft performance is at its peak in cold temperatures.

The stereographic projection was known as planisphere projection to the Egyptians. The oldest surviving document (*Planisphaerium*) that has explained this chart is attributed to Ptolemy, a Greek mathematician, astronomer, and geographer (about AD 100–170). The earliest world maps by Gualterius Lud in 1507 were also a stereographic projection. Its modern form was given in 1613 by François d'Aguilon, a Belgian mathematician, physicist, and architect. The conformality of it was proven in 1695 by Edmond Halley, a British astronomer, based on the calculus tools invented by Isaac Newton. This is also known as the Humphrey Gilbert Map, because it carries his name on the bottom-right corner. Although some may attribute the invention of this map to John Dee, a British mathematician and astronomer in the late 1500s, it is known that this association may not be valid. It is, however, remotely possible that Dee was not aware of the earlier use of this map and reinvented it; they also attribute this claim to his friendship with the cartographers such as Mercator and Ortelius.

To visualize how this projection works, imagine a sphere with a light bulb at the South Pole radiating light, projecting it onto a horizontal plane tangent to the North Pole. The projection of the light on the plane generates this map with a true scale at the North Pole (where the plane is in touch with the Earth). The latitudes therefore become concentric circles, centered at the Pole. The longitudes become straight lines, radiating from the Pole—Figure 104. Figure 105 is the Polar Stereographic Projection world map generated in MathWorks® Mapping Toolbox™.

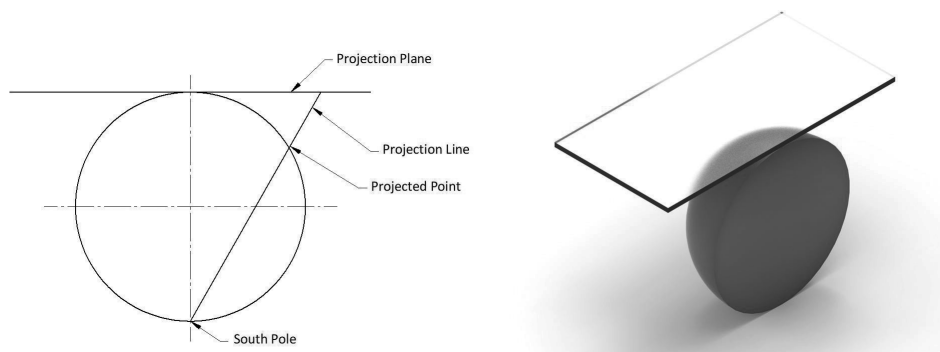


FIGURE 104 The Polar Stereographic Projection: (a) schematic (left), and (b) 3D visualization of the sectional view (right) (drawings created using Solid Edge CAD tool).

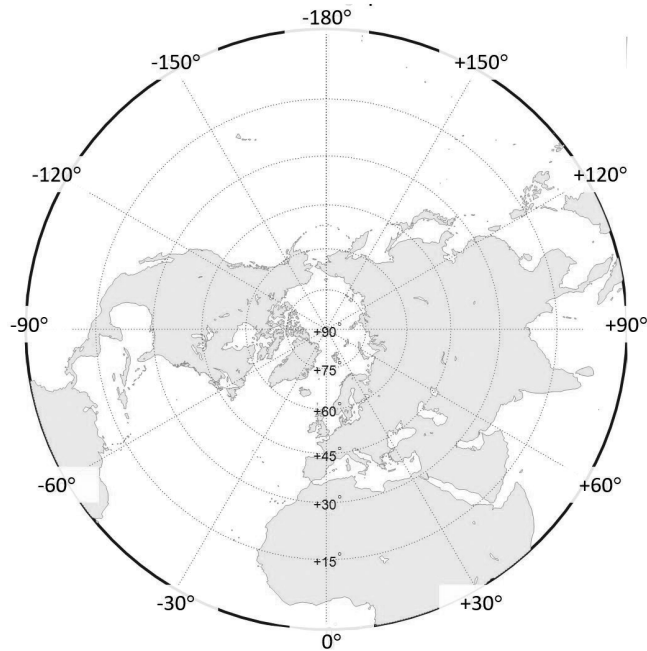


FIGURE 105 The Polar Stereographic Projection world map generated in MathWorks® Mapping Toolbox™.

The scale increases as one goes farther away from the Pole; the scale is proportional to the secant-squared of the half of the colatitude—equation (185). However, the perspective remains constant given the law of sines for triangles—which results in a constant convergence factor of one for all latitudes. Therefore, chart convergence is true and depends on the longitude—equation (186).

$$Scale_{Lat} = Scale_{Pole} \times Sec^2(0.5 CoLat) \quad (185)$$

$$Convergence = ChLong \quad (186)$$

Generally speaking, great circles curve toward the nearest Pole, but they are less curved compared to rhumb lines and are almost straight in the vicinity of the Pole. Rhumb lines curve to the nearest Pole. Assuming that the scale is true at the Pole, it is possible to obtain a relatively straight flight path on a great circle by connecting the departure and destination points by means of a straight line, and therefore flight distances may be calculated easily. The one-percent change of scale is still assumed acceptable in a navigation map, which makes it possible to cover a polar latitude down to 78° in a 2,317 km (1,440 miles) route that takes about 3.9 hrs by a jet traveling at 600 km/hr (372.8 miles/hr). Increasing the allowed change of scale from 1 to 3 percent will introduce the flexibility of flight down to the

70° latitude, equivalent to 3,861 km (2,400 miles) that takes about 6.4 hrs traveling by the same jet. It is useful to be able to fly for 6.4 hrs without needing to change the map.

The maximum travel time by a passenger jet in the polar regions as a function of the latitude (Y) and passenger jet speed (V) was modeled using regression analysis, and the result is equation (187), where the confidence level is above 82 percent. All predictors are statistically significant. The distance traveled is the product of the speed and the time traveled ($x = Vt$)—for example, for the latitude 70° and speed of 324 kt (600 km/hr) (as in the previous example), you can obtain 9 hrs ($87.75 - 0.75 \times 70 - 0.081 \times 324 = 9$ hr) using this equation.

$$\text{Time}(hr) = 87.75 - 0.75Y(^{\circ}) - 0.081V(kt) \quad (187)$$

9.6.4 Transverse and Oblique Mercator Conformal Projection Charts

The Transverse MC Projection Chart is also known as the Gauss Conformal, the Gauss-Krüger, and the Gauss-Krüger Transverse Mercator. This chart was introduced in 1825 by Carl Friedrich Gauss, a German mathematician and physicist, and further developed in 1912 by Johann Heinrich Louis Krüger, a German mathematician and surveyor. The Transverse MC Projection Chart is similar to the regular MC Projection Chart except that the cylinder is rotated 90° (in a vertical plane, about the center of the Earth), presenting a horizontal perspective versus the vertical one of the original MC Projection Chart (Figure 106). The light bulb is at the center of the Earth. Given the orientation of the cylinder, the Poles are in contact with the sphere, and so the scale is true at the Poles and along the meridian in between—the meridian of tangency. This proves advantageous given the large distance that it covers, all the way from the South Pole to the North Pole.

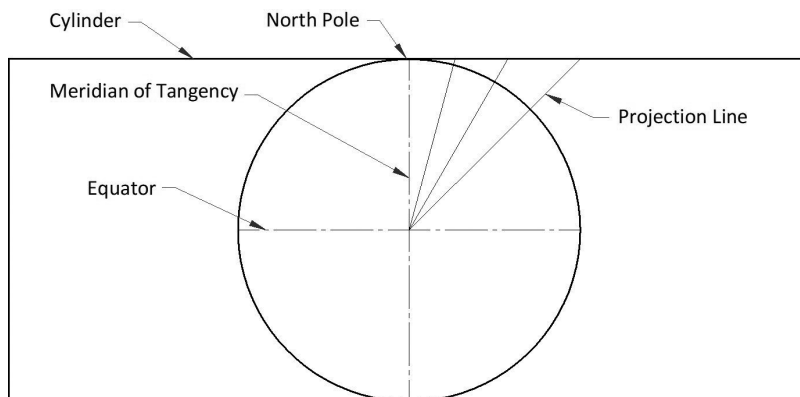


FIGURE 106 The Transverse MC Projection schematic
(drawings created using Solid Edge CAD tool).

Figure 107 is the Transverse MC Projection world map generated in MathWorks® Mapping Toolbox™. As mentioned earlier, the scale variation of one percent is within the acceptable limits when using the navigation charts; therefore, a distance of about 483 NM—(a band of 8.05°) on either side of the meridian of tangency of the Transverse MC Projection Chart (i.e., a total band of 16.1° equivalent to 966 NM or 1,789 km) can take advantage of the acceptable map, meaning that lines within this acceptable region may be connected by a straight line (Figure 108). For example, you are able to easily travel from the Gabon-Nigeria border to the west of Norway—see how powerful this map is in terms of area coverage! Outside the region, the scale increases toward the east and west by the secant of the longitude, and it gets really large on the far left of either the east or the west; therefore, the map is not suitable for applications in which large east-west distances are to be traveled. Note that in this scenario, instead of the latitude, the difference in degrees from the meridian of tangency is used for scale calculations. This map is particularly useful for travel inside countries with large aspect ratios (ratio of the north-south dimension to that of the east-west) such as Chile, located on the west coast of South America and elongated along the north-south direction. Another example is the Transverse MC Projection Chart presented for the United Kingdom at the vertical datum MSL at Newlyn (selected revision in 1984—scale: 1: 250,000).

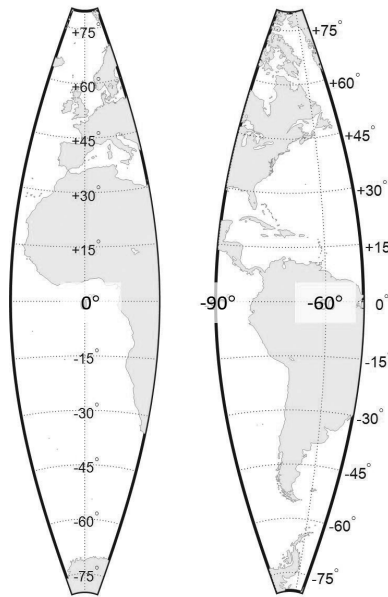


FIGURE 107 The Transverse MC Projection world map generated in MathWorks® Mapping Toolbox™: (a) centered on zero-degree meridian (left), and (b) centered on 70°W meridian (-70°), showing the west coast of South America (right).

To help you better visualize how this projection works, imagine you travel to an altitude of 20,000 ft and reflect down on the Earth—the “big picture.” You are situated so that your reflection falls over the Earth equally—your shape factor is the same for both the Eastern and Western Hemispheres. In thermal radiation topics in heat transfer, shape factor is the percent of energy that leaves the source (i.e., you) and reaches the destination (i.e., the surface of the Earth). When using this projection, looking down from the space, you are able to see both sides of the Earth—two Equators forming the top and bottom sides of a rectangle.

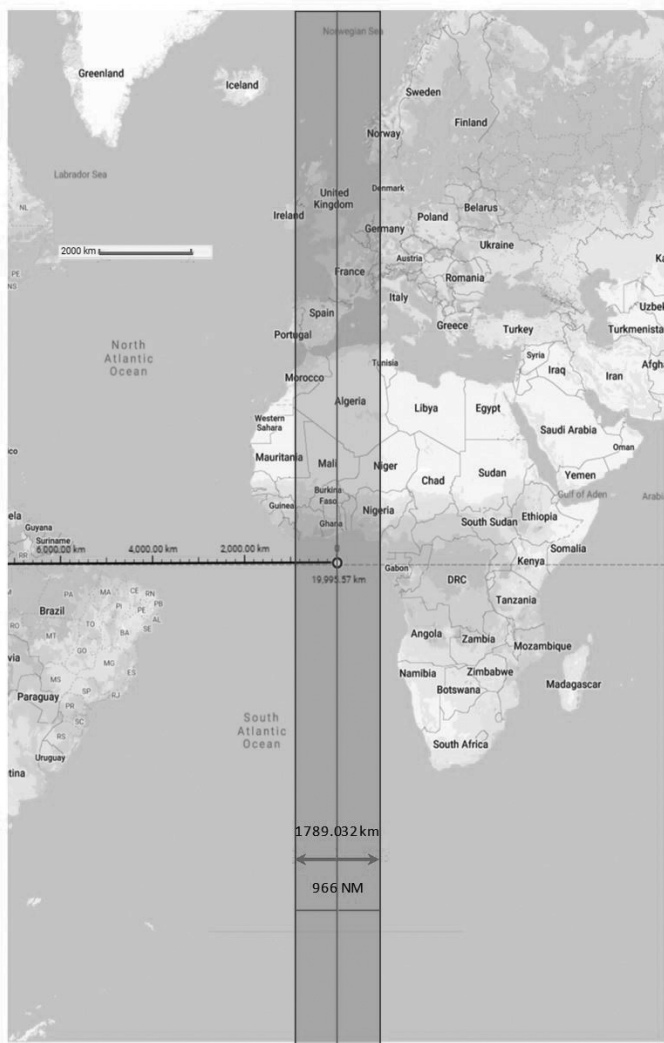


FIGURE 108 The band in which the Transverse MC Projection Chart is applicable within one-percent error; the map is courtesy of Google Maps.

The central horizontal line passes through the Pole, and the vertical left and right sides form the map length. The meridian of tangency that passes through the Pole forms a straight line, parallel to the map sides and perpendicular to the Equators. All meridians in between are to be perpendicular to the Equators and also pass through the Pole facing you. However, they are not parallel to one another—curves concaving away from the meridian of tangency. The latitudes form ellipses with a decreasing semi-minor axis as they get farther from the Poles. At regions close to the Poles, they converge to circles. Looking at this projection, you may be reminded of two garden-orb-weaving spiders touching their heads to greet (Figure 109). The convergency is then correct along the circle of tangency, the Poles, and the Equators. Rhumb lines are complex curves, and so are the great circles; however, they may be assumed as straight lines in the vicinity of the meridian of tangency. If you wanted to travel from point A to point B, and these points do not fall into the recommended longitude range to produce the maximum scale deviation of one percent, you need to select the great circle route in order to travel the least distance. You can then move the current meridian of tangency to this great circle (similar to the change of the origin of the coordinates by rotation and translation). In this scenario, the reference meridian is changed so that the Transverse MC Projection Chart touches the Earth at the points obtained from the extension of the line between A and B. Therefore, the cylinder is rotated or transported to merge its central axis with that of the meridian of tangency to produce a true scale (1: 1) along this line. The resultant projection is the oblique version of the Transverse MC Projection Chart. The properties of the oblique chart are similar to the non-oblique chart, except that the meridians of tangency are oriented as needed (Figure 110).

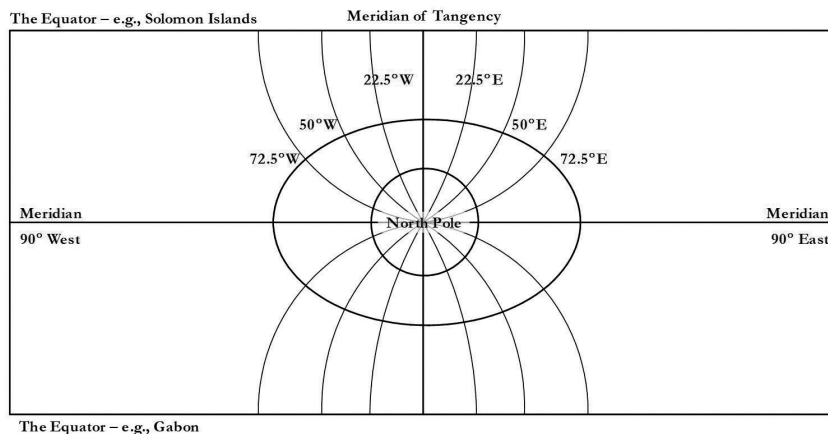


FIGURE 109 The Transverse MC Projection graticules (drawings created using Solid Edge CAD tool).

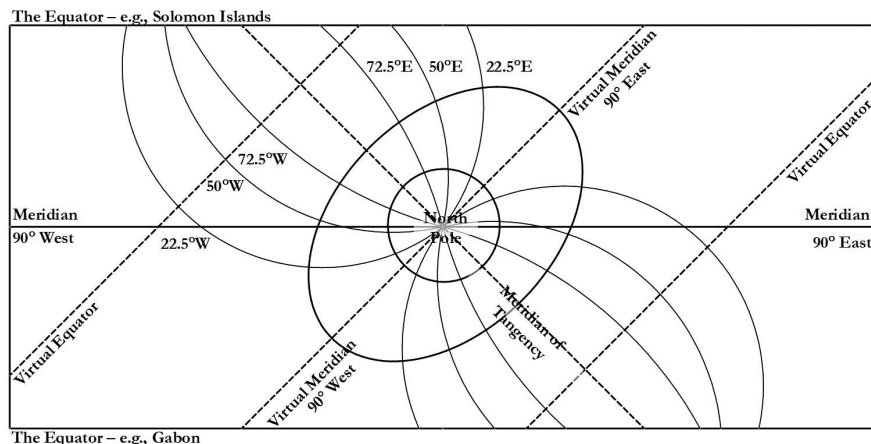


FIGURE 110 The Oblique Transverse MC Projection graticules
(drawings created using Solid Edge CAD tool).

9.7 Pilotage and Dead Reckoning

The most uncertain way of navigating happens when there is only one LOP available. In this situation, pilotage and Dead Reckoning (DR) are employed. In the former case, one uses geographical features and landmarks to find one's way; in the latter case, in addition to the pilotage, calculations based on the ground speed, distance, and time are employed.

Geographical features and landmarks employed for pilotage include bodies of water, roads, populated areas (cities and towns), hydro lines, rail tracks, and water towers. Generally, only the major roads are identified on the maps. Looking at your LOP, you may be able to approximately tell if you are to the left or right of the LOP or how far you are from the bend of the upcoming road using the width of the available “calibrated finger.” Is the lake on your right or left? Are you passing by the left, middle or right edge of the town? How many bodies of water are you expecting to cross? What towns are along the way? How about the important landmarks, factories with chimneys, railroads, water towers? All of these are guides that may help you identify your location.

You can use the map scale to measure distance between two points on the map. For example, use your index finger width and compare it with the size of the latitude and longitude it represents on your map. You may then employ this procedure when calculating navigation-related data such as distances and ground speeds. Although your index finger width could vary from that of your FIC (i.e., 4 miles versus 7 miles in a VNC chart), the method of calibration and use is the same.

When identifying the flight route on a chart, it is wise to avoid flying over large areas of uninhabited land or bodies of water, where one is away from civilization and unable to ask for timely and well-informed help if required. It is also prudent to study the map carefully when flying over restricted areas, such as military bases, and allow for the altitude and the radius within which the restrictions apply. When flying over extreme elevations such as mountainous areas, be cautious in selecting the appropriate elevations (i.e., allow for a minimum of 2,000 ft of clearance from the highest point) and also study the geographical features contributing to air currents and local weather in particular. You may encounter extreme updrafts and downdrafts in these areas, so getting specialized training will help. The survival kit and an additional portable Emergency Locator Transmitter (ELT), in case you lose the one in the aircraft, would be good precautions. An additional safety measure one may consider is to have an extra pilot on board when flying on your own or with family members. If an extra pilot is not available, it is never too late to teach your accompanying person some aircraft basics so that he or she can perform simple maneuvers and know how to operate the radio if required [198,199]. For more complex flights, such as the one around the world, you may get the help of expert navigators if available (e.g., A. M. Earhart and G. F. Mock used the help of experienced navy navigators when planning for their solo flight around the world).

If for whatever reason you are to plan your flight over such large bodies, it is wise to consider your aircraft gliding distance versus the altitude for given atmospheric conditions. The gliding distance decreases as altitude decreases; for example, a 2,000 ft altitude gives under 4 miles of gliding distance for the Cessna 172M compared to a 1,000-ft altitude giving under 2 miles of gliding distance. Glide slopes and the physics of engine failure are discussed in more detail later in Chapter 11 (Flight Plan).

Your smartphone, iPad, and other consumer technologies may be employed to assist with navigation and are literally in the palm of your hand; however, unless certified for this purpose, they cannot serve as primary navigational tools for IFR flights. Remember to take extra batteries to charge your device and also look into retrofitting your aircraft with electrical sockets fitted for charging your smart devices. Have extra connecting cables in case they are required. Try to update your smart devices and operating systems and make the plans when on the ground, prior to flying. You may use the debriefing applications such as CloudAhoy, which records a three-dimensional visual representation of your flight path along with other flight statistics such as airspeed, duration in the air and on the ground, total flight time, Hobbs time, distance, aerodrome, and runway from which you take

off and land, along with date, aircraft data, and persons in command. You are then able to replay the entire flight on your smart device and review it with the CFI (or FIC). Maneuvers such as takeoff, stall, and slow speed may also be recognized by these applications and are shown in different colors.

9.8 Examples

9.8.1 Case Study – Distances and Time Zones on the Equator

A South American family living in Ecuador is 8.0 hrs behind an African family living in Kenya. Discuss the time zones.

The South American family is 8.0 hrs behind the African family, meaning that they are geographically offset by 120.0° of longitude. Given that every Earth day on average consists of 24 hrs, and given that the line crossing the Earth at zero-degree latitude—passing through the Equator—makes a complete 360-degree arc, it may be deduced that each time zone (on average) comprises a 15-degree angle. Given the diameter of the Earth of 12,742 km (6,880 NM), each degree of longitude at the Equator is 111.2 km (60.0 NM). Each average time zone width of 15° is then equal in width 1,667.9 km (900.6 NM) at the Equator (Figure 90). The projected distance between the two families on the Equator in this example is 13,343.4 km (7,204.7 NM)—assuming identical time difference.

9.8.2 Case Study – Time Zones in North America

Discuss the North American time zones presented in Figure 111.

Figure 111 presents North American time zones. Time zones are associated with letters A through Z (e.g., P, Q, R . . .); Z is located in UTC—centered on the zero-meridian—and is not to be used interchangeably with Greenwich Mean Time (GMT). All reference times are measured based on the Earth's rotation with respect to stars and quasars and are calibrated for solar time. UTC is based on International Atomic Time (TAI) and is recommended over GMT for its accuracy since 1935 [200]. UTC is mainly based on an astronomical day associated with 24 hrs starting from noon versus the civil day that starts from midnight and is used in aviation. Zones may have local names (e.g., the alpha zone includes most western European countries). Daylight Saving Time (DST) is the first Sunday in April to the last Sunday in October based on the United States energy policy act of 2005, with the right to start earlier to meet energy-efficiency targets (Figure 111, light blue column). Time zones indicated by the difference from the UTC are shown with Standard Time (ST) and DT (Daylight Time) columns (Figure 111—dark purple column). The letters at the beginning of

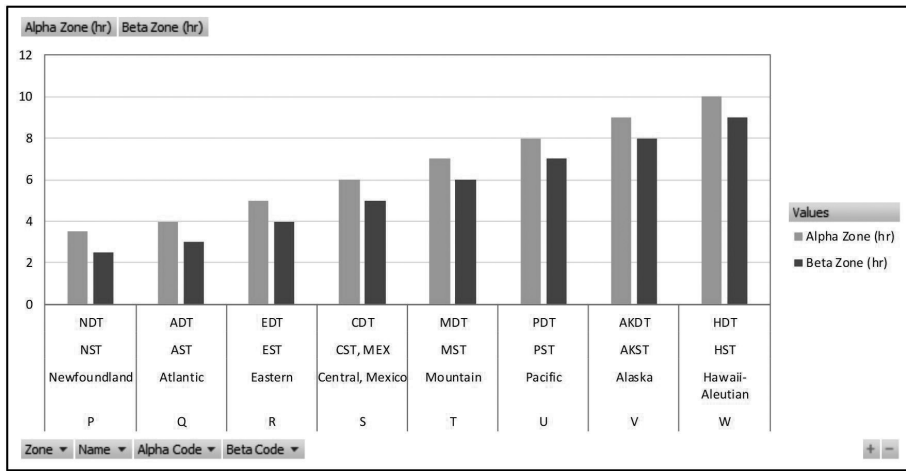


FIGURE 111 North American time zones.

the ST and DT are associated with the regions whose time is represented (e.g., PST for Pacific Saving Time and AKDT for Alaska Daylight Time). The international date line is the line approximately following the 180° longitude; you gain or lose a day by crossing this line. Hence, when you are given a task today that was due yesterday, it only makes sense if you are traveling from the West to the East, when your today practically has become tomorrow—that is, you are back to the future.

9.8.3 Case Study – Deviation from the Expected Time Zone Change

Are Thunder Bay Airport (CYQT) and Natashquan Airport (CYNA) within the same time zone?

One of these aerodromes is on the far-west side of Ontario (CYQT, $N48^\circ22.32'/W89^\circ19.30'$) and the other one is on the east side of Quebec (CYNA, $N50^\circ11.40'/W61^\circ47.33'$). The difference between their longitudes is $27^\circ31.97'$. There are 1.83 times 15-degree intervals in this longitude difference, which should be equivalent to a 1.83-hour time difference. However, looking at the time-zone map, you will find that these two airports are in the same time zone, UTC $-5(-4)$. The reason for this deviation from the expected time zone change is that time zones are actually a communal decision and must account for economic considerations.

9.8.4 Case Study – Departure, Scale, and Error on the Mercator Conformal (MC) Projection Chart

Assuming that the one degree of longitude at the Equator equals 60 NM, determine:

TABLE 40 Latitudes, changes in the longitudes, scales at the latitudes, and errors for the MC Projection Chart.

Case	Latitude		Cos (Latitude)	Departure NM (1°)	Departure NM (1/60°)	1/Scale	1/Scale at Latitude Given Equator's Scales	Error (%)	1/Scale at Equator Given Latitude's Scale
	°	Rad							
1	0	0.00	1.00	60.00	30.00	30,077,594	1,000,000	0.00	965,618
2	6	0.10	0.99	59.67	29.84	29,912,826	1,005,508	0.55	970,937
3	8	0.14	0.99	59.42	29.71	29,784,881	1,009,828	0.98	975,108
4	10	0.17	0.98	59.09	29.54	29,620,648	1,015,427	1.54	980,515
5	20	0.35	0.94	56.38	28.19	28,263,693	1,064,178	6.42	1,027,590
6	30	0.52	0.87	51.96	25.98	26,047,960	1,154,701	15.47	1,115,000
7	40	0.70	0.77	45.96	22.98	23,040,774	1,305,407	30.54	1,260,525
8	50	0.87	0.64	38.57	19.28	19,333,505	1,555,724	55.57	1,502,235
9	60	1.05	0.50	30.00	15.00	15,038,797	2,000,000	100.00	1,931,237
10	70	1.22	0.34	20.52	10.26	10,287,143	2,923,804	192.38	2,823,279
11	80	1.40	0.17	10.42	5.21	5,222,919	5,758,770	475.88	5,560,774
12	90	1.57	0.00	0.00	0.00	2,621,435	57,298,688	5,629.87	55,328,664

(a) departure at the latitudes indicated in Table 40 for one-degree change in longitude, (b) departure at the latitudes indicated for 0.5-degree change in longitude, (c) scale at the latitudes, assuming that the chart width is 133 cm from 180°W to 180°E at 65°N, (d) scale at the latitudes, knowing the scale at the Equator is 1,000,000, (e) scale error in percent obtained in parts “d”, and (f) given the highlighted scale at 30° latitude, the scales at other latitudes and the Equator.

Calculations have been performed with the results shown in Table 40. Figure 112 and Figure 113 depict the departure distance in nautical miles and scale versus the latitude for 10-degree latitude increments for one-degree- and 0.5-degree-longitude change. It is seen that the scale factor at the Equator is one, since this is where the dimensions remain true—the latitude is zero degrees—while the maximum variation from the true size is observed at the Poles—where the latitude is 90°. One method to ensure calculations are valid is to verify the trend in the magnitude: with increasing latitude, the scale is to increase. Figure 114 presents scale versus the latitude for the scenario in which the scale at the Equator is known (1,000,000) and where it is to be calculated based on the known scale at latitude 30° (1,115,000).

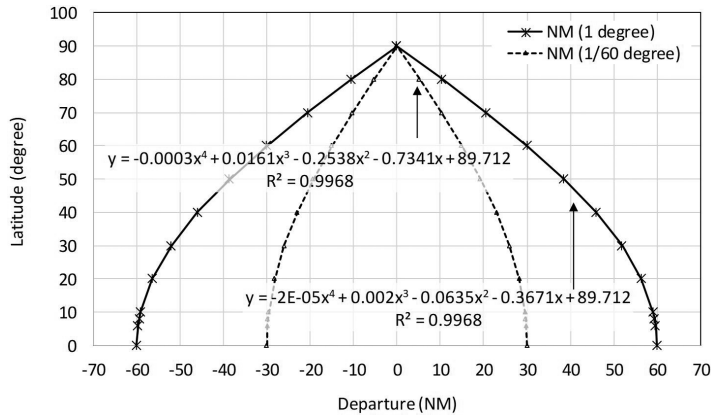


FIGURE 112 Latitude versus the departure in the MC Projection Chart, assuming 60 NM per one-degree change in longitude and 30 NM per 0.5-degree change in longitude.

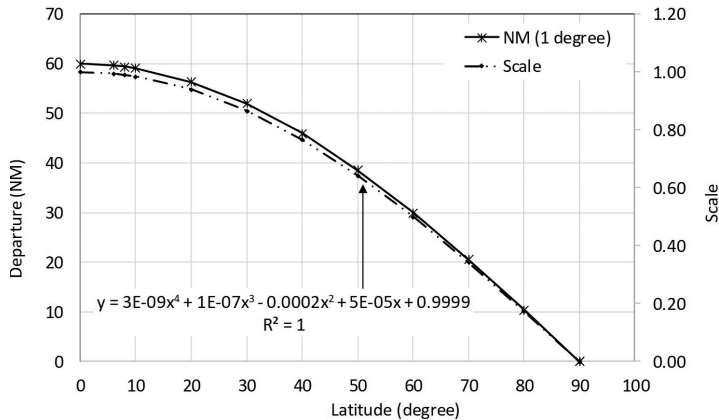


FIGURE 113 Departure versus the latitude in the MC Projection Chart, assuming 60 NM per one-degree change in longitude.

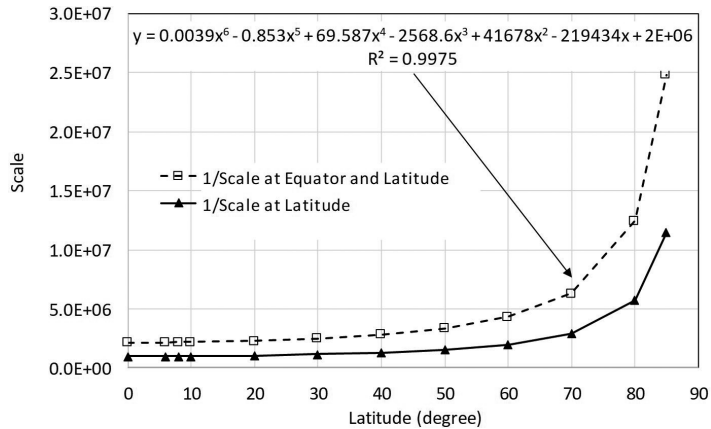


FIGURE 114 Scale versus the latitude in the MC Projection Chart.

9.8.5 Case Study – Mapping VOR Waves on the Mercator Conformal (MC) Projection Chart

An aircraft receives a true bearing from Norman Rogers Airport (CYGK) ATC NDB station (QTE) in Kingston (Canada) that is 263° T. Given the latitudes in Table 41, determine:

(a) convergency at the latitudes, (b) conversion angle at the latitudes, (c) bearings to be plotted from CYGK NDB on the MC Projection Chart, and (d) ADF bearings.

The bearing that the QTE NDB station sends is the great circle bearing (263°). The line that you draw from your current position on the MC Projection Chart is the rhumb line. The angle of this line with the true bearing station signal that the NDB sends and you receive by your ADF is the conversion angle. The angle that the tangents of the two lines at the station make is the convergency—half of that is the convergence angle.

TABLE 41 Convergency and convergence angles for the MC Projection Chart.

Case	Latitude		Mean Latitude		Sin (Latitude)	Convergency ($^{\circ}$)	Convergence Angle ($^{\circ}$)	NDB Bearing ($^{\circ}$)	ADF-Rhumb Line Bearing ($^{\circ}$)
	$^{\circ}$	rad	$^{\circ}$	rad					
2	0	0.00	-	-	-	-	0.00	263.00	83.00
3	6	0.10	3	0.05	0.05	3.14	1.57	261.43	81.43
4	8	0.14	7	0.12	0.12	7.31	3.66	259.34	79.34
5	10	0.17	9	0.16	0.16	9.39	4.69	258.31	78.31
6	20	0.35	15	0.26	0.26	15.53	7.76	255.24	75.24
7	30	0.52	25	0.42	0.42	25.36	12.68	250.32	70.32
8	40	0.70	35	0.57	0.57	34.41	17.21	245.79	65.79
9	50	0.87	45	0.71	0.71	42.43	21.21	241.79	61.79
10	60	1.05	55	0.82	0.82	49.15	24.57	238.43	58.43
11	70	1.22	65	0.91	0.91	54.38	27.19	235.81	55.81
12	80	1.40	75	0.97	0.97	57.96	28.98	234.02	54.02
13	90	1.57	82.5	0.99	0.99	59.49	29.74	233.26	53.26

9.8.6 Case Study – Convergence for the Lambert Conformal Conic (LCC) Projection Chart

Degrees of latitude are presented in Table 42 for the geographical locations in the Northern Hemisphere. For the given latitudes and one-degree change in longitude, calculate:

(a) convergence factor, and (b) chart convergence for a 100-degree change in longitude.

The convergence factor is the sine of the latitude (n), and the convergence is the change of the longitude multiplied by the convergence factor. Results are presented in Table 42 and Figure 115.

TABLE 42 Convergence factor and convergence for the LCC Projection Chart for a 100-degree change of longitude.

Case	Latitude		Sin (Latitude) n	Convergence for 100- degree change in longitude
	°	rad		
1	0	0.00	0.00	0.00
2	6	0.10	0.10	10.45
3	8	0.14	0.14	13.92
4	10	0.17	0.17	17.36
5	20	0.35	0.34	34.20
6	30	0.52	0.50	50.00
7	40	0.70	0.64	70.71
8	50	0.87	0.77	81.92
9	60	1.05	0.87	90.63
10	70	1.22	0.94	96.59
11	80	1.40	0.98	99.62
12	85	1.48	1.00	99.62

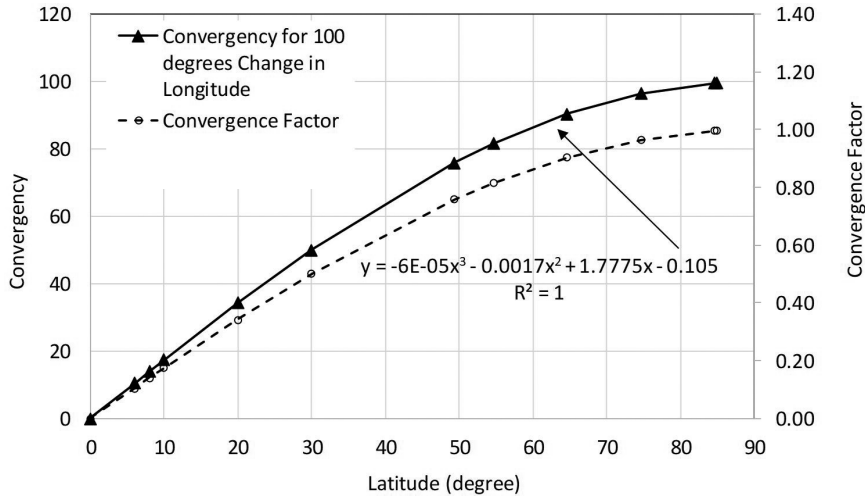


FIGURE 115 Convergency and convergence factor versus the latitude in the LCC Projection Chart.

9.8.7 Case Study – Longitude of a Point on the Lambert Conformal Conic (LCC) Projection Chart

A straight line on the LCC Projection Chart crosses the X° west meridian at point A, which is Y° north of a track angle of $Z^\circ T$. It crosses another meridian at point B, farther east on a track of $U^\circ T$ (Figure 116). The sine of the parallel of origin is v . Derive an expression for the longitude of point B ($W^\circ E$).

A regression analysis is performed for this case study to show the relationship between the difference of the longitudes of points A and B ($Z - U$) as well as the parallel (Y), and tracks passing through the two points— $X^\circ W_A$ and $W^\circ E_B$. The predictors are all statistically significant, and the longitude at the point B ($W^\circ E_B$), located east, may be expressed with a confidence level above 73 percent by equation (188), where δ is a factor

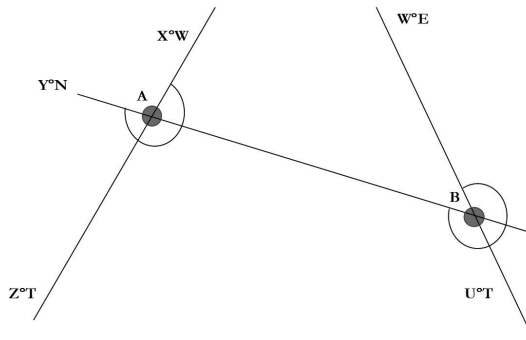


FIGURE 116 Longitudes along the tracks with the same latitude.

representing east-west scenarios (+1, -1) related to point B. As a hands-on exercise, you may attempt this problem and obtain the normal probability plot and summary analysis table.

$$W^{\circ}E_B = 114.4 + 0.4123(Z^{\circ} - U^{\circ}) - 0.319Y^{\circ} + 0.5686X^{\circ}W_A - 125.55\delta \quad (188)$$

A regression analysis is performed for the scenario where the east-west factor is ignored. It is seen that the relationship between the variables remains statistically significant and the longitude at point B ($W^{\circ}E_B$) located east may be expressed with a confidence level of about 52 percent by equation (189).

$$W^{\circ}E_B = 71.1 + 0.707(Z^{\circ} - U^{\circ}) - 0.429Y^{\circ} + 0.9830X^{\circ}W_A \quad (189)$$

9.8.8 Case Study – Geographical Coordinates for the Polar Stereographic Chart

Table 43 presents the latitude from the North Pole down to 60° north. For the latitudes presented, calculate:

(a) colatitude, (b) convergence, (c) maximum distance the Polar map may be employed, (d) scale error, and (e) maximum travel time within calculated distance range, given the passenger jet speed of 600 km/hr (370 kt).

Calculation results are presented in Table 43. Note that colatitude is the difference between the latitude and 90° latitude at the Pole.

TABLE 43 Maximum distances traveled for a passenger jet flying at 600 km/hr (370 kt) for the Polar Stereographic Chart.

Case	Latitude (°)	Colatitud e (°)	Cos (0.5CoL at)	Sec (0.5 × Col Lat) ²	Scale Error (%)	Maximum Distance (km)	Maximum Distance (NM)	Travel Time (hr)	Travel Time (hr: min)
1	90.00	0.00	1.000	1.000	0.00	0.0	0.0	0.00	00:00
2	88.00	2.00	1.000	1.000	0.03	444.5	240.0	0.65	00:39
3	86.00	4.00	0.999	1.001	0.12	889.0	480.0	1.30	01:18
4	84.00	6.00	0.997	1.003	0.27	1,333.4	720.0	1.95	01:57
5	82.00	8.00	0.995	1.005	0.49	1,777.9	960.0	2.59	02:36
6	80.00	10.00	0.992	1.008	0.76	2,222.4	1,200.0	3.24	03:15
7	78.00	12.00	0.989	1.011	1.09	2,666.9	1,440.0	3.89	03:54
8	76.00	14.00	0.985	1.015	1.49	3,111.4	1,680.0	4.54	04:32
9	74.00	16.00	0.981	1.020	1.94	3,555.8	1,920.0	5.19	05:11
10	72.00	18.00	0.976	1.025	2.45	4,000.3	2,160.0	5.84	05:50
11	70.00	20.00	0.970	1.031	3.02	4,444.8	2,400.0	6.49	06:29
12	68.00	22.00	0.964	1.038	3.64	4,889.3	2,640.0	7.14	07:08
13	66.00	24.00	0.957	1.045	4.32	5,333.8	2,880.0	7.78	07:47
14	64.00	26.00	0.949	1.053	5.06	5,778.2	3,120.0	8.43	08:26
15	62.00	28.00	0.941	1.062	5.85	6,222.7	3,360.0	9.08	09:05
16	60.00	30.00	0.933	1.072	6.70	6,667.2	3,600.0	9.73	09:44

9.8.9 Case Study – Geographical Reference System (GEOREF)

Provide the tables for the GEOREF for a location at AMPJ4028 for the following scenarios:

- (a) first two letters, (b) second two letters, (c) third 2 digits, and (d) interpret AMPJ4028.

Table 44 provides the complete coverage for the first two letters associated with the longitudes and latitudes. Table 45 provides the complete coverage for the second two letters associated with the longitudes and latitudes. Table 46 and Table 47 provide the portion of the two digits associated with the minutes of the longitudes and latitudes. Note that to find the degree fraction, the number is to be divided by 60.

TABLE 44 First two letters of GEOREF on the map with identified latitudes and longitudes in degrees.

Count			1	2	3	4	5	6	7	8	9	10	11	12
Latitude Longitude			-90	-75	-60	-45	-30	-15	0	15	30	45	60	75
Letter			A	B	C	D	E	F	G	H	J	K	L	M
24	165	Z	ZA	ZB	ZC	ZD	ZE	ZF	ZG	ZH	ZJ	ZK	ZL	ZM
23	150	Y	YA	YB	YC	YD	YE	YF	YG	YH	YJ	YK	YL	YM
22	135	X	XA	XB	XC	XD	XE	XF	XG	XH	XJ	XK	XL	XM
21	120	W	WA	WB	WC	WD	WE	WF	WG	WH	WJ	WK	WL	WM
20	105	V	VA	VB	VC	VD	VE	VF	VG	VH	VJ	VK	VL	VM
19	90	U	UA	UB	UC	UD	UE	UF	UG	UH	UJ	UK	UL	UM
18	75	T	TA	TB	TC	TD	TE	TF	TG	TH	TJ	TK	TL	TM
17	60	S	SA	SB	SC	SD	SE	SF	SG	SH	SJ	SK	SL	SM
16	45	R	RA	RB	RC	RD	RE	RF	RG	RH	RJ	RK	RL	RM
15	30	Q	QA	QB	QC	QD	QE	QF	QG	QH	QJ	QK	QL	PM
14	15	P	PA	PB	PC	PD	PE	PF	PG	PH	PJ	PK	PL	OM
13	0	N	NA	NB	NC	ND	NE	NF	NG	NH	NJ	NK	NL	NM
12	-15	M	MA	MB	MC	MD	ME	MF	MG	MH	MJ	MK	ML	LM
11	-30	K	LA	LB	LC	LD	LE	LF	LG	LH	LJ	LK	LL	KM
10	-45	J	KA	KB	KC	KD	KE	KF	KG	KH	KJ	KK	KL	JM
9	-60	I	JA	JB	JC	JD	JE	JF	JG	JH	JJ	JK	JL	IM
8	-75	H	HA	HB	HC	HD	HE	HF	HG	HH	HJ	HK	HL	HM
7	-90	G	GA	GB	GC	GD	GE	GF	GG	GH	GJ	GK	GL	GM
6	-105	F	FA	FB	FC	FD	FE	FF	FG	FH	FJ	FK	FL	FM
5	-120	E	EA	EB	EC	ED	EE	EF	EG	EH	EJ	EK	EL	EM
4	-135	D	DA	DB	DC	DD	DE	DF	DG	DH	DJ	DK	DL	DM
3	-150	C	CA	CB	CC	CD	CE	CF	CG	CH	CJ	CK	CL	CM
2	-165	B	BA	BB	BC	BD	BE	BF	BG	BH	BJ	BK	BL	BM
1	-180	A	AA	AB	AC	AD	AE	AF	AG	AH	AJ	AK	AL	AM

Interpretation of AMPJ4129: Located at the first 15-degree-longitude zone (A) and the twelfth 15-degree-latitude zone (M)—meaning it is located in a longitude band of between -180° to -165° and a latitude band of between 75° to 90° . It is located in the fourteenth 15-degree-longitude zone (P) and the ninth 15-degree-latitude zone (J)—meaning it is located at a longitude band from -167° to -166° and a latitude band from 83° to 84° . It is located at 40-min longitude and 28-min latitude (4028). This identifies a location at ($83^{\circ}28'0''\text{N}$, $166^{\circ}20'0''\text{W}$), which is a point located in the Arctic Ocean.

TABLE 45 Second two letters of GEOREF on the map with identified latitudes and longitudes in degrees.

Count		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Latitude	Longitude	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89
		A	B	C	D	E	F	G	H	I	J	K	L	N	P	Q
15	-166	P	PA	PB	PC	PD	PE	PF	PG	PH	PI	PJ	PK	PL	PN	PQ
14	-167	O	OA	OB	OC	OD	OE	OF	OG	OH	OI	OJ	OK	OL	ON	OQ
13	-168	N	NA	NB	NC	ND	NE	NF	NG	NH	NI	NJ	NK	NL	NN	NQ
12	-169	L	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LN	LQ
11	-170	K	KA	KB	KC	KD	KE	KF	KG	KH	KI	KJ	KK	KL	KN	KQ
10	-171	J	JA	JB	JC	JD	JE	JF	JG	JH	JI	JJ	JK	JL	JN	JQ
9	-172	I	IA	IB	IC	ID	IE	IF	IG	IH	II	IJ	IK	IL	IN	IQ
8	-173	H	HA	HB	HC	HD	HE	HF	HG	HH	HI	HJ	HK	HL	HN	HQ
7	-174	G	GA	GB	GC	GD	GE	GF	GG	GH	GI	GJ	GK	GL	GN	GQ
6	-175	F	FA	FB	FC	FD	FE	FF	FG	FH	FI	FJ	FK	FL	FN	FQ
5	-176	E	EA	EB	EC	ED	EE	EF	EG	EH	EI	EJ	EK	EL	EN	EQ
4	-177	D	DA	DB	DC	DD	DE	DF	DG	DH	DI	DJ	DK	DL	DN	DQ
3	-178	C	CA	CB	CC	CD	CE	CF	CG	CH	CI	CJ	CK	CL	CN	CQ
2	-179	B	BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL	BN	BQ
1	-180	A	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AN	AQ

TABLE 46 First two digits of GEOREF on the map with identified latitudes and longitudes in degrees (part 1).

Count			1	2	3	4	5	6	7	8	9	10	11
Longitude			0	1	2	3	4	5	6	7	8	9	10
Latitude													
60	-179.017	59	5900	5901	5902	5903	5904	5905	5906	5907	5908	5909	5910
59	-179.033	58	5800	5801	5802	5803	5804	5805	5806	5807	5808	5809	5810
58	-179.05	57	5700	5701	5702	5703	5704	5705	5706	5707	5708	5709	5710
57	-179.067	56	5600	5601	5602	5603	5604	5605	5606	5607	5608	5609	5610
56	-179.083	55	5500	5501	5502	5503	5504	5505	5506	5507	5508	5509	5510
55	-179.1	54	5400	5401	5402	5403	5404	5405	5406	5407	5408	5409	5410
54	-179.117	53	5300	5301	5302	5303	5304	5305	5306	5307	5308	5309	5310
53	-179.133	52	5200	5201	5202	5203	5204	5205	5206	5207	5208	5209	5210
52	-179.15	51	5100	5101	5102	5103	5104	5105	5106	5107	5108	5109	5110
51	-179.167	50	5000	5001	5002	5003	5004	5005	5006	5007	5008	5009	5010
50	-179.183	49	4900	4901	4902	4903	4904	4905	4906	4907	4908	4909	4910
49	-179.2	48	4800	4801	4802	4803	4804	4805	4806	4807	4808	4809	4810
48	-179.217	47	4700	4701	4702	4703	4704	4705	4706	4707	4708	4709	4710
47	-179.233	46	4600	4601	4602	4603	4604	4605	4606	4607	4608	4609	4610
46	-179.25	45	4500	4501	4502	4503	4504	4505	4506	4507	4508	4509	4510
45	-179.267	44	4400	4401	4402	4403	4404	4405	4406	4407	4408	4409	4410
44	-179.283	43	4300	4301	4302	4303	4304	4305	4306	4307	4308	4309	4310
43	-179.3	42	4200	4201	4202	4203	4204	4205	4206	4207	4208	4209	4210
42	-179.317	41	4100	4101	4102	4103	4104	4105	4106	4107	4108	4109	4110
41	-179.333	40	4000	4001	4002	4003	4004	4005	4006	4007	4008	4009	4010
40	-179.35	39	3900	3901	3902	3903	3904	3905	3906	3907	3908	3909	3910
39	-179.367	38	3800	3801	3802	3803	3804	3805	3806	3807	3808	3809	3810
38	-179.383	37	3700	3701	3702	3703	3704	3705	3706	3707	3708	3709	3710
37	-179.4	36	3600	3601	3602	3603	3604	3605	3606	3607	3608	3609	3610
36	-179.417	35	3500	3501	3502	3503	3504	3505	3506	3507	3508	3509	3510
35	-179.433	34	3400	3401	3402	3403	3404	3405	3406	3407	3408	3409	3410
34	-179.45	33	3300	3301	3302	3303	3304	3305	3306	3307	3308	3309	3310
33	-179.467	32	3200	3201	3202	3203	3204	3205	3206	3207	3208	3209	3210
32	-179.483	31	3100	3101	3102	3103	3104	3105	3106	3107	3108	3109	3110
31	-179.5	30	3000	3001	3002	3003	3004	3005	3006	3007	3008	3009	3010
30	-179.517	29	2900	2901	2902	2903	2904	2905	2906	2907	2908	2909	2910
29	-179.533	28	2800	2801	2802	2803	2804	2805	2806	2807	2808	2809	2810
28	-179.55	27	2700	2701	2702	2703	2704	2705	2706	2707	2708	2709	2710
27	-179.567	26	2600	2601	2602	2603	2604	2605	2606	2607	2608	2609	2610
26	-179.583	25	2500	2501	2502	2503	2504	2505	2506	2507	2508	2509	2510

TABLE 47 First two digits of GEOREF on the map with identified latitudes and longitudes in degrees (part 2).

Count			1	2	3	4	5	6	7	8	9	10	11
Latitude	Longitude		0	1	2	3	4	5	6	7	8	9	10
			0	1	2	3	4	5	6	7	8	9	10
25	-179.6	24	2400	2401	2402	2403	2404	2405	2406	2407	2408	2409	2410
24	-179.617	23	2300	2301	2302	2303	2304	2305	2306	2307	2308	2309	2310
23	-179.633	22	2200	2201	2202	2203	2204	2205	2206	2207	2208	2209	2210
22	-179.65	21	2100	2101	2102	2103	2104	2105	2106	2107	2108	2109	2110
21	-179.667	20	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
20	-179.683	19	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910
19	-179.7	18	1800	1801	1802	1803	1804	1805	1806	1807	1808	1809	1810
18	-179.717	17	1700	1701	1702	1703	1704	1705	1706	1707	1708	1709	1710
17	-179.733	16	1600	1601	1602	1603	1604	1605	1606	1607	1608	1609	1610
16	-179.75	15	1500	1501	1502	1503	1504	1505	1506	1507	1508	1509	1510
15	-179.767	14	1400	1401	1402	1403	1404	1405	1406	1407	1408	1409	1410
14	-179.783	13	1300	1301	1302	1303	1304	1305	1306	1307	1308	1309	1310
13	-179.8	12	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210
12	-179.817	11	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110
11	-179.833	10	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010
10	-179.85	9	0900	0901	0902	0903	0904	0905	0906	0907	0908	0909	0910
9	-179.867	8	0800	0801	0802	0803	0804	0805	0806	0807	0808	0809	0810
8	-179.883	7	0700	0701	0702	0703	0704	0705	0706	0707	0708	0709	0710
7	-179.9	6	0600	0601	0602	0603	0604	0605	0606	0607	0608	0609	0610
6	-179.917	5	0500	0501	0502	0503	0504	0505	0506	0507	0508	0509	0510
5	-179.933	4	0400	0401	0402	0403	0404	0405	0406	0407	0408	0409	0410
4	-179.95	3	0300	0301	0302	0303	0304	0305	0306	0307	0308	0309	0310
3	-179.967	2	0200	0201	0202	0203	0204	0205	0206	0207	0208	0209	0210
2	-179.983	1	0100	0101	0102	0103	0104	0105	0106	0107	0108	0109	0110
1	-180	0	0000	0001	0002	0003	0004	0005	0006	0007	0008	0009	0010

9.8.10 Case Study – Geographical Coordinates— Distances on the Earth

Table 48 presents examples of some of the geographical coordinates worldwide along with their elevations. For the locations presented in Table 48, calculate:

TABLE 48 Examples of geographical coordinates along with their elevations [201].

Case	Code	Aerodrome	Location	Latitude	Longitude	Elevation MSL (ft)
1	EGTK	Oxford	Oxford, UK	N051°50.22'	W001°19.20'	270
2	EDDM	München	Munich, Germany	N048°21.23'	E011°47.17'	1,487
3	LSZC (ex LSMU)	Buochs	Luzern, Switzerland	N046°58.47'	E008°23.82'	1475
4	CYBK	Norman Rogers Airport	Kingston, Canada	N044°13.55'	W076°35.80'	303
5	KBOS	General Edward Lawrence Logan International	Boston, USA	N042°21.78'	W071°00.38'	19
6	RJAA	Narita International	Tokyo, Japan	N035°45.92'	E140°23.13'	135
7	OHH	Mehrabad International	Tehran, Iran	N035°41.33'	E051°18.88'	3,962
8	YSSY	Sydney International	Sydney, Australia	S033°56.77'	E151°10.63'	21
9	CCB5	Goose Bay Airport	Labrador, Canada	N053°21.00'	W060°25.00'	0
10	BIKF	Keflavik Airport	Suðurnes, Iceland	N063°59.10'	W022°36.33'	169

(a) spherical coordinate, (b) distance from location 4 (i.e., CYBK) to i , ($i = 1$ to 10) assuming 60 NM for each degree on average, (c) accuracy of measurements by comparing the results with the ones from well-known sources (e.g., SkyVector®), and (d) average length of one degree between the two points.

Calculations are presented in Table 49. Note that to calculate the distances, you are to ensure the θ and ϕ are properly identified. ϕ is the colatitude in these calculations. Also note the easterly and southerly positions of the locations are presented with a 180° reference point separating the east and west. Note the potential change of the cosine to sine when adding odd multipliers of 90° as well as a change in its sign (\pm). There are a number of formulae that may be used to obtain the distance between the two points on the great circle. Use of spherical coordinates in this case study proves to be a good choice with small deviations from the values presented in SkyVector®—a well-known online source of flight planning with extensive data on the world's aerodromes.

In Table 49, ϕ is the colatitude and is calculated by ($\phi = 90 \mp \text{Latitude}$), with the negative sign used for the northerly latitudes and the positive sign used for the southerly latitudes. θ is the reciprocal of the longitude and is calculated by ($\theta = 180 \mp \text{Longitude}$), with the negative sign used for the westerly longitudes and the positive sign for the easterly longitudes.

TABLE 49 Distances calculated for the locations presented in Table 48.

Case	Latitude	Longitude	Colatitude ϕ (°)	θ (°)	(X_i, Y_i, Z_i)	Dot Product	aCos (°)	Distance d_{cal} (NM)	SkyVector® d_{sv} (NM)	Error e (%)	1°/NM
1	N051° 50.22'	W001° 19.20'	38.2	178.7	(-0.62, 0.01, 0.79)	0.66	48.6	2,917.7	2,928.1	0.4	59.79
2	N048° 21.23'	E011° 47.17'	41.7	191.8	(-0.65, -0.14, 0.75)	0.53	57.7	3,460.8	3,473.0	0.4	59.79
3	N046° 58.47'	E008° 23.82'	43.0	188.4	(-0.68, -0.1, 0.73)	0.55	56.5	3,387.4	3,399.2	0.3	59.79
4	N044° 13.55'	W076° 35.80'	45.8	103.4	(-0.17, 0.7, 0.7)	1.00	0.0	0.0	0.0	0.0	0.00
5	N042° 21.78'	W071° 00.38'	47.6	109.0	(-0.24, 0.7, 0.67)	1.00	4.5	2,68.4	269.2	0.3	59.83
6	N035° 45.92'	E140° 23.13'	54.2	320.4	(0.63, -0.52, 0.58)	-0.06	93.3	5,595.5	5,612.2	0.3	59.82
7	N035° 41.33'	E051° 18.88'	54.3	231.3	(-0.51, -0.63, 0.58)	0.05	87.2	5,230.5	5,246.4	0.3	59.82
8	S033° 56.77'	E151° 10.63'	124.0	331.2	(0.73, -0.4, -0.56)	-0.79	142.1	8,525.5	8,529.7	100	59.97
9	N053° 21.00'	W060° 25.00'	36.7	119.6	(-0.29, 0.52, 0.8)	0.97	14.0	838.5	840.5	100.2	59.85
10	N063° 59.10'	W022° 36.33'	26.0	157.50	(-0.4, 0.17, 0.9)	0.81	35.8	2,144.8	2,151.8	100.3	59.81

The coordinates of the location (X_i, Y_i, Z_i) are found using the spherical coordinates formula $(\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi)$ for a unit vector. The dot product of the two coordinates is $(X_1, Y_1, Z_1)(X_2, Y_2, Z_2) = (X_1X_2 + Y_1Y_2 + Z_1Z_2)$. The reference location from which the relative measurements are made is at CYGK. The distance between the two coordinates in nautical miles is the angle between the two locations in degrees that is the arc(cosine) of the dot product of the coordinates multiplied by the length of each degree (60 NM at the Equator)— $\arccos(X_1X_2 + Y_1Y_2 + Z_1Z_2) \times 60$. To validate the calculated data, it is possible to input the latitude and longitude of the coordinates in a flight plan calculation software (e.g., SkyVector®) and compare the suggested distance with the calculated one. The Error is then obtained by comparing the two values— $Error = |(d_{sv} - d_{cal})/d_{sv}|$, where d_{sv} is the distance from SkyVector® and d_{cal} is the calculated distance. It is then possible to calculate the distances between the longitudes in nautical miles per degree of longitude given the geographical location, considering the original assumption that was based on 60 NM per degree of longitude

$(60 \times d_{Cal}/d_{SV})$. Note that the length of degree longitude is not constant and changes depending on the location's latitude. The assumed 60 NM is valid at the Equator and decreases toward the north and south. The distance for one degree of latitude is 60 NM at the Equator (and all other latitudes). This means that at the Equator, the area of a one-degree by one-degree grid is 3,600 NM². Each degree is divided into 60 min of distance and each minute is further divided into 60 seconds of distance. The tenth of a latitude-minute equals 185 m (600 ft). A nautical mile (NM) is 1,852 km (6,076.12 ft). A statute mile is about 1,609 km (5,280 ft). Therefore, a nautical mile is 15 percent more than a statute mile.

9.8.11 Case Study – Calculating the Sun's Azimuth Using the Sun's Position in the Sky

G. F. Mock was the first female aviator to fly solo around the world. She departed from Columbus, Ohio in the United States on March 19, 1964 and ended the flight there on April 17, 1964. She traveled in the Cessna 180 aircraft, called *Spirit of Columbus*, also known as *Charlie*. Columbus, Ohio Airport is at coordinates N39°59.82' and W82°53.53'. Multiple observations were made at 12:00 Pacific Standard Time (PST) on March 19, 1964 to find the accurate values for the Sun's position above the horizon; the Sun's declination of S000°23.9' and GHA of W028°03.7' were reported on an almanac table. For the locations presented in Table 50, calculate:

(a) decimal (dec.) values of the longitudes and latitudes, (b) differences in longitudes from the Sun's, (c) sines of the azimuths, and (d) azimuths in degrees.

Calculations are presented in Table 50. Note that to calculate the distances, you are to ensure the minutes in the longitudes and latitudes are properly converted to decimals. Also note the easterly and westerly positions of the locations presented with a 180° reference point separating the east and west and north and south positions are to start at zero degrees as the reference point—resulting in negative values in the Southern and Eastern Hemispheres.

In Table 50, the longitude difference is the difference between the longitudes of the location (Case “i”, “i” varying from 1 to 13) and that of the Sun (Case 12)— $Longitude_{Difference} = Longitude_{Sun} \mp Longitude_{Location}$, the negative sign used for the westerly longitudes and the positive sign for the easterly longitudes. The sine of the azimuth is then calculated by $Sin(Azimuth) = Cos(Latitude_{Sun}) \times Cos(Latitude_{Location}) \times Cos(Longitude_{Difference}) + Sin(Latitude_{Sun}) \times Sin(Latitude_{Location})$. The

TABLE 50 Locations to calculate the azimuth.

Case	Latitude	Longitude	Latitude (dec.)	Longitude (dec.)	Longitude difference (°)	Longitude difference (rad)	Sin (Azimuth)	Azimuth (rad)	Azimuth (°)	Azimuth
1	N051° 50.22'	W001° 19.20'	51.8	178.7	-26.74	-0.47	0.5463	0.58	33.12	33°7.2'
2	N048° 21.23'	E011° 47.17'	48.4	191.8	343.72	6.00	0.6327	0.69	39.25	39°15'
3	N046° 58.47'	E008° 23.82'	47.0	188.4	340.34	5.94	0.6374	0.69	39.60	39°36'
4	N044° 13.55'	W076° 35.80'	44.2	103.4	48.54	0.85	0.4696	0.49	28.01	28°0.6'
5	N042° 21.78'	W071° 00.38'	42.4	109.0	42.94	0.75	0.5362	0.57	32.42	32°25.2'
6	N035° 45.92'	E140° 23.13'	35.8	320.4	112.32	1.96	-0.3123	-0.32	-18.20	-19°48'
7	N035° 41.33'	E051° 18.88'	35.7	231.3	23.25	0.41	0.7421	0.84	47.91	47°54.6'
8	S033° 56.77'	E151° 10.63'	-34.0	331.2	123.12	2.15	-0.4493	-0.47	-26.70	-27°18'
9	N053° 21.00'	W060° 25.00'	53.4	119.6	32.36	0.56	0.4987	0.52	29.91	29°54.6'
10	N063° 59.10'	W022° 36.33'	64.0	157.4	-5.46	-0.10	0.4304	0.44	25.49	25°29.4'
11	N041° 00.00'	W070° 00.00'	41.0	110.0	41.94	0.73	0.5568	0.59	33.84	33°50.4'
12	S000° 23.90'	W028° 03.70'	-0.4	151.9	0.00	0.00	1.0000	1.57	90.00	90°0'
13	N039° 59.82'	W082° 53.53'	40.0	97.1	54.83	0.96	0.4368	0.45	25.90	25°54'

azimuth in radians is the arc(sine) of this calculated sine of the azimuth— $Azimuth = \arcsin(\sin(Azimuth))$. The decimal part of the azimuth can then be divided by 60 to calculate the minute portion of the azimuth.

9.8.12 Case Study – Calculating the Sun’s Azimuth Using an Observation Latitude

G. F. Mock, the first female aviator who flew solo around the world, departing from Columbus, Ohio in the United States, started at the coordinates N35°59.82' and W82°53.53'. Multiple observations were made at 14:00 UTC on March 19, 1964 to find the accurate values for the Sun’s position above the horizon; the Sun’s declination of S000°23.9' and GHA

of W028°03.7' were reported on an almanac table. The observation latitude was reported as 24°35'. For the locations presented in Table 51, calculate:

(a) decimal (dec.) values of the longitudes and latitudes, (b) differences in longitudes from the Sun's, (c) sines of the azimuths, and (d) Sun's azimuths in degrees.

Calculations are presented in Table 51. Note that to calculate the distances, you are to ensure the minutes in the longitudes and latitudes are properly converted to decimals. Also note the easterly and southerly positions of the locations presented with a 180° reference point separating the east and west and zero-degree north and south positions—resulting in negative values in the Southern and Eastern Hemispheres.

In Table 51, the longitude difference is the difference between the longitudes of the location (Case “i”, “i” varying from 1 to 13) and that of the Sun (Case 12)— $Longitude\ Difference = Longitude_{Sun} \mp Longitude_{Location}$, the negative sign being used for the westerly longitudes and the positive sign for the easterly longitudes. The cosine of the azimuth is then calculated by a numerator presented by $Numerator = \sin(Latitude_{Sun}) \times \cos(Latitude_{Location}) - \cos(Latitude_{Sun}) \times \sin(Latitude_{Location}) \times \cos(Longitude\ Difference)$ and by a denominator that is the cosine of the observation altitude— $Denominator = \cos(Latitude_{Observation})$. The azimuth

TABLE 51 Locations to calculate the Sun's azimuth.

Case	Latitude	Longitude	Latitude (dec.)	Longitude (dec.)	Longitude difference (°)	Longitude difference	Cos (Azimuth)	Azimuth (rad)	Azimuth (°)	Azimuth
1	N051°50.22'	W001°19.20'	51.84	178.68	-26.74	-0.47	-0.7746	2.46	140.77	140°46.2'
2	N048°21.23'	E011°47.17'	48.35	191.79	343.72	6.00	-0.7916	2.48	142.33	142°19.8'
3	N046°58.47'	E008°23.82'	46.97	188.40	340.34	5.94	-0.7600	2.43	139.46	139°27.6'
4	N044°13.55'	W076°35.80'	44.23	103.40	48.54	0.85	-0.5119	2.11	120.79	120°47.4'
5	N042°21.78'	W071°00.38'	42.36	108.99	42.94	0.75	-0.5464	2.15	123.12	123°7.2'
6	N035°45.92'	E140°23.13'	35.77	320.39	112.32	1.96	0.2372	1.33	76.28	76°16.8'
7	N035°41.33'	E051°18.88'	35.69	231.31	23.25	0.41	-0.5939	2.21	126.43	126°25.8'
8	S033°56.77'	E151°10.63'	-33.95	331.18	123.12	2.15	-0.3408	1.92	109.93	109°55.8'
9	N053°21.00'	W060°25.00'	53.35	119.58	32.36	0.56	-0.7476	2.42	138.39	138°23.4'
10	N063°59.10'	W022°36.33'	63.99	157.39	-5.46	-0.10	-0.9842	2.96	169.81	169°48.6'
11	N041°00.00'	W070°00.00'	41.00	110.00	41.94	0.73	-0.5408	2.14	122.74	122°44.4'
12	S000°23.90'	W028°03.70'	-0.40	151.94	0.00	0.00	0.0000	1.57	90.00	90°0'
13	N039°59.82'	W082°53.53'	40.00	97.11	54.83	0.96	-0.4118	2.00	114.32	114°19.2'

in radians is the arc(cosine) of this calculated cosine of the azimuth— $Azimuth = \arccos(Numerator / Denominator)$. The decimal part of the azimuth can then be divided by 60 to calculate the minute portion of the azimuth.

9.8.13 Case Study – Distance from Longitude versus the Latitude

Using the latitudes listed in Table 52, calculate the distances for a single-degree longitude and an average time zone (i.e., 15°-longitude) using the first- and the second-order regression formulae.

Table 52 presents examples in which the latitude data are employed to obtain the distances for the latitudes, using the formulae for both the First Order (FO) and the Second Order (SO) regression fits. Figure 117 compares the results in a single diagram. The results for the 15°-longitude spacing follow similar trends that further confirm the accuracy of the formula for larger-spaced longitudes. The first order and the second order models are presented at the bottom of Table 52. The results are compared with the data from the formula: $Distance = 60 \times Longitude \times Difference \cos(latitude)$

TABLE 52 Case studies to obtain longitude distance versus the latitude using suggested formulae.

Case	Latitude	First Order		Second Order	
		FO-Space (NM) per 15°	FO-Space (NM) per 1°	SO-Space (NM) per 15°	SO-Space (NM) per 1°
1	10	1029.30	68.62	912.70	60.85
2	20	923.19	61.55	882.71	58.847
3	30	817.08	54.47	834.58	55.64
4	40	710.97	47.40	768.30	51.22
5	50	604.86	40.32	683.88	45.59
SO-Space (NM) per 15° : $y = 912.70 - 2.092 \text{ Latitude}(\circ) - 0.09071 \text{ Latitude}(\circ) \times \text{Latitude}(\circ)$ FO-Space (NM) per 15° : $y = 1029.3 - 10.611 \text{ Latitude}(\circ)$					
SO-Space (NM) per 1° : $y = 60.847 - 0.1395 \text{ Latitude}(\circ) - 0.006048 \text{ Latitude}(\circ) \times \text{Latitude}(\circ)$ FO-Space (NM) per 1° : $y = 68.62 - 0.7074 \text{ Latitude}(\circ)$					

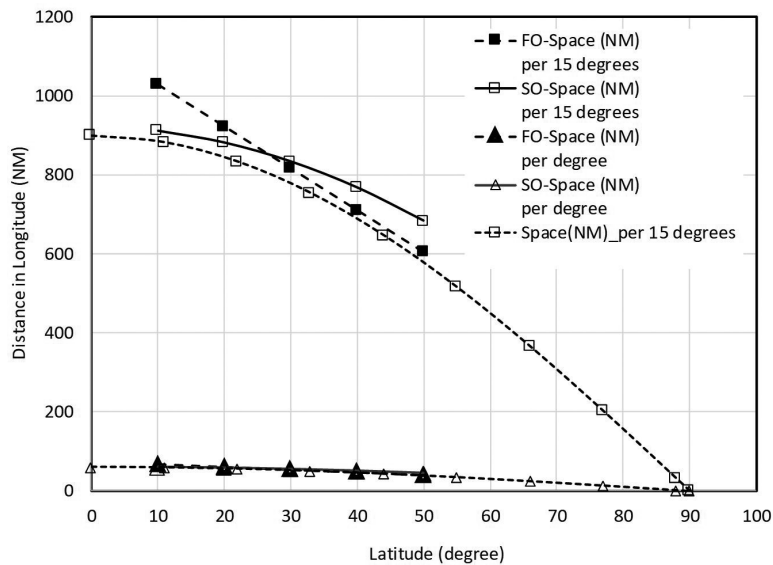


FIGURE 117 The comparison between the second-order (SO) and first-order (FO) relations between the latitude versus the distance in longitude for one-degree longitude.

FLIGHT PLAN

This section provides you with the tools required to make that memorable flight plan you have been dreaming of. While working on your own flight plans, in some sense, you will be following in the footsteps of extraordinary aviators of the past, such as C. A. Lindbergh (an American aviator, first transatlantic solo flight in 1927) [202,203] and his wife Anne Morrow Spencer Lindbergh (an American aviator, first woman solo glider pilot in 1930) [204,205]; A. M. Earhart (an American aviator, first woman solo to fly cross the Atlantic in a single engine in 1928) [206,207]; B. C. Markham (a British-born Kenyan aviator, first woman solo to fly west across the Atlantic in 1936) [208,209]; and G. F. Mock (an American aviator, first woman aviator to fly solo around the World in 1964) [210,211]. These exceptional people—one may call them “outliers”—wrote fascinating autobiographies worth reading and studying; reading them, you can learn about their flight preparation methods and the tough challenges they overcame.

Not all received the recognition they deserved. A. M. S. Lindbergh accompanied C. A. Lindbergh on all the exploratory flights to map the potential airline routes in the 1930s, but her daughter only learned about her mother being a certified glider pilot accidentally while looking among her office papers. The history books have not given enough credit to this modest woman who was trained technically and revered by her husband and occasionally teased by A. M. Earhart. Even Wikipedia neglects to list this individual in the article entitled “List of Women Aviators” [212] (last checked in January 2019) [213,214,215,216,217].

10.1 Introduction

Preparing a flight plan is perhaps one of the most time-consuming activities required before each flight by the FIC and the Second-In-Command (SIC). Each one is to be fully prepared for the flight ahead, starting preparation well ahead of time, sometimes days in advance, especially for longer hauls. The minimum requirement to make a cross-country flight (in addition to wearing a watch preferably with a chronometer) is to be familiar with the taxiways and runways of the departure aerodrome, flight path, and taxiways and runways of the destination aerodrome as well as the provisional aerodromes along the route where it would be appropriate to land if required. The reason for this diversion may be to use the facilities, have a sick person on board, take fuel, or check the weather data.

No matter the reason, you still need to be fully prepared. This preparation requires collection of all kinds of information: radio station data, both for weather information and air traffic control; runway information, such as surface type, direction, and length; available fuel types; landing fees; service centers; facilities on-site; places to eat or stay; the closest city or medical center to the aerodrome; and hours of operation.

Flight briefing is a conversation prior to any flight where crew with all levels of responsibility take part, be it a training exercise, a cross-country flight, or a formation flight. If it is a training flight, the student is to know what is done, how it is done, and the safety-related precautions. The debrief after a training flight is not the same as the groundwork and presentations; it is a must for all flights including the student solo flights that do not include new exercises. The areas to be covered are: (a) NOTAM, meteorological, and aerodrome situations; (b) information regarding the aircraft such as its airworthiness, fuel, and working fluids; (c) location of the exercises; (d) takeoff and landing information along with the duration of the flight; (e) maneuvers and exercises that are intended to be performed; and (f) review of the applicable knowledge and skills.

During the flight briefing, you need to review the route and all potential issues that may affect the planned trip. Earlier sections of this book talked about the weather resources available, METAR, FSS (Flight Service Station), and FIC being the most widely used. There are regular periodicals published by the countries hosting the aerodromes, such as Canada Flight Supplement (CFS) published for Canadian land and sea aerodromes. Learn to use traditional pen and paper techniques and master the calculations at early stages. As the FIC, no matter how miserable it makes you feel, you

are to know how to calculate the information you require from takeoff to landing. There are also electronic planning tools available in addition to the pen and paper techniques that provide the weather data, occasionally near-real-time radar information, aerodrome information, VNC and VTA charts, VFR and IFR procedures, arrival and departure procedures, and other information required for a successful flight. You have all the information you need to carry on your flight plan successfully except for the population of mosquitos, their camaraderie, their active hours when they bite less defensive prayers, and the way your allergy will manifest—either in the form of cold or skin rashes.

Assume you have been dreaming of taking a cross-country trip on your birthday or another special occasion. Assume you have decided on specific constraints such as time, fuel requirements (e.g., travel on one tank for a round trip or one way), and travel costs. You would need to draw isochrones (lines of constant travel time to a given location to find the acceptable locations fulfilling the aircraft endurance given the set fuel requirements); isotims (lines of constant transport costs as high lodging costs in some destinations may not fit within the set budget); or isodapanes (lines of constant cost of travel time since you may still be able to travel on the set budget if you economize the itinerary, from the air travel to the boarding), to shortlist your dream destination. Imagine you do not do your homework by making these plans beforehand; you may end up making an emergency stop in an unfamiliar aerodrome only to find out that their FBO does not have the 100LL fuel on which your hungry Cessna 172M needs to feed.

10.2 Airworthiness

For an aircraft to be considered airworthy, a number of documents are required. A memory aid, AROWJILI, may be used to help you remember what you will need: (a) Airworthiness certificate, (b) aircraft Registration documents, (c) Pilot's Operating Handbook (POH), (d) Weight-and-balance calculations and certificate, (e) Journey logbooks, (f) Insurance documents, (g) crew Licenses (i.e., medical documentation, flying licenses, radio operator licenses), and (h) Interception procedures. In the United States, the FAA issues the Federal Aviation Regulations (FAR), which identify the airworthiness requirements for the normal and transport category airplanes [218,219,220]. The documents are almost always placed inside a single bag and are to be carried on board. Review the documents before flying, paying particular attention to the dates. If new work has been done, such as the weight-and-balance, and new documents have been issued, the old versions

are always superseded by the new documents with the word *superseded* written obliquely across the old versions. The very first page of a journey logbook has an attachment in which issues and defects are to be reported either by the FIC or a flight crew. These are called snags and are to be addressed before the start of the flight by a certified Aircraft Maintenance Engineer (AME). In some cases where electronic components are to be certified, the AME is to be certified for aircraft electronics as well. If for whatever reason you encounter a mechanical problem, you are to report it immediately to the AME (you are to have the required contact information) and record it on the snags paper attached to the journey logbook. The aircraft may not be airworthy as a result; however, the AME may generate special permission or allow for a ten-hour extension in case the maintenance time has expired but the safety is not jeopardized.

There are regular oil checks (e.g., 50 hrs), maintenance checks (e.g., 100 hrs), and Time Between Overhauls (TBO) checks (e.g., 1,200 hrs) in addition to the engine checks and electronics calibrations that need to be done either annually (e.g., first aid, ELT, and compass swing) or biannually (e.g., pitot-static, altimeter, and transponder). Some equipment or sub-components, such as the stall system, exterior pitot-static system sensors, or ELT battery, are to be checked regularly or as recommended by the manufacturer. Note that you may be able to perform some preventive maintenance work yourself on your non-commercial aircraft on the items specified in FAR (Title 14, Chapter 1, Subchapter C, Part 43, Section 3) contingent upon completing an approved training program that has been authorized in writing by the training agency. The work also may be approved by a certified AME, excluding any work related to the airframe, engine, and propeller, which are all considered major repairs [221,222].

10.3 Weight-and-Balance

Calculating the weight-and-balance and payloads is one of the most important elements of flight preparation. Recall that for an aircraft to be airworthy, you need to have properly completed weight-and-balance calculations. As was shown in Chapter 8 (Performance Charts), even within the allowed limits, having the forward or rearward COG will affect your stall speed. These calculations are especially important on warm and humid days, when the aircraft performance is most affected; for example, your minimum runway length would need to be longer on such a day for a safe takeoff and landing. For an airline, their ultimate objective is to maximize the revenue, which is directly related to the payload (also known as the

traffic load). However, since safety comes first, the traffic load may need to be redistributed or reduced to ensure the flight will be safely completed.

Before further discussion of this subject, it should be noted that the terms *mass* and *weight* are often used interchangeably but, by definition, weight is a force and is technically different from mass. Weight is expressed in Newtons (N), pound-force (lbf), or their derivatives. Thus, your *weight* is different on the Earth and on the Moon. A mass, on the other hand, is an inherent property of the matter and would be the same, no matter where you are. Mass is expressed in kilograms (kg), pounds (lb), or their derivatives. So, a 100-kg object is the same 100 kg everywhere, but it weighs 981 N on the Earth and 162 N on the Moon (where acceleration gravity is 1.62 m/s^2 on the Moon versus 9.81 m/s^2 on the Earth). Dividing the weight by the local gravitational acceleration gives the object's mass.

When addressing the aircraft weight-related topic, one needs to first define the following terms:

1. Standard empty weight—the standard weight of an empty aircraft, including the usable and unusable operating fluids (e.g., engine oil), except for fuel.
2. Basic empty weight—the standard empty weight of an aircraft plus the standard operational equipment.
3. Gross weight—the weight of a fully loaded aircraft.
4. Maximum Take-off Weight (MTOW)—the maximum airworthy weight allowed for the aircraft.
5. Takeoff weight—the weight at which the aircraft takes off.
6. Useful weight—the difference between the basic empty weight and takeoff weight.
7. Maximum landing weight—maximum allowable weight for landing.
8. Dry Operating Mass (DOM)—the mass of the aircraft ready to carry out certain types of operations; includes the crew and their baggage, catering-related equipment, and items such as food and beverages and removable passenger service equipment such as lavatory chemicals, but excluding the useful fuel and traffic load.
9. Traffic Load (TL)—the weight of passengers, baggage, cargo, and any non-revenue load.

10. Zero-Fuel Mass (ZFM)—the dry mass plus the traffic load excluding the useful fuel.
11. Maximum Zero-Fuel Mass (MZFM)—the maximum allowed airplane mass with no useful fuel; note that for an aircraft carrying most of the fuel in its wings, the weight distribution changes if the amount of fuel carried reduces to its minimum while traffic load correspondingly increases. The extra load in the fuselage may put excessive mechanical stress on the wing roots; therefore, it is paramount for the aircraft to meet the MZFM for structural safety reasons.
12. Maximum Take-off Mass (MTOM) is the maximum allowable mass as the takeoff is performed under normal conditions and is equivalent to the MTOW divided by the gravitational acceleration.
13. Maximum Landing Mass (MLM), also known as maximum structural landing mass, is the maximum allowable landing mass in normal conditions and is equivalent to the maximum landing weight divided by the gravitational acceleration.

Among the aforementioned masses, maximum traffic load is the most important, and calculations are to be performed to find this value and to identify the restricting mass that imposes it—equation (190)—where *DOM* is dry operating mass under the given conditions, *FUEL* is fuel required, column *MTOM* is maximum take-off mass, column *MLM* is maximum landing mass, and column *MZFM* is maximum zero-fuel mass. The maximum traffic load (*TL*) is the smallest of the three limits (methods) calculated in the equation (190), and the restriction that limits the maximum traffic load is the column heading in which the smallest number is calculated (Table 53).

$$Max(TL) = Min [MTOM - (DOM + FUEL), MLM - (DOM + FUEL), MZFM - DOM] \quad (190)$$

TABLE 53 Maximum take-off load (MTOL) obtained from other critical masses and mass limits.

Item	Column 1 – Method 1	Column 2 – Method 2	Column 3 – Method 3
Limits	<i>MTOM</i>	<i>MLM</i>	<i>MZFM</i>
<i>DOM</i>	<i>DOM</i>	<i>DOM</i>	<i>DOM</i>
<i>FUEL</i>	<i>FUEL</i>	<i>FUEL</i>	
<i>TL</i>	$TL_1 = MTOM - (DOM + FUEL)$	$TL_2 = MLM - (DOM + FUEL)$	$TL_3 = MZFM - DOM$
Maximum <i>TL</i>	<i>Minimum(TL₁, TL₂, TL₃)</i>		

For example, for a 2007 Piper Cherokee Archer III, given *MTOM* (2,550 lb), *DOM* (1,680 lb), *FUEL* (50 gal), *MLM* (2,250 lb), and *MZFM* (1,950 lb), the maximum traffic load ($Max(TL)$) is calculated as the minimum ($Min(TL_1, TL_2, TL_3)$) of the three load criteria presented in Table 53.

$$\begin{aligned} Max(TL) &= Min(2,550 - 1,680 - 50 \times 6, 2,250 - 1,680 - 50 \times 6, 1,950 - 1,680) \\ &= Min(570, 270, 270) = 270lb \end{aligned}$$

Note that in this scenario, the maximum take-off and landing weights are the same; additionally, the fuel density is assumed to be 6 lb/gal.

The weight-and-balance sheet is part of the airworthiness document requirements that you keep on board, provided by the manufacturer at the time of releasing the aircraft into service. It usually includes the aircraft manufacturer, type (private, commercial, military), category (normal, utility), and the aircraft model number and serial number. The unusable and useable amounts of oil are also indicated on the sheet as well as the total useful fuel capacity. For the Cessna 172M, the total useful fuel capacity is 52 gallons with a long-range tank, equally distributed between the left and right tanks, of which 48 gallons is the useful fuel. Nine quarts of oil are used, from which one quart is unusable. You will need to have a minimum of 5 gallons of fuel in each tank to operate the aircraft.

Assume that you decide to add some extra equipment to the aircraft such as a cool new Multi-Function Display (MFD), a set of the leather Carrera seats, a fancy radar, or paint the aircraft in bright pink to be seen from far away by the surrounding traffic. If additions increase the aircraft weight by 2 or more percent, the aircraft is to be reweighed. You may be able to measure the weight of each piece of new equipment and add it to the total weight (to be approved by the company), but for cases where measuring the individual components is not easy, such as a fresh coat of paint with sparkles to represent stars on a dark night, you are to actually weigh the aircraft based on recommended standards in the POH. That usually includes deflating the tires to a certain pressure and elevating each aircraft tire and securing it using tares on the scales, reading the measurements on each scale, adding them up, and subtracting the weight of the auxiliary parts to measure the aircraft weight, and voilà, you have your weight. You are even able to calculate the COG given the horizontal and vertical distances between each scale center-point and the axis datum from which the COG is to be measured (e.g., the engine firewall)—normally, the lateral spacing between the left and right tires and the datum axis crossing the datum reference point are equal, so you only need to consider the longitudinal distances from the reference point in order to calculate the moments and

the COG as a result. You are also then able to tell if you are within rearward and forward limits of the COG required to demonstrate the airworthiness of the aircraft. Generally speaking, an aircraft weighing over 12,500 lb is to be re-weighted every five years.

Depending on the category in which the aircraft is to be operated, namely normal or utility, you are to make sure the weight-and-balance calculations are done appropriately. Aircraft performance charts usually include diagrams which may be used to identify the category in which you are operating when knowing the weight and the COG data. For example, the MTOM for the utility category for the Cessna 172M is 2,000 lb, while for the normal category it is 2,300 lb. Note that in the utility category, you are allowed to perform the “5S” maneuvers—namely, Stall, Spin, Spiral, Steep turn, and Slow flight—that all require HASEL (Height, Area, Security, Engine, and Lookout) checks. On your check ride, you are to be within the utility category to be permitted to perform the 5S maneuvers.

If you and your FIC weigh 110 and 210 lb, respectively, assuming that you are sitting side by side, you may add up your total weights to 320 lb and use that data as the equivalent for the front passengers’ weight—moments can be combined assuming the arm distances are the same for the two front seats. If the weight for whatever reason increases, such as eating the Christmas turkey dinner or taking your pet beagle with you on the trip, the weight is to be adjusted.

The difference between the maximum allowed takeoff weights (either for the normal or utility categories) and the persons’ weights in addition to the luggage equals the maximum fuel you are allowed to carry on board. You may allocate the entire available weight to fuel. Under standard conditions, every gallon of AVGAS weighs about 6 lb. However, recall that fuel weight varies depending on the type of fuel used and variations in the fuel. Therefore, in most cases, especially for the passenger jets, this information is provided as part of the fuel data. Hence, for the Cessna 172M example, you can carry total fuel of 80 gal ($2,300 - 1,500 - 110 - 210 = 480$ lb, $480 / 6 = 80$ gal) or 30 gal ($2,000 - 1,500 - 110 - 210 = 180$ lb, $180 / 6 = 30$ gal) for normal and utility categories, respectively, assuming that the basic empty weight is 1,500 lb for both categories.

10.4 Fuel

Fuel planning is a vital part of the flight planning process; with excess fuel, the aircraft may fail to take off, and with less than the required amount

of fuel, the aircraft may not make it to the destination. Given the high cost of fuel, there is a lot of pressure to reduce fuel consumption and also carry as little as possible for each flight. This is one reason why large jets are prioritized for takeoffs and landings and you, in a small-sized aircraft, may be asked to hold short at the runway or to extend the downwind leg to let the “heavies” land first.

The calculations for fuel required to be carried on board are made based on the international agreement (e.g., European Aviation Safety Agency—EASA—fuel policy, Civil Air Patrol (CAP) 697) on fuel policy and must cover the groundwork, trip, and reserve (i.e., the Contingency, Alternate, Final reserve, and Extra—CAFE [223]). When calculating the fuel requirements, you are to consider the fuel required for start-up of the engine, pre-flight routine checks, taxiing, pre-takeoff routine checks, taking off, cruising, landing, and taxiing back to the hangar or a place to secure the aircraft, and fuel reserve. The total fuel on board before taking off is also known as the ramp fuel. You need to consider contingency (i.e., 5 percent of trip fuel or 5 min of holding flight) as well as additional or discretionary fuel reserve for icing conditions, requirements to make a precautionary landing, or taking a scenic detour. Be prepared to review or defend your calculations. Changes in weather, rerouting, and fuel consumption greater than expected result in requirements for contingency fuel in addition to the trip fuel. If the aircraft is not able to land at the destination field, if it must execute a missed approach, experiences a balked (go-around) landing, or needs to divert to another airport (i.e., alternate airport), alternate fuel is used. If there is no contingency allowance or alternate fuels available on board in this scenario, landing is to be performed with the minimum amount of fuel in the tank (final reserve fuel), which is the amount of fuel required for a turbine engine aircraft and piston engine aircraft to fly 30 min (at 1,500 ft over the airfield) and 45 min (at endurance speed), respectively during the day and 45 min during the night for the latter scenario. Extra fuel is carried on board under special circumstances. For example, G. F. Mock, the first female aviator who traveled around the world in her Cessna 180, carried more fuel on board so that her takeoff weight exceeded the MTOM by 500 lb, making her aircraft not airworthy. However, she had acquired special ferry permit from the Federal Aviation Agency (FAA)—the last letter “A” now stands for Administration [213].

The operations flight plan includes an itemized list of the fuel requirements indicated previously, excluding the ground handling fuel. To make sure the fuel data used in the fuel plan are correct, as part of fuel monitoring during the flight, fuel checks are performed and the

consumption is then compared with that in the fuel plan. The old-fashioned method to monitor the fuel usage is to use a diagram that presents the fuel versus the distance from the departure point to the destination point. The reserve and alternate fuels are identified on the diagram (the two of them are the fuel requirement for the aircraft to land) as well as the trip fuel. The fuel quantity then is recorded hourly and the predicted remaining fuel at the destination is extrapolated on the same diagram. In case this predicted remaining fuel goes below the alternate fuel requirement, meaning that the reserve fuel would have to be used when landing at the destination airport, the trip either needs to be shortened by picking an alternate airport or the aircraft is to return to the point of departure.

Nowadays, when available, flight computers, also known as Flight Management Systems (FMS), do the fuel monitoring calculations based on the same principle. If the calculated number goes below the fuel requirements, warnings are issued. It is also possible to monitor the required remaining fuel recorded in the fuel plan and compare it with that indicated by aircraft fuel gauges to look for deviations.

Figure 118 illustrates the aforementioned discussion. The filled circles in the figure are associated with two fuel consumption per distance scenarios: one in which the fuel requirements are met (black) and one in which they are not (red). In each case, the circles represent the fuel remaining records collected along the route showing the trend for fuel consumption. Extrapolating the line trend to the destination aerodrome in one case results in compliance with the fuel requirements (black line ending at the alternate fuel limit) and in another case does not (red line extending below the alternate fuel limit). Depending on the fuel consumption trend and how far the aircraft is from the departure aerodrome, the FIC makes the decision whether to continue the flight, return, or land at an alternate aerodrome.

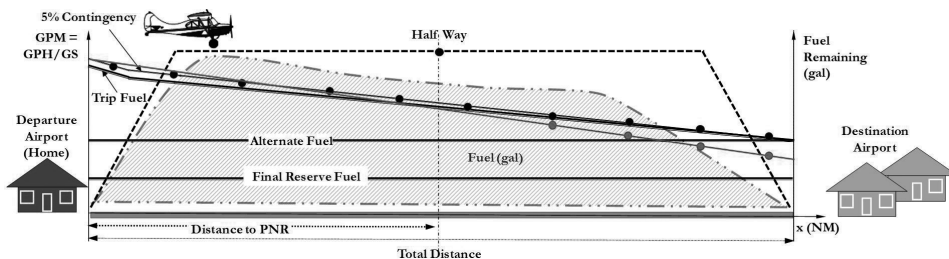


FIGURE 118 Altitude variation (dashed lines) and fuel consumption per distance versus the distance from the destination (dashed-dotted lines) (drawings created using Solid Edge CAD tool).

No matter which leg of your flight plan is to be completed, the amount of fuel required for a given time duration is only affected by the fuel mixture ratio (for a piston engine) and power settings. The cruise, ascent, and descent performance data, depending on the atmospheric conditions, are provided in the POH. It is useful to take a more conservative approach when making flight plans. It is customary to calculate for 30 min of extra fuel in cruise conditions for a medium weight aircraft for the day with VFR conditions and 45 min for the night VFR flight, based on ICAO regulations. IFR conditions require reserve fuel for 45 min of extra flight time after having reached the destination. While in a hold, this number is 45 min for the piston engine and 30 min for jet engine aircraft. Consult with the POH and your regional regulations.

Be careful and double check the calculation steps, as a small error in one step or a mistake in units may lead to incorrect results and an emergency situation during the flight. This is especially the case where two different units (e.g., metric versus imperial: liters versus gallons or kilometers versus miles) are used interchangeably, as not all countries follow the same standard of units, while some use both. For example, in Australia the metric system is used, while in the United States the imperial system is used.

One prudent approach to fuel calculations is to consider extra fuel for the pre-flight ground idle where you are to wait for the aircraft in front of you to take off before you do; for example, during preparation for a formation flight or when expecting busy hours at the airport.

Recall that unless you are to disembark the passengers off the plane by parachutes, the weight allowed for the fuel is the remaining one after the weight-and-balance calculations, where the weight of aircraft, crew, operating fluids, passengers, and luggage are deducted from the maximum allowable takeoff weight—your last unknown variable with enough constraints to be calculated. Recall that the weight of fuel is approximately 6 lb/gal for AVGAS and 6.8 lb/gal for jet fuels. Fuel weight is a function of the fuel density for a constant volume, and fuel density is a function of temperature and fuel type, varying from 6.55 lb/USG to 6.80 lb/USG.

Think of the Boeing 777 (e.g., 777-300ER) that burns 7.50 tons of fuel per hour during cruise at 892 km/hr (554 MPH) at 30,000 ft. Boeing was able to decrease the fuel consumption by 2 percent in 2016 by introducing an upgrade package resulting in better fuel efficiency of 7.35 tons of fuel per hour. 0.1 lb/USG variation means a difference of up to 1.4 tons for an aircraft fueled for a 6,000 NM trip. Given the reduction of fuel consumption to 1,784 liters, the total associated cost is about \$2,051. Note that fuel

consumption for the 777-300ER is 9,206 L/hr, which can be estimated to cost \$10,586.90 per hour (assuming \$1.15/L). That is about \$28.70 per seat per hour for a full plane (396 passengers).

Therefore, one fuel density number does not apply at all times, even for the same fuel type. Some places receive a daily report to inform the crew of the current fuel density so that they can use that for their weight-and-balance calculations. Some airlines measure fuel density for each single operation for the accurate calculations and therefore correct fuelling of tanks.

One method to quickly convert from pounds to gallons without a calculator is to first divide the weight by 10 and then add half of that number. So, if you have 4,000 lb of fuel, dividing by 10 gives 400. Add half of that (200) to get 600 gal. You are just multiplying 4,000 by 0.15. Taking the calculator and dividing 4,000 by 6.55 lb/USG to 6.80 lb/USG gives 588 gal to 611 gal, averaging the results, 599 gal is obtained, which is close to the estimated value of 600 gal.

👉 Rule of Thumb 👈

$$\text{AVGAS (US gal)} = (\text{AVGAS (lb)}/10) \times 1.5$$

10.4.1 Fuel Consumption per Distance Traveled

The fuel consumption rate expressed in fuel volume or weight per unit time, such as lb/hr or gal/hr (GPH), helps to determine how long an aircraft can stay in the air. This can be used in trip planning if the distance and the ground speed are known—the time in the air can be obtained by dividing the trip distance by the ground speed. The fuel consumption over time is also needed to estimate how long an aircraft can stay in a hold; for example, where one is forced to remain at the same location on ATC orders while waiting for landing clearance.

Another useful way to describe the fuel efficiency of an aircraft is by the rate of fuel use per unit distance traveled. This quantity is a function of the aircraft's engine power setting, ground speed, weight, elevation, and outside air temperature. When planning a trip in a turbine-powered aircraft, you have a choice of altitude and power setting. The fuel consumption rate over time is reduced at higher altitudes and with lower temperatures. Higher cruise speeds require higher power settings and thus higher fuel consumption over time but allow one to travel further over the same time. Thus, it is not obvious which setting will give you the most economical cruise. To help a pilot plan a trip, some POHs will include charts describing how the fuel consumption over unit length varies.

For example, for the Pilatus PC12, a ten-passenger, single-engine, turboprop aircraft, the POH includes charts of specific air range. Charts for all combinations of three aircraft weights (7,000, 8,000, and 9,000 lb) and three temperatures (ISA-20C, ISA, ISA+20C) are provided. On each chart, you can find a point corresponding to a combination of pressure altitude (ft) and torque (psi). Torque is how the engine power output is set in a turboprop engine; it equals the rotational force applied by the propeller as it pushes through the air. Maximum safe torque is about 37 psi for PC12. This altitude-torque grid is overlaid on top of the rectangular grid of a specific air range (NM/lb) along the vertical and TAS (kt) along the horizontal axes. Thus, one can find a point on the altitude-torque grid and then find the corresponding point on the air range-TAS grid [224].

For example, consider planning a trip from CYQH (Watson Lake, YK) to CYFB (Iqaluit, NU). The distance is 1,660 NM and temperature is at ISA standard value. The aircraft weight with maximum fuel and a single pilot is 8,300 lb. A maximum of 2,692 lb of usable Jet-A fuel can be carried on board. As per fuel calculations described earlier, you can subtract from this amount fuel to taxi and climb (180 lb), and reserve at the destination (300 lb). A separate chart is available in the PC12 POH giving the fuel and distance to climb. The latter distance to climb to 30,000 ft is found to be 80 NM.

From the previous calculations, you get the fuel for cruise of 2,212 lb and distance of 1,580 NM. Dividing the distance by the fuel quantity gives the minimum specific air range value (0.71 NM/lb) required to reach the destination after consuming the previous fuel quantity. The previous calculation disregards the fact that during descent the fuel consumption will be reduced; this produces a more conservative estimate. Knowing the minimum specific air range, you can use the charts to determine the best trade-off between faster cruise speed and fuel economy for this flight that will leave you with sufficient fuel at the destination while also getting you there as soon as possible.

For the trip in the example, using the 9,000 lb (ISA) chart, at 30,000 ft and 20 psi torque setting, a specific air range of 0.82 NM/lb and 234 kt TAS is predicted. The trip duration, assuming ground speed equal to TAS, is thus predicted at 6.75 hrs. Dividing the trip distance by the specific air range predicts use of 1,904 lb of fuel, leaving 308 lb in addition to the 300 lb reserve. This additional reserve may be required if strong headwinds are encountered on the flight. For example, a 30 kt headwind would increase the trip duration to 7.75 hrs and can be estimated to require 2,184 lb of

fuel, nearly equal the allotted quantity of 2,212 lb just leaving the aircraft with the 328 lb reserve at the destination. A 56 kt headwind will consume just about all of the 300 lb reserve at the destination and will require a diversion to an en route aerodrome for refueling.

The author can report that after the previous flight was carried out (in a simulation), the actual total fuel consumed was 2,008 lb, and the average specific air range figure was 0.82 NM/lb, with 724 lb of fuel left at the destination. On this flight with real-world weather of June 23, 2018, departing at 10 am local time, some moderate tailwinds (~20 kt) were encountered, resulting in better performance. The simulation was carried out in Prepar3d® V4.2 Simulation Software by Lockheed Martin.

A modern FMS or a built-in GPS unit can provide instantaneous readout of the fuel consumption per unit distance traveled. These units can interface with the onboard electronics to monitor the fuel flow rate and the aircraft's ground speed to provide this fuel economy reading. Observing this number, you can then adjust the engine power setting to achieve the fuel economy desired.

Even if your aircraft does not have such an FMS or GPS unit, you can quickly gauge your fuel economy by dividing your estimated ground speed by the fuel flow rate. For example, consider the Cessna 172M performance charts provided in Section 8.3. For a 6,000 ft altitude and 2,400 RPM, Figure 74 predicts a TAS of 106 kt and Figure 72 predicts a fuel consumption rate of 7 GPH. Calculating the ratio gives 15.1 NM/gal. Dividing the distance remaining to the destination by this quantity will predict the volume of fuel that will be consumed.

10.5 Communication

Air-to-ground communications in flight are normally carried out by radio transmission. These transmissions employ Electro-Magnetic (EM) waves. An antenna is used on the transmission end to convert the associated electrical current into an EM wave, which is then picked up by the receiving antenna, where the wave induces electric current from which the useful signal is extracted. The EM waves can travel over long distances through the air. But any matter with density greater than air will normally interfere with or block the transmission. The interaction of the electromagnetic waves with the environment depends on the wave's frequency. Different communication protocols use different frequency ranges.

Depending on how the EM waves propagate through space, they are described as either ground, sky, or space waves. Ground waves propagate by bouncing between the ground and the ionosphere and can travel over long distances. They are the lowest frequency radio waves, up to one MHz. Sky waves range in frequency from one MHz to 30 MHz, and their propagation path is curved back to the Earth when they pass through the ionosphere. They can travel long distances, but their reliability is limited to only 80 to 90 percent due to the unpredictable conditions in the ionosphere affecting the propagation; thus, they are not used for navigation but only for long-range voice communication via High Frequency (HF) radio. This technology is now being superseded by satellite-based communication.

Space waves pass through the ionosphere into space and are at frequencies from 15 MHz up to several GHz. These radio frequencies are used for navigational purposes, as they are not interfered with by the reflections from the ionosphere. However, at these high frequencies, the radio signals are reflected from or blocked by hard solid objects like mountains or buildings. These EM frequencies are employed for transmission by the GPS satellites, ILS, and VOR stations. This explains the “Very high” in the “Very high frequency Omnidirectional Range” that VOR stands for. VORs operate in a 108 MHz to 118 MHz frequency range.

Space waves (by their nature) can be only transmitted while there is the LOS between the signal sender and receiver. Whether you are within the LOS depends on the height of both sender’s and receiver’s antennas. Equation (191) predicts the LOS distance d for the case where the transmitter is ground-based and a receiver is at height H , R is the radius of the Earth (6,378 km in the Equator), and k is the dimensionless refractive index of the medium ($k_{air} = 4/3$). Note that the input dimensions determine the output dimensions. In reality, the radio waves do not travel in a perfectly straight line; depending on their environment, the k -factor (the refractive index), which is the ratio of the speed of light in a vacuum to that in the environment (air), they bend to some degree. This makes their service range usually greater by approximately 15 percent compared to that in the vacuum. For a normal range of heights, the magnitude of the antenna height squared (H^2) term in the equation should be much smaller than the antenna height multiplied by the radius of the Earth ($2RH$), that is, $H^2 \ll 2RH$, and so this term may be ignored in the equation (191). If this approximation is assumed, equations (192) and (193) result, for both imperial and metric systems. Note that in order to obtain these formulae, the dimensions of the right side of the equations must be expressed in feet—equation (192)—or

meters—equation(193)—and the answer in nautical miles or kilometers is obtained at the next step by choosing the appropriate conversion factors. If the LOS between two or more antennas (stations) with different heights is of interest, equation (194) is to be used. This equation is the summation of the distances for each individual antenna presented by equations (192) and (193)—where d is the LOS in miles or kilometers and h_i is the height associated with antenna “ i ” in feet or meters.

$$d = k^{0.5} \sqrt{(R + H)^2 - R^2} = k^{0.5} \sqrt{H^2 + 2RH} \quad (191)$$

$$d(NM) = 1.064k^{0.5} \sqrt{H(ft)} \quad (192)$$

$$d(km) = 3.570k^{0.5} \sqrt{H(m)} \quad (193)$$

$$d(NM, km) = (1.064, 3.570) \sum_{i=1}^n \sqrt{k_i H_i(ft, m)} \quad (194)$$

10.6 Wind

Wind is an important factor when making a flight plan. Wind is a vector and so is identified by magnitude and direction. The wind vector varies with elevation—meaning that you should not expect the same wind vector at an elevated altitude as on the ground. There are also days with calm winds but bumpy flights on which you may feel stomach upset; therefore, the stomach upset is not fully dependent upon the turbulence conditions either. During the routine pre-flight checks, you are to park the aircraft facing into the wind so that the engine is kept cool by the airflow that enters at the upper engine cowling, passes by the cylinders, and exits the bottom cowling in a piston engine. The only disadvantage of such arrangement is that if the ground is sand-covered, the rotation of the propeller may lift the pieces of sand and damage the engine components or propeller since each small mass has a momentum capable of creating dents on the propeller, fuselage, or front window. The small particles may also find their way inside the cowling if the carburetor heat is on, leading to unfiltered air entering the engine cowling. This scenario is more probable during the cold winter months and in remote locations where aerodrome surfaces are not in good condition. You may want to decrease the engine RPM to reduce some of the adverse effects this may lead to.

Wind is also a key factor in selection of the runway. It is desirable to land and take off into the wind to reduce the ground speed in both cases; the minimum required runway length increases with tailwind. If you choose to take off or land with the tailwind, make sure you are prepared and have

completed all necessary calculations and have good reasons to do it. Wind socks installed on aerodromes may be used to roughly assess instantaneous wind magnitude and direction as you are lining up for takeoff or just coming in for a landing. The tip of the sock shows the direction of the wind, and you can estimate the wind magnitude by observing the fraction of the sock length which is inflated by air. For a regular orange-white striped wind sock—imagine the hat from *The Cat in the Hat* [225,226]—each additional stripe is associated with an increase of 3 kt of wind magnitude, with 15 kt maximum at the full expansion to the 5th stripe.

On aeronautical and marine weather charts, wind information may be displayed using special symbols called *wind barbs* (Figure 119). The wind barb symbol consists of a line with a set of “feathers” on one end and a small circle or arrow at the other end (which is omitted on some charts). On weather charts, the north would normally be at the top. The barb represents an arrow indicating the wind direction; the small circle (when present) is the end of the arrow pointing to where the wind is blowing toward and the “feathers” are at the tail of the arrow. The pattern of the “feathers” indicates the wind magnitude. A small feather (short line) is associated with 5 kt, a longer line with 10 kt, and a filled triangle with 50 kt wind speed. For example, an easterly wind with 75 kt speed (represented in METAR as 09075KT) would be shown as a horizontal line with a circle on the left end and, on the opposite end, a filled triangle (50 kt), two long feathers (20 kt), and one short feather (5 kt).

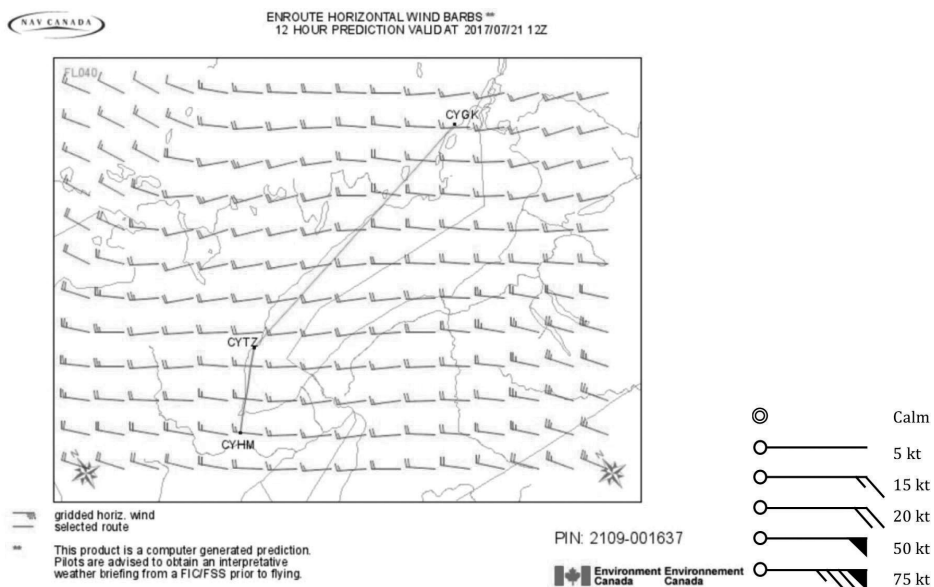


FIGURE 119 (a) En route horizontal wind barbs (left), and (b) Wind barb symbols (right).

(2×10 kt), and one short feather (5 kt), to give $50 + 20 + 5 = 75$ kt. A small double circle with no line indicates calm wind conditions. Figure 119 also shows the en route horizontal wind barb diagram for the flight plan in Section 10.8 “Examples” in this chapter. In this diagram, the wind at CYGK is blowing from the northwest with a magnitude of 15 kt.

Meteorological organizations usually publish the weather data and make it accessible to interested groups such as sailors, pilots, engineers (e.g., civil and energy engineers who design, operate, and maintain wind turbines, all of whom are interested in the integrity of structures exposed to wind), and those in charge of airports, wind turbine manufacturers, and wind farm sites. The data may be published in different forms, and may include information such as wind power class, wind power density, and its speed at 10 m (33 ft) and 50 m (164 ft) heights above the ground in addition to its direction and magnitude at other elevations, including low-altitude surface winds. The data may also present the time of the day when the conditions are applicable. The wind data files would also be made available for extraction and further manipulation to study the wind’s socioeconomic and ecological impacts. Among databases, the one provided by the National Renewable Energy Laboratory (NREL) of the Geospatial Data Science group is among the most extensive and organized [227].

Wind data records are available for many United States states for over-the-ground 50-m wind turbine hub heights, and the offshore data for 90-m hub heights are also assessed for their energy level on an annual basis. Wind data records in Western and Eastern regions in addition to the Pacific, Hawaii, the Atlantic, the Gulf, and the United States Virgin Islands have been organized and mapped. Maps pertaining to a multitude of wind-related effects are available. Among the items that have been mapped are: (a) Areas of critical environmental concerns (e.g., critical habitats for the southwest fauna and flora); (b) The corridor at which the price per megawatt-hour of electricity may be harvested within a certain range; (c) Since most of the wind harvest is to be done on land, the status of lands in terms of private ownership, restricted areas, brownfields, and national parks and trails in addition to the tribal lands; (d) Military areas and their usage, with data on special use airspace in high and low altitudes, boundaries, training routes (e.g., requiring instrument guidance, speed regulated, or visual conformation); (e) Areas with special topography such as hill-shade, land cover, and slope under 20 percent along with hydrology data (e.g., lakes and rivers); and (f) Potential wind capacity at different hub heights—80 m (263 ft) to 140 m (459 ft) and distributed historical wind resource data

such as pressure, temperature, speed, and direction. Some of the previous information may be useful when planning a flight (both for aircraft and UAV devices) in order to avoid the tactical areas under military operations and populated bird habitats.

Although as a pilot you may not need to collect and interpret in detail the wind data, familiarity with the resources available to obtain the wind data and to have a basic understanding of these data are relevant. This is particularly important for flying in the hurricane-prone seasons and locations. Figure 120 presents wind speed (both in m/s and mph) versus the wind power density (wind power per area) for a 50-m hub. It is seen that power density increases sharply versus the wind speed for speeds higher than 6 m/s. Figure 121 shows the wind power density as a function of the wind speed in m/s. These profiles are almost identical for the 10-m (33-ft) and 50-m (164-ft) hubs. Wind class (a standard method to categorize the average of the measured wind speed, gust, and turbulence for an area) versus the wind speed is also identified in the diagrams, with a strip that follows a step function following the wind speed magnitude closely.

The wind direction in METAR and TAF is given as true heading, which you will need to convert to the magnetic direction. This is required so that you can compare the wind direction to the runway heading, which is also magnetic. POH often includes a plot to aid in calculation of the crosswind and headwind components as shown in Figure 122. Using this figure, one locates the arc matching the wind magnitude and then traces this arc to

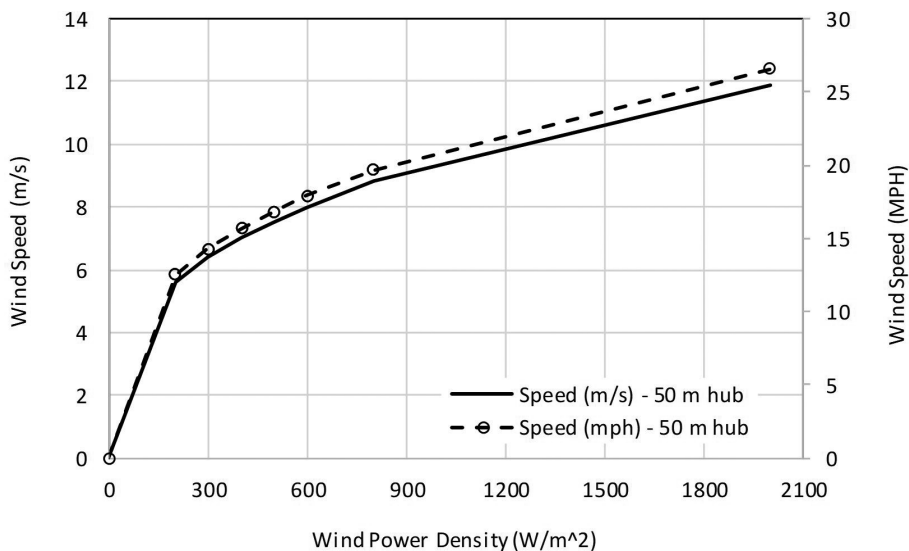


FIGURE 120 Wind speed versus the wind power density.

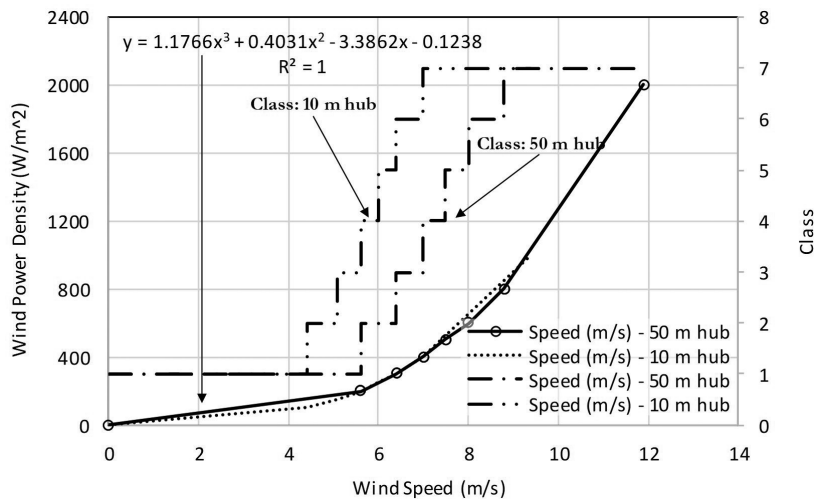


FIGURE 121 Wind power density and class versus the wind speed.

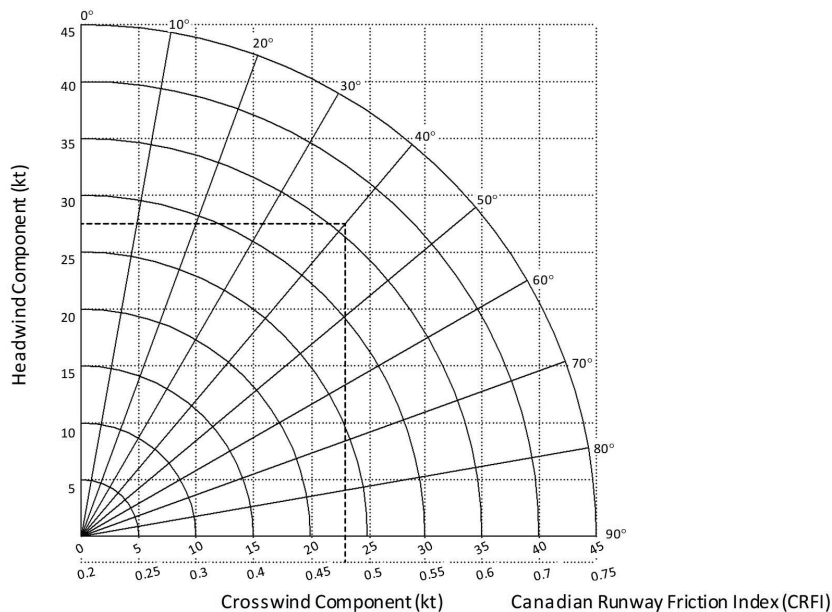


FIGURE 122 Crosswind and headwind components along with the Canadian Runway Friction Index (CRFI) (drawings created using Solid Edge CAD tool).

the point of intersection with the radial line of the appropriate angle. From that point, follow the line vertically straight down to find the crosswind component value along the horizontal axis and follow the line horizontally from the same point to the left to intersect the vertical axis at the headwind component value. To use the chart for relative angles greater than 90°,

meaning for a tailwind, subtract the relative angle from 180° and use the resultant angle on the chart. Instead of using the chart, one can also make the calculations directly. To calculate the crosswind component, multiply the wind magnitude by the sine of its angle relative to your intended course, which would be the runway heading for takeoffs and landings. To calculate the headwind component, multiply the cosine of the previous angle by the wind magnitude.

Consider the scenario where the runway heading is 90° (runway 09) and the wind is reported as coming from 120° at 36 kt. As you recall, winds are reported as true headings. The local magnetic variation is 10°W . As the variation is westerly, you need to add it to the reported true wind direction to get the magnetic direction of $120^\circ + 10^\circ = 130^\circ$. The relative angle to the runway heading is 40° (i.e., $130^\circ - 90^\circ = 40^\circ$). Using the chart in Figure 122, you would locate the arc matching the 36 kt and trace it to the 40° radial to find the crosswind of about 23 kt and the headwind of about 28 kt. Performing the calculations for the same conditions, you get a crosswind of $36 \times \sin(40^\circ) = 23.1$ kt and a headwind of $36 \times \cos(40^\circ) = 27.6$ kt.

After calculating the crosswind component, the next step is to make sure the aircraft is airworthy—meaning that the crosswind component needs to be assessed relative to the maximum demonstrated crosswind component specified in the POH. This is the maximum crosswind speed at which landings and takeoffs have been carried out successfully by the plane manufacturer's test pilots and which they expect pilots with ordinary flight skills to be able to complete safely. The POH does not claim this to be the maximum allowable value. For the Cessna 172M, the maximum demonstrated crosswind component is 15 kt; however, there are cases in which you may need to fly in stronger crosswinds given only a single runway at a number of aerodromes if you have no other safe options available. For example, G. F. Mock had to land in Bermuda in the Atlantic in a 40 kt crosswind during her solo flight around the world in the Cessna 180 taildragger. Whatever decision you make, you are to feel comfortable with it both technically and emotionally.

The wind data are also used when calculating the takeoff and landing distances. Performance charts usually provide tables or graphs from which you can obtain the adjustments to these distances as a function of the headwind or tailwind. For example, for the Cessna 172M, for every 9 kt headwind, the takeoff and landing distances are decreased by 10 percent, while for every 2 kt tailwind up to 10 kt, the takeoff and landing distances are increased by 10 percent. The other use of the wind information, especially

the crosswind component, is to calibrate the Canadian Runway Friction Index (CRFI). The CRFI is a standard measurement, reported for each runway during or after the precipitation occurring, that classifies runway conditions such as snow, ice, slush, and so on by assigning recommended landing distances—a scalar between zero and one—with zero corresponding to no braking capability on a solid ice surface and one corresponding to the maximum braking capability on a dry and clear runway. The maximum crosswind component to take off and land is affected on a runway with a CRFI of less than one. The *Mu* number in the United States is the equivalent of the braking friction that has been superseded by the numerical Runway Condition Codes (RwyCC) that are based on the Runway Condition Assessment Matrix (RCAM). RwyCC may be calibrated for each third of the runway and is consistent all over the runway if the surface contaminant is the same throughout. Crosswind and headwind components along with the CRFI are presented in Figure 122.

10.7 Cross-Country Flight

This section discusses making VFR flight plans. Planning a cross-country flight can take anywhere from under a couple of hours to a few days, depending on the flight plan complexity, distance, weather, and your desired level of preparation. As stated before, there are numerous tools and applications available online and on smart devices (tablets and cellphones) that have made this activity simpler and more efficient. However, the author recommends you do not treat these tools and applications—tempting though this may be—as black boxes. You should learn the old-fashioned way of extracting the data required to perform your calculations—perfectionism is a good approach to follow in these situations. After you have completed your flight planning using the traditional methods, such as using flight computers (e.g., E6B), you may graduate to use the digital flight computers. Keep in mind that only some of these digital computers are FAA-approved, allowing you to use them for your ground test. The next step would be to read some reviews on the flight plan application programs available on the market and decide which one is right for you. Some of them require a subscription plan (e.g., ForeFlight and Navigraph) and some are available free-of-charge (e.g., SkyVector® and FltPlan.com). With these applications, you can learn about the terrain features, Flight Information Services (FIS), FBOs, and voluminous information on the aerodromes—almost everything you can find in a CFS or an A/FD.

The following steps are involved in making a VFR cross-country plan:

1. Collect weather data using METAR, TAF, and other resources available. This includes weather, cloud conditions and ceilings, turbulence, ice and freezing rain, temperature and pressure at elevations, wind conditions such as magnitude and direction at elevations, and headwind-tailwind data along the route.
2. Collect information about the departure and destination aerodromes, such as pattern (circuit) altitude and procedures, frequencies (e.g., terminal, tower, and ground), taxiways, FBOs, fuel availability along with its type, and suitable parking or hangar locations. Also collect Flight Information Region (FIR) data to become familiar with the activities in the vicinity of the flight route.
3. Collect flight route data such as elevations, geographical features, landmarks, restricted areas, via points, aerodromes that may be used during the flight, variations, and nearest towns.
4. Assemble all the information on the map, draw the LOP by connecting the departure and destination points along with the via points to obtain the true bearings, identify the half-way point as well as the one-quarter and three-quarter points, identifying the 10-NM points starting from the destination.
5. Collect all the frequencies needed to make the radio calls. Be vigilant in making the radio calls—make all the necessary radio calls at the mandatory and nonmandatory position report locations—recall the filled and non-filled triangles on a VNC chart that are associated with the former and latter scenarios.
6. A flight following may be requested if the tower's work schedule permits it, contingent on the transponder and radios being operative, so make sure they are working well before taking off. Note that a transponder is required for distances over 25 NM. Make sure the radios and antennas work properly and their protective cover is intact. Become familiar with the No-Radio (NORDO) procedures along the routes and over the aerodromes in case of communication equipment malfunction.
7. Be familiar with the flight routes and review the plan several times along with the frequency data. Identify the club whose aircraft is being rented, the hangar where the aircraft is being kept, and the flight community that may provide some reliable and thoughtful assistance if required. If

you are a student working toward your Private Pilot License (PPL), you can expect to be supported by your flight school.

8. Make sure a paper copy of the CFS (Canada Flight Supplement) or A/FD (American version) is available on board to review in case the electronic devices do not function as expected for whatever reason.
9. Process the collected information by:
 - (a) converting the true bearings to magnetic bearings (include the variations); to help you remember, the Sun rises *FROM* the East (subtract *FROM*) and sets *TO* the West (add *TO*).
 - (b) incorporating the wind data into the magnetic bearing to obtain the magnetic heading based on the TAS of the choice from the aircraft performance chart and data—it is then possible to calculate the compass heading from the magnetic heading by calibrating this data using the compass card.
 - (c) using the wind data to obtain the crosswind and headwind components.
 - (d) calculating the takeoff and landing distances—ground roll and distance to clear a 50-ft obstacle—given the weight, pressure altitude, temperature, and headwind-tailwind data.
 - (e) calculating the cruise data, such as brake horsepower, TAS, and rate of fuel usage as a function of the temperature, pressure altitude, and RPM using the performance charts.
 - (f) calculating the climb performance numbers, such as distance to climb, time to climb, and rate of fuel usage to climb as a function of the pressure altitude and temperature—the rate of climb and climb speed using the climb performance chart data can also be predicted.
 - (g) calculating the TAS, IAS, and CAS speeds, and the ground speed from the wind data, the total leg time as well as the fuel required can be calculated using the data and performance chart information for the pressure altitude and temperature.
 - (h) calculating the weight-and-balance, considering the weight of the passengers and belongings. If not certain as to what weight to include in the calculations, the following assumptions may be made: 200 lb (90.7 kg) without heavy clothing and an additional 6 lb (2.7 kg) for winter clothing, 165 lb (74.8 kg) for females without heavy clothing and additional 6 lb (2.7 kg) for winter clothing, 75 lb (34 kg)

for children over 2 years and under 12 years old, and 30 lb (13.6 kg) for infants under the age of 2. Remember that utility versus the normal categories are to be taken into consideration when deciding on the total weight allowed for the journey.

- (i) calculating the fuel required for each leg as well as the total fuel required to complete the flight plan that includes the reserve fuel (30–45 min) and other additional fuel requirements (e.g., 20 percent extra fuel) while satisfying the weight-and-balance requirements.
- (j) entering all the data on the flight planning sheet, including the via points, and identifying the distances between the points and the remaining distances to the destination point. This is used to calculate the ground speeds when approaching the via points and immensely helps to estimate the time to the destination point. The time to the destination is to be checked periodically to make sure there is sufficient fuel to complete the journey; if for whatever reason, such as changing wind direction and magnitude, which happens often, more fuel than planned is consumed, the flight plan is to be modified and the aircraft is to be landed in the safest location or as soon as practicably possible to obtain the required fuel.
- (k) opening a flight plan with the CFS or FSS or a flight itinerary with the dispatch or a responsible person at the flying club; the purpose is to enable the search-and-rescue team to provide help if required (60 min after the reported flight-closing time); feel free to ask the FSS personnel for any information related to the flight route (e.g., weather data, occurrence of flight formations, military maneuvers, or parachute activities over the nearby fields).
- (l) making sure that the ELT is on *auto* and check that it works by putting it into the *on* mode and sending the signal at the beginning of every hour for the first 5 min on the Ultra High Frequency (UHF) or VHF frequencies (121.5 Hz); recall to switch it to the *auto* mode afterward—the return is to be reported to the FSS or the responsible person within one hour after the flight plan is closed or as soon as applicable within the time limit identified in the flight itinerary.

10.7.1 Diversion

A *diversion* can be: (a) the act of redirecting something from its original path, (b) the act of transferring some resource from one place to another, (c) doing something that takes one's mind away from another tedious activity or grave concern, and (d) something meant to take one's attention

away from something else of greater importance [228]. All of these may happen at any stage of flight planning, executing, and reporting—either pleasantly invited or exasperatingly uninvited and intruding. A diverted flight is one that changes from its original destination to a new one, usually on a temporary basis. The leg of the flight that consists of the route followed from the alternate destination to the next waypoint in the original plan is known as the recovery leg.

Recall the wind information Air Traffic Service (ATS) provides is true north while the headings of the runways and the aircraft headings are magnetic. The true wind direction is to be converted to the magnetic wind direction by incorporating the variation when you are doing short as well as long-distance cross-country navigation. When executing a diversion, follow this checklist:

1. Set the en route frequency on the COM radio and report your diversion intention before starting it.
2. Establish altitude—1,500-ft AGL is usually a safe altitude in almost all conditions as you are to fly 1,000-ft AGL over the populated areas; nevertheless, check the en route map for the highest obstacle's elevation—the flight is to be made 500 ft above the highest obstacle en route. There are exceptions when you are to fly over antennas exceeding this height or restricted areas that impose their own regulations.
3. Set cruise power—for the Cessna 172 or a similar fixed-pitch prop aircraft—at about 2,200 rpm. Note that you may decide to use a 20-degree flap setting and lower the speed to about 70 kt to complete your diversion—this may also prepare you for the precautionary landing that follows.
4. Set carburetor to heated position—full out for the aircraft with this feature available or cowl flap closed for other models—so that the heating is provided in cold or icing conditions. Keep in mind that the icing may occur even at temperature ranges of up to 21 °C (70°F) with high humidity levels in some carburetor designs.
5. Set mixture to rich to get as much performance as possible, even if some fuel is wasted.
6. Enter a race pattern over the Set Heading Point (SHP) to give yourself some time in order to identify the bearing to the diversion point; with the SHP on your left, follow a left-pattern (circuit) along any convenient

ground-reference direction such as a road and approach the SHP as if you were approaching the runway to land. SHP is the point at which you set your heading so that you follow the planned course to the destination.

7. Set heading—by making sure that the heading on the compass matches that of the directional gyro indicator; this is to be adjusted every 15 min (in 3-degree increments).
8. Set the course—this is what you have calculated based on the true track data, converting it to the magnetic track by adding or subtracting the variation, incorporating wind data, and estimating distance and time.
9. Identify the opening angle—the angle of the bearing from a selected set point you are approaching and the diversion destination, not to be mistaken with the opening-closing angles to correct the drift—before getting to the point where your diversion is to start, review the map, identify the main roads, geographical features, or landmarks as much as possible, and try to align yourself with an easily identifiable feature so that you are as close to the track you calculated as possible.
10. Make a drift correction if you have deviated from the desired course for whatever reason; either choose the double-error method before half-distance completion, opening-closing method after half-distance completion, or fly directly to the destination in case you know the bearing. This will be discussed further in Chapter 12 (Special Scenarios).
11. Record time—right over the starting point, record the time so that you may use that to predict the time of arrival at the diversion destination as well as the halfway point.
12. Advise FSS if there is a change in plan. Note that when you get to the halfway point, you are to review the calculated time again and see if it is different from the original prediction; if the two times are different, you are to recalculate the arrival time and fuel required, and inform the FSS of the changes that may also affect the available fuel.
13. Communicate your decisions with the FIC adhering to the sterile cockpit rules by the FAA presented in the following section.

Remember that doing spontaneous diversions while in the air with no prior preparation is challenging, especially if your only tools are your fingertips and your basic mathematics. Here is a technique that may help you do this more efficiently. Identify the start point and destination by circling the two and connect a line in between, measure the line length by

using your calibrated finger width, and estimate the heading by identifying the direction of the line with respect to the north or horizontal line (i.e., you do: circle-circle-line-distance-heading). Now add the westerly variation (or deduct the easterly variation) to obtain the magnetic track. Depending on the direction of the wind (e.g., 0° , 30° , 60° , 90°), adopt the related portion of the wind magnitude (e.g., 1, $2/3$, $1/3$, and 0, respectively), x . Crab into the wind by the calculated magnitude (x). Your distance remains the same no matter what the direction of the wind is. The time will increase or decrease depending on the direction of the wind, headwind, or tailwind status (180° , 90° , 60° , 30° , and 0°) to the distance portion of ($1/2$, 0, $1/3$, $2/3$, and 1, respectively). This approximation is based on an assumed airspeed of 100 kt. Hence, it means if you are flying to the north (i.e., true course is zero degrees), and you are facing a headwind (0° northerly wind), the flight time is longer (doubled) than if you face a tailwind (180° southerly wind), depending on the wind magnitude.

10.7.2 Flight Operation—Very Good Practices and Regulations

Here are some suggestions to ease the cross-country flight:

1. Review the map beforehand—if practicable.
2. Use the same calibrated (index) finger at all times for consistency.
3. Give yourself one to two miles for the final approach and treat the start point as the start of the runway with calculated heading as the runway heading.
4. If you happen to be off-course by half a mile or so, adjust as required.
5. Report your location to the traffic as often as possible so you do not end up surprising others—recall the poor airmanship of some pilots, their aircraft appearing below or above your aircraft with no prior notice through radio communications.
6. Do a number of practice diversion math exercises before flying, as painful as they may be.

Sterile cockpit rules regulation by the FAA requires the pilots to refrain from non-essential activities during critical portions of flight operations. The regulation started in 1981 due to a number of accidents that were caused by flight crew being distracted from their duties by engaging in non-essential activities and conversations (e.g., idle chatter) at inappropriate times. The importance of this regulation—and diligently adhering to it in all conditions—cannot be overemphasized.

Section 121.542, Part 121 of Title 14 Code of Federal Regulations (Amdt. 121-369, Eff. 4/14/2014) on operating requirements for domestic, flag, and supplemental operations states that, during the critical phases of flight, crewmembers must not be asked to nor should they themselves perform any duties that are unrelated to the safe operation of the aircraft. Examples of activities unrelated to safety include ordering galley supplies, confirming passenger connections, announcements related to promoting the air carrier or sights of interest, or doing paperwork such as filling timesheets. Any actions resulting in distraction of the crewmembers, which may affect their performance, or affect proper conduct of tasks—during the critical phases of flight—are strictly prohibited. This includes eating meals, nonessential communications among the crewmembers or on the radio, or reading non-related publications. Use of personal wireless communication devices such as cellphones and laptops is prohibited during the flight unless it is directly related to the flight or is related to the flight safety in accordance to the air carrier procedures. Critical phases of flight include ground operations, taxi, takeoff, landing, and flight operations performed below 10,000 ft, except when in cruise flight [229].

Note that the FAA has organized user-friendly databases under Regulatory and Guidance Library, which includes all the Advisory Circulars, CARs/CAMs/Aero-Bulletins, Codes of Federal Regulations (Title 14), Equivalent Levels of Safety, Exemptions, Orders/Notices, Parts Manufacturer Approvals, Policies, Regulatory Basis Tools, Special Airworthiness Information Bulletins, Special Conditions, Supplemental Type Certificates, Technical Standard Orders and Indices, and Type Certificate Data Sheets [230]. Advisory Circulars (AC) are intended to inform the public on non-regulatory information and may be ordered from government printing offices or departments of transportation—the public is not obligated to follow these circulars unless they are referenced within the regulations. The information on subject matters is recorded under different numbering systems (e.g., airmen, air traffic control and general operations, and airspace related information can be found under numbers 60, 90, and 70, respectively).

10.8 Examples

10.8.1 Case Study – Line of Sight (LOS) – Single Station

A typical mast of a medium-wave antenna is 1,000 ft. Assuming this value for the height of the antenna, calculate the LOS for:

(a) a vacuum, and (b) air.

For clear vacuum conditions, the LOS is 38.8 mi (62.4 km) and can be obtained by evaluating:

$$\begin{aligned} LOS_{vacuum} &= \sqrt{\left(0.19 + \frac{7,926}{2}\right)^2 - \left(\frac{7,926}{2}\right)^2} \\ &= \sqrt{0.19^2 + 0.19 \times 7,926} \approx \sqrt{0.19 \times 7,926} = 38.8 \text{ mi} \end{aligned}$$

For clear air conditions, the LOS is 44.8 mi (72.1 km) and can be obtained using the following equation. See Figure 123 for the related diagram.

$$LOS_{air} = \left(\frac{4}{3}\right)^{0.5} LOS_{vacuum} = \left(\frac{4}{3}\right)^{0.5} \times 38.8 = 44.8 \text{ mi}$$

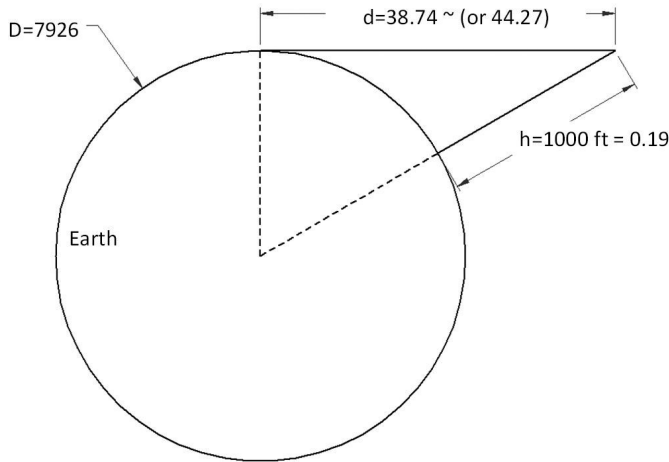


FIGURE 123 Line of sight (LOS) (dimensions in miles; not to scale)
(drawings created using Solid Edge CAD tool).

10.8.2 Case Study – Line of Sight (LOS) – Multistation

There are two stations which broadcast medium frequency radio waves. They are located in the area surrounding the city of Kingston (Canada), in the proximity of the towns Selby (874 ft MSL, 415 ft AGL elevation) and Odessa (745 MSL, 295 AGL elevation). Calculate:

(a) LOS for each antenna, and (b) combined (total) LOS for these antennas. Data are presented in Table 54 for both metric and imperial systems.

TABLE 54 Line of sight (LOS) for two antennas located at different locations with different elevations.

Station 1					Station 2			
Location	MSL (ft)	AGL (ft)	Elevation (ft)	LOS (ML)	MSL (m)	AGL (m)	Elevation (m)	LOS (km)
Selby	874	415	459	28.8	262.2	124.5	137.7	46.0
Odessa	745	295	450	24.3	223.5	88.5	135	38.8
Total LOS				53.1				84.8

10.8.3 Case Study – Crosswind-Headwind Components

For the scenarios presented in Table 55, assuming that the angle provided is the relative angle between the aircraft course and the wind direction (as specified in weather reports), calculate:

(a) headwind component, and (b) crosswind component.

Note that the headwind component will result in a positive value, while the tailwind component will result in a negative value; wind direction in this case is the direction wind is coming from. That means the ground speed with headwind is smaller than that with the tailwind. The crosswind component will be positive coming from the right side and negative from the left side when calculating the crab angle. The angles are to be converted to radians ($\lambda\pi/180$) to be used in the headwind and crosswind relationships. For example, for Case 1, $Headwind = w \cos \lambda = 25 \times \cos(30) = 21.7$ kt, $Crosswind = w \sin \lambda = 25 \times \sin(30) = 12.5$ kt. The previously calculated crosswind and headwind may be compared with those from the chart in Figure 122.

TABLE 55 Examples of crosswind and headwind components.

Case	Direction λ (°)	Magnitude w (kt)	Headwind $w \cos \theta$ (kt)	Crosswind $w \sin \theta$ (kt)
1	30	25	21.7	12.5
2	10	35	34.5	6.1
3	50	10	6.4	7.7
4	280	25	4.3	-24.6
5	330	30	26.0	-15.0
6	120	40	-20	34.6
7	300	10	5.0	-8.7
8	200	45	-42.3	-15.4

10.8.4 Case Study – Weather Brief

A student pilot is to complete her cross-country solo flight from Norman Rogers Airport (CYGK) in Kingston (Canada) to Hamilton Airport (CYHM) in Hamilton (Canada) located west of Kingston; the planned flight altitude is 4,500 ft. Billy Bishop Toronto City (CYTZ) in Toronto (Canada) is the en route airport and the via point to avoid direct fly over Lake Ontario and to stay within Canadian borders. Wind magnitude and direction along with the temperature and pressure altitude at the destination, departure, 3,000 ft, and 6,000 ft elevations based on Figure 124 are presented in Table 56. Current wind data (magnitude and direction) as a function of the elevation are presented at the bottom of Figure 124 (Part 3). Based on the METAR and TAF reports for CYGK (Kingston)-CYHM (Hamilton) data presented in Figure 124, extract:

(a) wind data (magnitude and direction) at the departure, destination, and at flight elevations 3,000 ft and 6,000 ft, (b) temperatures, (c) pressure altitude, (d) justify the choice of the altitude (4,500 ft) to complete the cross-country trip, and (e) headwind and crosswind components—if the aircraft is to take off from CYGK runway 25.

TABLE 56 Departure and destination aerodromes information along with the temperatures, pressures, and en route data.

Case	Location	Elevation H (ft)	Variation (°)	Temperature T (°C)	Pressure P (inHg)	Pressure Altitude PA (ft)	Wind	
							Magnitude w (kt)	Direction ϕ (°)
1	CYGK	303	13W	21.0	29.89	333	5	250
2	CYTZ	252	10W	22.0	29.93	242	4	210
3	CYHM	780	10W	21.0	29.96	740	5	260
4	Enroute CYTZ	6,000	11W	14.0	29.93	5,990	20	280
5	Enroute CYHM	4,500	10W	15.8	29.96	4,460	18.5	300
6	Enroute CYGK	3,000	12W	17.7	29.89	3,030	17	320

Data in Figure 124 are extracted to Table 56. Sample calculations for the CYGK aerodrome runway are as follows:

$$PA_{ft} = H_{ft} + 1,000(29.92 - P_{inHg}) = 303 + 1,000(29.92 - 29.89) = 333 \text{ ft}$$

To calculate the runway crosswind and headwind for a no-gust scenario, use the following relations:

$$\text{Runway}(25) = 250^\circ (\text{Magnetic}), \text{Var} = 13^\circ \text{W},$$

$$\text{Wind} = 250^\circ (\text{True}) + 13^\circ = 263^\circ (\text{Magnetic})$$

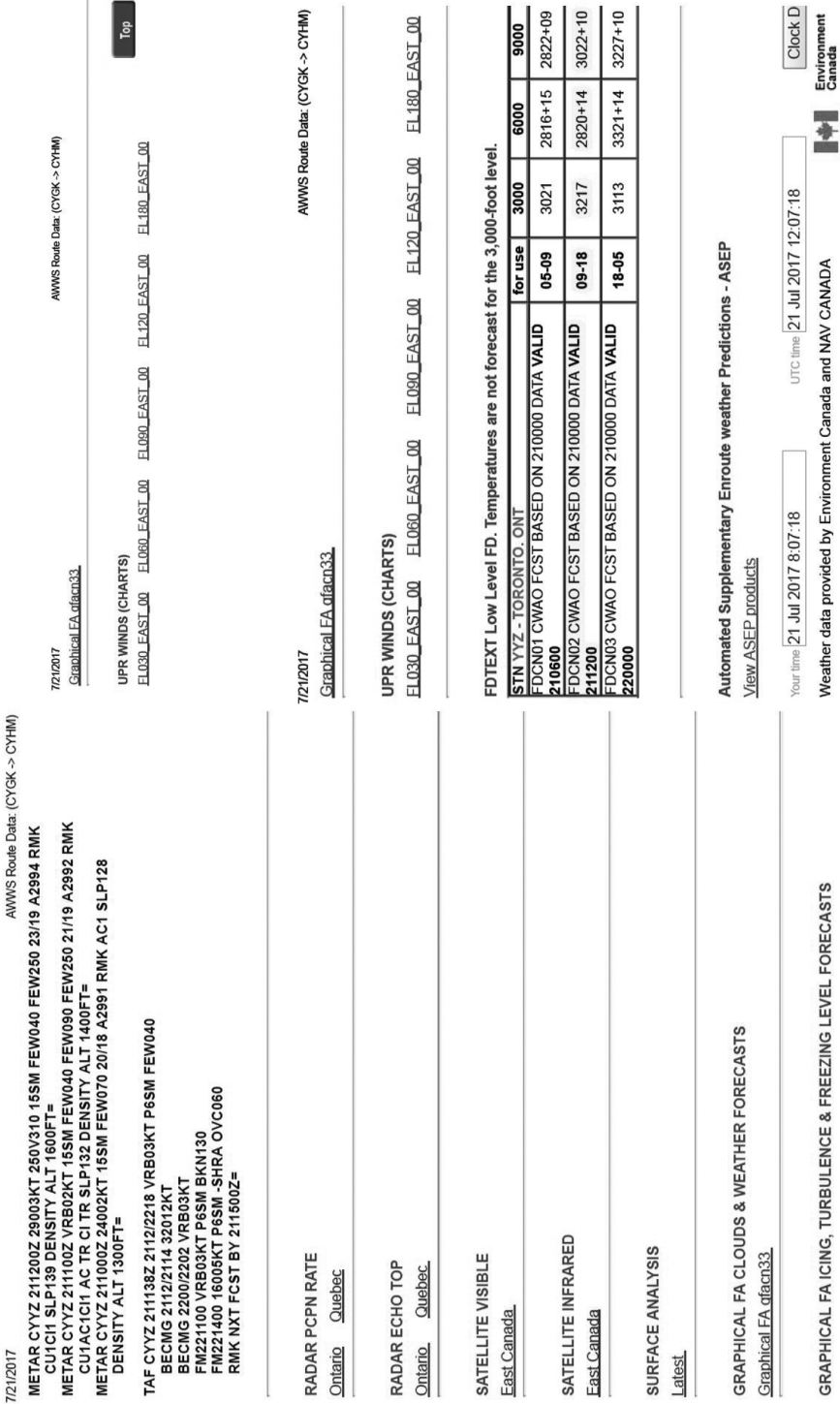
7/21/2017	AWWS Route Data: (CYGK -> CYHM)	7/21/2017	AWWS Route Data: (CYGK -> CYHM)
PIREP		PETERBOROUGH/ON [show WxCam]	
FIR_MONTREAL (UL)			METAR CYQO 211200Z AUTO 25004KT 200V290 9SM CLR 21/19 A2991 RMK SLP128 DENSITY ALT 1400FT=
UL - PIREP not available			SPECI CYQO 211111Z AUTO 00000KT 6SM BR CLR 18/18 A2991 RMK SLP129 DENSITY ALT 1100FT=
FIR_TORONTO (YZ)			METAR CYQO 211100Z AUTO 00000KT 4SM BR CLR 17/17 A2991 RMK SLP128 DENSITY ALT 1000FT=
UACN10 CYXU 211141			SPECI CYQO 211047Z AUTO 00000KT 3SM BR CLR 17/17 A2990 RMK SLP127 DENSITY ALT 1000FT=
UA JOV CYQA 150024 /TM 1140 /FLO65 /TP BE36 /TA 25 /RM 0250VC TO THE NE			SPECI CYQO 211037Z AUTO VRB02KT 1 3/4SM BR CLR 16/15 A2990 RMK SLP127 DENSITY ALT 800FT=
			SPECI CYQO 211036Z AUTO VRB02KT 2 1/2SM BR CLR 16/15 A2990 RMK SLP127 DENSITY ALT 800FT=
			SPECI CYQO 211033Z AUTO VRB02KT 5SM BR CLR 15/15 A2990 RMK SLP127 DENSITY ALT 800FT=
			SPECI CYQO 211029Z AUTO 25003KT 1 1/2SM BR CLR 15/15 A2990 RMK SLP127 DENSITY ALT 800FT=
METAR/TAF			SPECI CYQO 211023Z AUTO 25002KT 1 1/4SM BR CLR 16/16 A2990 RMK VIS VRB 1/4-3 SLP127 DENSITY ALT 800FT=
KINGSTON/ON			SPECI CYQO 211022Z AUTO 25002KT 2SM BR CLR 16/16 A2990 RMK VIS VRB 3/4-3 SLP126 DENSITY ALT 800FT=
METAR CYGK 211200Z 25005KT 15SM FEW220 21/20 A2989 RMK C11 SLP123			SPECI CYQO 211018Z AUTO 24002KT 4SM BR CLR 16/16 A2990 RMK SLP125 DENSITY ALT 900FT=
DENSITY ALT 1200FT=			SPECI CYQO 211014Z AUTO 24003KT 6SM BR CLR 17/16 A2990 RMK SLP125 DENSITY ALT 900FT=
METAR CYGK 211100Z 26007KT 15SM FEW200 19/18 A2988 RMK C11 SLP120			METAR CYQO 211000Z AUTO 23003KT 5SM BR CLR 16/16 A2989 RMK SLP123 DENSITY ALT 900FT=
DENSITY ALT 1000FT=			TAF AMD CYQO 211030Z 2110/2122 VRB03KT P6SM FEW020 TEMPO 2110/2112 11/2SM BR BKN020
METAR CYGK 211000Z 25004KT 15SM FEW038 18/17 A2986 RMK SC1 SC TR SLP114 DENSITY ALT 800FT=			FM211200 27008KT P6SM FEW050
TAF CYGK 211138Z 2112/2124 26007KT P6SM FEW030 TEMPO 2112/2113 2SM BR BECMG 2113/2115 20008KT			RMK FCST BASED ON AUTO OBS. NXT FCST BY 211400Z=
FM211700 22012G32KT P6SM SCT040			TORONTO/OSHAWA EXECUTIVE AIRPORT/ON [show WxCam]
BECMG 2121/2123 24008KT			METAR CYQO 211200Z AUTO 25003KT 9SM CLR 21/18 A2992 RMK SLP133 DENSITY ALT 1200FT=
RMK NXT FCST BY 211400Z=			METAR CYQO 211100Z AUTO VRB02KT 9SM CLR 17/16 A2991 RMK SLP131 DENSITY ALT 800FT=
TRENTON/ON			METAR CYQO 211000Z AUTO 00000KT 6SM BR CLR 15/15 A2990 RMK SLP127 DENSITY ALT 600FT=
METAR CYTR 211200Z 23005KT 8SM FEW220 22/19 A2990 RMK C11 SLP127=			CYOO - No TAF is issued for this station
METAR CYTR 211100Z 23005KT 8SM FEW025 FEW220 20/19 A2990 RMK SC1C11 SLP124=			TORONTO/BUTTONVILLE MUNICIPAL/ON
METAR CYTR 211000Z 24004KT 8SM FEW025 19/18 A2988 RMK SC1 SLP120=			https://flightplanning.navcanada.ca/cgi-bin/route.cgi
TAF CYTR 211130Z 2112/2212 25008KT P6SM FEW025			
BECMG 2114/2116 25012KT			
BECMG 2202/2204 VRB03KT			
RMK NXT FCST BY 211800Z=			

FIGURE 124 METAR-TAF weather reports for CYGK-CYHM route recorded on July 21, 2017 at 7:18 a.m. (Part 1).

7/21/2017	AWNS Route Data: (CYGK -> CYHM)	7/21/2017	AWNS Route Data: (CYGK -> CYHM)			
METAR CYKZ 211100Z 00000KT 15SM FEW070 FEW240 19/18 A2993 RMK ACC1C11 ACC TR SLP133 DENSITY ALT 1300FT= METAR CYKZ 211000Z 00000KT 15SM FEW040TCU FEW240 18/17 A2991 RMK TCU1C11 TCU TR CL TR SLP128 DENSITY ALT 1200FT= METAR CYKZ 210900Z 00000KT 15SM SKC 18/17 A2990 RMK SLP126 DENSITY ALT 1300FT= TAF CYKZ 211138Z 211212Z 2124 VRB03KT P6SM SKC BECMG 21132115 32012KT RMK NXT FCST BY 211800Z= ST. CATHARINES/NIAGARA DISTRICT/ION	METAR CYKZ 211100Z 00000KT 15SM FEW070 FEW240 19/18 A2993 RMK ACC1C11 ACC TR SLP133 DENSITY ALT 1300FT= METAR CYKZ 211000Z 00000KT 15SM FEW040TCU FEW240 18/17 A2991 RMK TCU1C11 TCU TR CL TR SLP128 DENSITY ALT 1200FT= METAR CYKZ 210900Z 00000KT 15SM SKC 18/17 A2990 RMK SLP126 DENSITY ALT 1300FT= TAF CYKZ 211138Z 211212Z 2124 VRB03KT P6SM SKC BECMG 21132115 32012KT RMK NXT FCST BY 211800Z= ST. CATHARINES/NIAGARA DISTRICT/ION	METAR CYKZ 211138Z 211222Z 25005KT P6SM SKC BECMG 21132115 31010KT BECMG 22032205 20005KT FM21100 15005KT P6SM BKN130 RMK NXT FCST BY 211800Z= KITCHENER/WATERLOO/ION [show WxCam]	METAR CYKZ 211200Z AUTO 31005KT 8SM CLR 21/20 A2997 RMK SLP147 DENSITY ALT 1900FT= SPECI CYKZ 211147Z AUTO 30005KT 6SM BR CLR 21/19 A2997 RMK SLP144 DENSITY ALT 1900FT= METAR CYKZ 211100Z AUTO VRB02KT 4SM BR CLR 18/18 A2996 RMK SLP142 DENSITY ALT 1800FT= SPECI CYKZ 211046Z AUTO VRB02KT 5SM BR CLR 17/17 A2996 RMK SLP141 DENSITY ALT 1400FT= SPECI CYKZ 211039Z AUTO 21004KT 6SM BR CLR 17/17 A2995 RMK SLP139 DENSITY ALT 1500FT= SPECI CYKZ 211032Z AUTO VRB02KT 3SM BR CLR 17/17 A2995 RMK SLP139 DENSITY ALT 1500FT= SPECI CYKZ 211018Z AUTO 30004KT 2SM BR CLR 16/16 A2995 RMK SLP140 DENSITY ALT 1400FT= SPECI CYKZ 211013Z AUTO 29004KT 3/4SM BR CLR 16/16 A2995 RMK VIS VRB 1/8-3 SLP139 DENSITY ALT 1300FT= METAR CYKZ 211000Z AUTO 34002KT 1/8SM FG CLR 16/16 A2994 RMK SLP135 DENSITY ALT 1400FT= TAF CYKZ 210738Z 210821Z 20 VRB03KT P6SM SKC TEMPO 21082112 11/2SM BR PROB40 21082112 1/4SM FG FM211500 32012KT P6SM SKC RMK FCST BASED ON AUTO OBS. NXT FCST BY 211400Z= TORONTO/BILLY BISHOP TORONTO CITY AIRPORT/ION [show WxCam]	METAR CYBN 211200Z AUTO 25003KT 220V280 9SM CLR 20/19 A2994 RMK SLP138= METAR CYBN 211100Z AUTO 00000KT 9SM CLR 17/16 A2993 RMK SLP133= METAR CYBN 211000Z AUTO 25002KT 9SM CLR 14/14 A2992 RMK SLP133= TAF CYBN 211130Z 211212Z 2124 VRB03KT P6SM FEW040 BECMG 21142116 31010KT RMK FCST BASED ON AUTO OBS. NXT FCST BY 211800Z= HAMILTON/ION	METAR CYTZ 211200Z AUTO 21004KT 9SM CLR 22/20 A2993 RMK SLP135 DENSITY ALT 1000FT= METAR CYTZ 211100Z AUTO 30002KT 9SM CLR 21/19 A2991 RMK SLP129 DENSITY ALT 1000FT= METAR CYTZ 211000Z AUTO 00000KT 7SM CLR 20/19 A2990 RMK SLP125 DENSITY ALT 800FT= TAF CYTZ 210738Z 21082208 27005KT P6SM SKC TEMPO 21082112 2SM BR PROB40 21082112 3/4SM BR FM211300 32006KT P6SM SKC BECMG 22002202 22005KT RMK FCST BASED ON AUTO OBS. NXT FCST BY 211400Z= TORONTO/LESTER B. PEARSON INTL/ION	METAR CYTZ 211200Z AUTO 21004KT 9SM CLR 22/20 A2993 RMK SLP135 DENSITY ALT 1000FT= METAR CYTZ 211100Z AUTO 30002KT 9SM CLR 21/19 A2991 RMK SLP129 DENSITY ALT 1000FT= METAR CYTZ 211000Z AUTO 00000KT 7SM CLR 20/19 A2990 RMK SLP125 DENSITY ALT 800FT= TAF CYTZ 210738Z 21082208 27005KT P6SM SKC TEMPO 21082112 2SM BR PROB40 21082112 3/4SM BR FM211300 32006KT P6SM SKC BECMG 22002202 22005KT RMK FCST BASED ON AUTO OBS. NXT FCST BY 211400Z= TORONTO/LESTER B. PEARSON INTL/ION

<https://flightplanning.navcanada.ca/cgi-bin/route.cgi>

FIGURE 124 METAR-TAF weather reports for CYGK-CYHM route recorded on July 21, 2017 at 7:18 a.m. (Part 2).



$$\text{Wind Relative Angle} = 263^\circ - 250^\circ = 13^\circ$$

$$\text{Headwind} = w \cos \lambda = 5 \times \cos(13^\circ) = 4.87 \text{ kt}$$

$$\text{Crosswind} = w \sin \lambda = 5 \times \sin(13^\circ) = 1.12 \text{ kt}$$

The previously calculated crosswind and headwind may be compared with those from the chart in Figure 122. Since the aircraft is flying to the west on a VFR flight plan, an even-thousand flight altitudes plus 500 ft is to be selected.

10.8.5 Case Study – Selection of Runway Based on METAR-TAF Data

A student pilot is to complete her cross-country solo flight from Norman Rogers Airport (CYGK) in Kingston (Canada) to Hamilton Airport (CYHM) in Hamilton (Canada) with the Cessna 172M aircraft. Current METAR and TAF reports for the CYGK (Kingston)-CYHM (Hamilton) route are presented in Figure 124 (Parts 1 to 3). For the departure, via point, and destination, for non-gusty and gusty conditions (using the TAF data provided in Figure 124—parts 1, 2, and 3—presented in Table 57), calculate:

(a) headwind and crosswind components, (b) runway to use for takeoff and landing, (c) justify the choice of runway, and (d) runway length correction factor for the headwind or tailwind scenarios (assuming that every 9 kt headwind decreases the takeoff and landing distances by 10 percent and every 2 kt tailwind increases the takeoff and landing distances by 10 percent).

Data extracted from Figure 124 is shown in Table 57. Sample calculations for CYHM are as follows—data are presented in Table 58

TABLE 57 Departure and destination aerodromes information along with temperatures, pressures, and en route data.

Location	Runway (Length in ft)	Elevation <i>H</i> (ft)	Variation (°)	Temperature <i>T</i> (°C)	Pressure <i>P</i> (inHg)	Pressure Altitude <i>PA</i> (ft)	Wind		Runway	Runway
							Magnitude <i>w</i> (kt)	Direction <i>φ</i> (°)		
CYGK	19/01 – 07/25 (4,929 × 100–3,909 × 100)	303	13W	21	29.89	333	5G0	250	25	0.95
							5G8	200	19	0.93
CYTZ	16/24 – 08/26 (2,460 × 100–3,988 × 150)	252	10W	22	29.93	242	4G0	210	24	0.96
							6G10	320	26	0.97
CYHM	16/24 – 12/30 (6,010 × 150–10,006 × 200)	780	10W	21	29.96	740	5G0	260	24	0.95
							7G10	310	30	0.91

(no-gust conditions) and Table 59 (gust conditions) for CYGK and CYHM aerodromes.

$$PA_{ft} = H_{ft} + 1,000(29.92 - P_{inHg}) = 780 + 1,000(29.92 - 29.96) = 740 \text{ ft}$$

Since every 9 kt headwind decreases the takeoff and landing distances by 10 percent, the calculated headwind (kt) is to be multiplied by $10\% / 9(\text{kt})$ to obtain the percentage of its contribution toward the runway length requirements. The same logic is applied to the tailwind data except the required length of the runway is increased by 10% for every 2 kt tailwind. Note that the correction factor is to be less than one for the headwind case and larger than one for the tailwind case.

$$\text{Runway Correction Factor (Headwind)} = \left(100\% - \text{Headwind (kt)} \times \frac{10(\%)}{9(\text{kt})} \right) / 100$$

$$\text{Runway Correction Factor (Tailwind)} = \left(100\% + \text{Tailwind (kt)} \times \frac{10(\%)}{2(\text{kt})} \right) / 100$$

To calculate the runway crosswind and headwind for the no-gust scenario, the following relations are used:

$$\text{Runway}(24) = 240^\circ (\text{Magnetic}), \text{Var} = 10^\circ \text{W},$$

$$\text{Wind} = 260^\circ (\text{True}) + 10^\circ = 270^\circ (\text{Magnetic})$$

$\lambda = \text{Angle Difference} = 270^\circ - 240^\circ = 30^\circ$ (relative angle of wind with respect to the runway)

$$\text{Crosswind} = w \sin \lambda = 5 \times \sin(30^\circ) = 2.50 \text{ kt}$$

$$\text{Headwind} = w \cos \lambda = 5 \times \cos(30^\circ) = 4.33 \text{ kt}$$

$$\begin{aligned} \text{Runway Correction Factor} &= \frac{100\% - \text{Headwind (kt)} \times \frac{10(\%)}{9(\text{kt})}}{100} \\ &= \frac{4.33 \times 10}{9 \times 100} = 0.95 \end{aligned}$$

To calculate the runway crosswind and headwind, given gust conditions, the following relations can be developed (half of the gust factor is added to the wind magnitude when calculating the headwind and crosswind components):

$$G_f = \frac{10 - 7}{2} = 1.5 \text{ kt}$$

$$\text{Runway}(30) = 300^\circ (\text{Magnetic}), \text{Var} = 10^\circ \text{W},$$

$$\text{Wind} = 310^\circ (\text{True}) + 10^\circ = 320^\circ (\text{Magnetic})$$

$\lambda = \text{Angle Difference} = 320^\circ - 300^\circ = 20^\circ$ (relative angle of wind with respect to the runway)

$$\text{Crosswind} = w \sin \lambda = (7 + 1.5) \times \sin(20^\circ) = 2.91 \text{ kt}$$

$$\text{Headwind} = w \cos \lambda = (7 + 1.5) \times \cos(20^\circ) = 7.99 \text{ kt}$$

$$\begin{aligned} \text{Runway Multiplier} &= \frac{100\% - \text{Headwind (kt)} \times \frac{10(\%)}{9(\text{kt})}}{100} \\ &= \frac{7.99 \times 10}{9 \times 100} = 0.91 \end{aligned}$$

CYBK and CYHM data for no-gust (Table 58) and gust conditions (Table 59) are provided. The reader is encouraged to repeat the exercise for CYTZ and analyze the choice of runways for CYTZ without and with gust conditions (Table 60).

As another approach, the chart in Figure 122 may be used to obtain crosswinds and headwinds; these can be compared to the previous calculations.

TABLE 58 Crosswind and headwind components, without gust (CYBK and CYHM).

Not Gusting Condition					
		CYGK			
		Wind Direction	250	TAF CYGK 211338Z 211400/211600 P6SM FEW040 20005G8kt	
		Wind Mag. Gusting (kts)	5	0	
Length	3,909	Runway	25	Runway: 25/ Variation: 13 Wind: 250 @ 5 G 0	
Width	100	Variation	13		
		degrees difference		13	
		kts (headwind)		4.872	
		kts (crosswind)		1.125	
			Multiplier	0.946	
			Head/Tail	Headwind	
		CYHM			
		Wind Direction	260	TAF CYHM 211738Z 211800/212000 P6SM FEW050 31007G10kt	
		Wind Mag. Gusting (kts)	5	0	
Length	6,010	Runway	24	Runway: 24/ Variation: 10 Wind: 260 @ 5 G 0	
Width	150	Variation	10		
		degrees difference		30	
		kts (headwind)		4.330	
		kts (crosswind)		2.500	
			Multiplier	0.952	
			Head/Tail	Headwind	

TABLE 59 Crosswind and headwind components, with gust (CYGK and CYHM).

Gusting Condition				
		CYGK		
		Wind Direction	200	TAF CYGK 211338Z 211400/211600 P6SM FEW040 20005G8kt
		Wind Mag. Gusting (kts)	5	8
Length	4,929	Runway	19	Runway: 19/ Variation: 13 Wind: 200 @ 5 G 8
Width	100	Variation	13	
		degrees difference		23
		kts (headwind)		5.983
		kts (crosswind)		2.540
			Multiplier	0.934
			Head/Tail	Headwind
		CYHM		
		Wind Direction	310	TAF CYHM 211738Z 211800/212000 P6SM FEW050 31007G10kt
		Wind Mag. Gusting (kts)	7	10
Length	10,006	Runway	30	Runway: 30/ Variation: 10 Wind: 310 @ 7 G 10
Width	200	Variation	10	
		degrees difference		20
		kts (headwind)		7.987
		kts (crosswind)		2.907
			Multiplier	0.911
			Head/Tail	Headwind

TABLE 60 Crosswind and headwind components, with and without gust (CYTZ).

Not Gusting Condition				
		CYTZ		
		Wind Direction	210	TAF CYTZ 211338Z 211400/211600 P6SM FEW025 32006G10kt
		Wind Mag. Gusting (kts)	4	0
Length	2,460	Runway	24	Runway: 24/ Variation: 11 Wind: 210 @ 4 G0
Width	100	Variation	11	
		degrees difference		19
		kts (headwind)		3.782
		kts (crosswind)		1.302
			Multiplier	0.958
			Head/Tail	Headwind
Gusting Condition				
		CYTZ		
		Wind Direction	320	TAF CYTZ 211338Z 211400/211600 P6SM FEW025 32006G10kt
		Wind Mag. Gusting (kts)	6	10
Length	3,988	Runway	26	Runway: 26/ Variation: 11 Wind: 320 @ 6 G10
Width	150	Variation	11	
		degrees difference		71
		kts (headwind)		2.605
		kts (crosswind)		7.564
			Multiplier	0.971
			Head/Tail	Headwind

10.8.6 Case Study – Takeoff and Landing Distances Using the Performance Charts

A student pilot is to complete her 162-mile (one-way) cross-country solo flight from Norman Rogers Airport (CYGK) in Kingston (Canada) to Hamilton Airport (CYHM) in Hamilton (Canada) located west of Kingston with the Cessna 172M aircraft at a planned altitude of 4,500'. Billy Bishop Toronto City (CYTZ) in Toronto (Canada) is the en route airport and a via point to avoid flight over Lake Ontario and to stay within Canadian borders. METAR and TAF reports for CYGK (Kingston)-CYHM (Hamilton) are presented in Figure 124. En route data along with the ground pressure and temperature are presented in Table 61. Runway length correction factors may be adopted from Case Study 10.8.5 “Selection of Runway Based on METAR-TAF Data”. For non-grass and grass runways, calculate:

(a) takeoff and landing distances for ground rolls and distances to clear 50-ft obstacles for departure and destination aerodromes (CYGK and CYHM)—for asphalt runways, (b) confirm choice of takeoff and landing runway for lengths in Case Study 10.8.5 “Selection of Runway Based on METAR-TAF Data”, (c) headwind and crosswind components at the via aerodrome CYTZ along with the preferred runways to land or take off, (d) takeoff and landing distances for the grass runways—assuming that the ground roll distances increase by 15 percent, and (e) takeoff and landing distances assuming a safety margin of 50 percent for both asphalt and grass runways for the CYGK and CYHM aerodromes.

TABLE 61 Departure and destination aerodromes information along with temperatures, pressures, and en route data.

Case	Location	Runway (Length in ft)	Elevation <i>H</i> (ft)	Variation (°)	Temperature <i>T</i> (°C)	Pressure <i>P</i> (inHg)	Pressure Altitude <i>PA</i> (ft)	Wind		Runway	Runway	Runway Correction Factor
								Magnitude <i>w</i> (kt)	Direction <i>φ</i> (°)			
1	CYGK	19/01 – 07/25 (4,929 × 100- 3,909 × 100)	303	13 W	21	29.89	333	5G0	250	25	25	0.95
								8G5	200	19	19	0.93
2	CYTZ	16/24 – 08/26 (2,460 × 100- 3,988 × 150)	252	10 W	22	29.93	242	4	210	24	24	0.96
								6G10	320	26	26	0.97
3	CYHM	16/24 – 12/30 (6,010 × 150- 10,006 × 200)	780	10 W	21	29.96	740	5G0	260	24	24	0.95
								10G7	310	30	30	0.91
4	En Route CYTZ	-	6,000	11 W	14	29.93	5,990	20	280	NA	NA	NA
5	En Route CYHM	-	4,500	10 W	15.8	29.96	4,460	18.5	300			
6	En Route CYGK	-	3,000	12 W	17.7	29.89	3,030	17	320			

TABLE 62 Takeoff and landing distances, without grass (CYGK and CYHM).

CYBK	Temp (°C)	21		0.946 Gust 0.934
No Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	980	1725	0.946	
Wind	926.95 Gust 914.85	1631.62 Gust 1610.32	0.934	
Grass	926.95 Gust 914.85	1631.62 Gust 1610.32	1.15	
Safety	1390.43 Gust 1372.27	2447.43 Gust 2415.48	1.5	
Runway Length (ft)	3909 , 4929	3909 , 4929	GOOD	
No Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	565	1330	0.946	
Wind	534.42 Gust 527.44	1258 Gust 1241.58	0.934	
Grass	534.42 Gust 527.44	1258 Gust 1241.58	1.45	0.946 Gust 0.934
Safety	801.62 Gust 791.16	1887.01 Gust 1862.37	1.5	
Runway Length (ft)	3909 , 4929	3909 , 4929	GOOD	
CYHM	Temp (°C)	21		
No Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	980	1725	0.952	
Wind	932.85 Gust 893.03	1642.01 Gust 1571.91	0.911	
Grass	932.85 Gust 893.03	1642.01 Gust 1571.91	1.15	
Safety	1399.27 Gust 1339.54	2463.01 Gust 2357.86	1.5	
Runway Length (ft)	6010 , 10006	6010 , 10006	GOOD	
No Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	565	1330	0.952	
Wind	537.82 Gust 514.86	1266.01 Gust 1211.96	0.911	
Grass	537.82 Gust 514.86	1266.01 Gust 1211.96	1.45	0.952 Gust 0.911
Safety	806.72 Gust 772.29	1899.02 Gust 1817.95	1.5	
Runway Length (ft)	6010 , 10006	6010 , 10006	GOOD	

TABLE 63 Takeoff and landing distances, with grass (CYGK and CYHM).

CYGK	Temp (°C)	21		0.946 Gust 0.934
Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	980	1725	0.946	
Wind	926.95 Gust 914.85	1631.62 Gust 1610.32	0.934	
Grass	1065.99 Gust 1052.08	1770.67 Gust 1747.55	1.15	
Safety	1598.99 Gust 1578.11	2656 Gust 2621.32	1.5	
Runway Length (ft)	3909 , 4929	3909 , 4929	GOOD	
Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	565	1330	0.946	
Wind	534.42 Gust 527.44	1258 Gust 1241.58	0.934	
Grass	774.9 Gust 764.79	1498.49 Gust 1478.93	1.45	
Safety	1162.35 Gust 1147.18	2247.74 Gust 2218.39	1.5	
Runway Length (ft)	3909 , 4929	3909 , 4929	GOOD	
CYHM	Temp (°C)	21		0.952 Gust 0.911
Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	980	1725	0.952	
Wind	932.85 Gust 893.03	1642.01 Gust 1571.91	0.911	
Grass	1072.78 Gust 1026.98	1781.93 Gust 1705.86	1.15	
Safety	1609.17 Gust 1540.47	2672.9 Gust 2558.79	1.5	
Runway Length (ft)	6010 , 10006	6010 , 10006	GOOD	
Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	565	1330	0.952	
Wind	537.82 Gust 514.86	1266.01 Gust 1211.96	0.911	
Grass	779.83 Gust 746.54	1508.03 Gust 1443.65	1.45	
Safety	1169.75 Gust 1119.81	2262.04 Gust 2165.47	1.5	
Runway Length (ft)	6010 , 10006	6010 , 10006	GOOD	

Detailed calculations are presented in Table 62 (with no grass) and Table 63 (with grass). Note that for a grass runway, the multiplier 1.15 is to be included only for the ground roll calculations, where the tire (wheel) is in direct contact with the ground. The moment the tire (wheel) lifts off the ground, the length of the runway is not affected by the runway surface condition anymore. Therefore, this portion does not influence the total length to clear a 50-foot obstacle. Thus, this length (i.e., the difference between the total length to clear the 50-foot obstacle and the ground roll, considering wind effects) is to be calculated to obtain the no-ground portion of the obstacle clearance required runway length. This length is then to be added to the revised ground roll value (extended by a factor of 1.15) to obtain the required distance to clear a 50-foot obstacle. The safety margin of 50 percent (correction factor of 1.5) is then to be included in the calculations (i.e., the runway length is to increase by 50 percent for safety).

10.8.7 Case Study – Weight-and-Balance

The Cessna 172M aircraft is to take a cross-country flight. The front passenger and rear passenger (child and puppy) as well as luggage information are presented in Table 64. Assume the density of fuel is 6 lb per (US) gallon. Assume that rate of fuel usage for ground handling is 7.8 gph and the entire period (from the start of the engine to lining up for the takeoff) takes about 8.5 min. For the normal and utility categories, calculate:

(a) mass of the fuel required for ground handling, (b) moment each weighted item produces for the given arms, (c) total weight, (d) total moment, (e) COG, (f) confirm that the COG is within the required limits, and (g) repeat items “a” to “f,” assuming no extra fuel for ground handling.

Table 64 presents the passenger and luggage requirements for both utility and normal categories along with the requirements for the location of the COG. For the aircraft to be airworthy, the calculated data are to be within the specified range. The COG may not exceed the rearward or forward range identified in the normal and utility mass charts. Calculations are presented in Table 65 and Table 66 for both normal and utility categories

TABLE 64 Passengers and luggage mass data.

Category	Pilot (SIC) (lb)	Pilot (FIC) (lb)	Rear Passenger 1 (lb)	Rear Passenger 2 (lb)	Baggage 1 (lb)	Baggage 2 (lb)	Empty Weight (lb)	COG Limit Forward (in) <1950 lb, 2,000 lb	COG Limit Aft (in) all weights	MTOW (lb)
Normal	110	210	50	20	7	3	1,515	35.0, 38.5	47.3	2,300
Utility	110	210	0	0	0	0	1,515	35.0, 35.5	40.5	2,000

TABLE 65 Weight-and-balance data for normal and utility categories with extra ground handling fuel (part 1).

Item	Normal Mass (lb)	Utility Mass (lb)	Arm (in)	Normal Moment/g (lb.in)	Utility Moment/g (lb.in)
Maximum Take-off	2300	2000	-	-	-
Empty	1515	1515	40.02	60630.30	60630.30
SIC and FIC	320	320	37.00	11840.00	11840.00
Rear Passenger	70	0	73.00	5110.00	0.00
Baggage Area 1	7	0	95.00	665.00	0.00
Baggage Area 2	3	0	112.00	336.00	0.00
Subtotal	1915	1835	41.03	78581.30	72470.30
FUEL	391.6	171.6	48.00	18796.80	8236.80
TOTAL	2306.6	2006.6	42.22	97378.10	80707.10
COG (in)				42.22	40.22

TABLE 66 Weight-and-balance data for normal and utility categories without extra ground handling fuel (part 2).

NORMAL and Utility WEIGHT AND BALANCE—Ramp Off (A-B) No Extra Ground Handling Fuel					
Item	Normal Mas (lb)	Utility Mass (lb)	Arm (in)	Normal Moment/g (lb.in)	Utility Moment/g (lb.in)
Maximum Take-off	2300	2000	-	-	-
EMPTY	1515	1515	40.02	60630.30	60630.30
SIC and FIC	320	320	37.00	11840.00	11840.00
Rear Passenger	70	0	73.00	5110.00	0.00
Baggage Area 1	7	0	95.00	665.00	0.00
Baggage Area 2	3	0	112.00	336.00	0.00
Subtotal	1915	1835	41.03	78581.30	72470.30
FUEL	385	165	48.00	18480.00	7920.00
TOTAL	2300	2000	42.20	97061.30	80390.30
COG (in)				42.20	40.20

with and without ground handling fuel, respectively. The tables list the normal and utility category weights and moments along with the calculated COGs. Note that for the utility category, the rear passenger and luggage areas are to be unoccupied for the Cessna 172M. The calculated COG falls within the maximum and minimum allowed location for the COG for both categories.

To obtain the fuel required for the ground handling, the rate of fuel usage during the ground handling (gph) is to be multiplied by the total time dedicated to the activity (hr), which is $1.1 \text{ gal } (7.8 \times 8.5 / 60 = 1.1 \text{ gal})$. The moments are calculated by multiplying the weight by the arm (the horizontal distance from the reference datum). Note that for this aircraft, the

performance data includes the moment per gravity acceleration ($g = 9.8 \text{ m/s}^2$) presented in lb.in. The reader is to ensure units of mass and distance are correctly implemented.

Note that the extra ground handling fuel on board (1.1 gal) shifts the COG by a very small amount (0.2 in.). Thus, increasing the fuel increases the distance of the COG from the reference datum.

10.8.8 Case Study – Itemized Fuel

For the fuel figures for a piston-engine aircraft presented in Table 67, calculate:

(a) TAS assuming that CAS is the same as IAS and FL55 is the pressure altitude, (b) ground speed, (c) true heading, (d) TOC to TOD time, (e) TOC to TOD fuel consumption in gallons, (f) trip fuel, (g) contingency fuel, (h) fuel reserve, (i) takeoff fuel, and (j) landing fuel.

To calculate the extra ground handling fuel on board, the rate of fuel usage during the ground handling (gph) is to be multiplied by the total time (hr). This is shown by the fuel used for taxi, which is 5 gal, given the fuel consumption of 17.5 gph and a duration of 17 min ($17.5 \times 17.0 / 60 = 5.0$ gal). If the ground handling time is 8.5 min and the rate of fuel usage is 7.8 gph, the total fuel used is 1.1 gal.

TABLE 67 Fuel data for a piston-engine aircraft.

Itemized Fuel	Quantity	Unit	Flight Plan	Quantity	Unit
Taxi	5	gal	TOC to TOD (Cruise)	180	NM
Climb	15	gal	FL55	5,500	ft
Descent	7	gal	OAT	12	°C
Cruise	50	GPH	IAS	140	kt
Alternate	20	gal	Course (True)	60	°
Endurance	40	GPH	Wind	010@20	°@kt
Fuel – Cruise	64.286	gal	TAS	153.9	kt
Fuel – Trip	86.286	gal	GS	140.3	kt
Contingency	4.314	gal	THdg	54.3	°
Fuel Reserve (45 min @ endurance speed)	30	gal	Time (TOC to TOD)	77.143	min
Fuel Uplift at Departure (Takeoff Fuel)	145.600	gal			
Fuel Load at Destination (Landing Fuel)	54.314	gal			

10.8.9 Case Study – Fuel Requirements

The Cessna 172M aircraft is to take a cross-country flight from Norman Rogers Airport (CYGK, Point A) to Hamilton Airport (CYHM, Point B) via Billy Bishop Airport (CYTZ) (no landing at the latter aerodrome is intended). The total flight is to take under 1.8 hrs to be back in time to take the puppy-in-training for a walk. The fuel quantity and weight are adopted from Case Study 10.8.7 “Weight-and-Balance.” The fuel data and tank capacities are provided in Table 68. For normal and utility categories, assuming that the ramp fuel may be added to the initial fuel on board, based on the ground handling period of 8.5 min and rate of fuel consumption of 7.8 gph, calculate:

(a) total allowable fuel in gallons (assuming that fuel density is 6 lb/gal)—Table 68, (b) total fuel requirements based on the known highlighted fuel data in Table 69, (c) fuel endurance given an average fuel consumption of 7 GPH, and (d) fuel usage after a 1.8-hour flight time given the average fuel consumption in part “c.”

TABLE 68 Allowable fuel and tank capacity based on the weight-and-balance data.

Fuel	Category		Tank Capacity (gal)	
	Normal	Utility	Long	Short
Pounds (lb)	391.6	171.6	270	228
Gallons (gal)	65.27	28.60	45	38

TABLE 69 Fuel requirements for the en route flight for the normal and utility categories.

Requirement	Fuel (gal)	Itemized Fuel	Normal Category	Utility Category
			Fuel Requirement (gal)	
Endurance (GPH)	7.5	CYGK (Point A)-Taxi-Take Off-TOC	2.9	2.9
Maximum (GPH)	8.8	Top of Climb (TOC)-Set Heading Point (SHP)	0.1	0.1
Cruise (GPH)	6.5	SHP-CYHM (Point B)	8.6	8.6
Taxi (gal)	1.1	Point B-Landing-Taxi	3.6	3.6
		Total	15.2	15.2
		Reserve (30-min Cruise)	3.25	3.25
		Total (including Reserve)	18.45	18.45
		20% Extra	3.69	3.69
		Required	22.14	22.14
		Useful Carried	45.0	28.6
		Remainder	22.9	6.46
		Takeoff (including 1.1-gal Groundwork)	43.9	27.5
		Landing (including 1.1-gal Groundwork; no reserve data are considered)	28.7	12.3

TABLE 70 Fuel requirements along with the performance and endurance data.

Category	Fuel (gal)					
	Capacity	Takeoff	Landing	Requirements	Endurance (hr)	Consumption (gal) / 1.8 (hr)
Normal	45.0	43.9	28.7	22.14	6.4	12.6
Utility	28.60	27.5	12.3	22.14	4.1	12.6

Total allowable fuel amounts in gallons for the normal and utility categories with the assumed density are provided in the last row of Table 68. Table 69 presents data for the fuel assuming a 30-min cruise reserve requirement and 20 percent extra precautionary fuel. Based on the allowable weights and requirements in Table 68 and requirements in Table 69, the fuel requirements for the cross-country trip can be predicted (Table 70).

10.8.10 Case Study – Maximum Traffic Load

For the data presented for two different aircraft in Table 71, calculate:

- (a) traffic load using three methods, (b) maximum traffic load, and
- (c) traffic load restricting factor.

Recall the traffic load calculations presented in Table 53. The Maximum Traffic Load (MTL) is the lowest figure of the three calculated values in the relation $Max(TL) = Min[MTOM - (DOM + FUEL), MLM - (DOM + FUEL), MZFM - DOM]$. Additionally, the restricting factor that limits the traffic load corresponds to the column heading under which the traffic load (TL) is identified. For the table on the left (Aircraft 1), the limit is due to the MZFM, while for the table on the right (Aircraft 2), the limit is due to the MLM.

TABLE 71 Calculating maximum traffic load using mass limits.

Aircraft 1				Aircraft 2			
Fuel (kg)	MTOM	MLM	MZFM	Fuel (kg)	MTOM	MLM	MZFM
Limits	77,000	69,000	62,000	Limits	18,000	17,000	16,000
DOM	46,000	46,000	46,000	DOM	11,000	11,000	11,000
FUEL	6,000	6,000		FUEL	2,100	2,100	
TL	25,000	17,000	16,000	TL	4,900	3,900	5,000
Max(TL)			16,000	Max(TL)		3,900	
Contingency	Not used			Contingency	Not used		
Alternate	Not used			Alternate	Not used		

10.8.11 Case Study – Flight Plan and Itinerary Plan

A student pilot is to complete her 162-mile (one-way) cross-country solo flight with the Cessna 172M from Norman Rogers Airport (CYGK) in Kingston (Canada) to Hamilton Airport (CYHM) in Hamilton (Canada). Billy Bishop Toronto City (CYTZ) in Toronto (Canada) is the en route airport and a via point to avoid flight over Lake Ontario and to stay within Canadian airspace. She is to change the course before entering the Trenton airspace by passing via Consecon, a town south of Trenton, and continue with the rest of her trip via Brighton, after clearing the Trenton airspace. The flight plan itinerary details are presented in Table 72. METAR and TAF reports for CYGK (Kingston) – CYHM (Hamilton) as well as the en route weather data are incorporated in Figure 125. Complete a flight plan sheet by obtaining the following information:

TABLE 72 Cross-country flight plan table of contents.

Table of Contents	
Title	Page
Flight Plan	2
Take Off and Landing Distances	5
Wind Data	6
Weight and Balance Data	9
Performance Graphs	13
Weather Updates	26
- Weather reports (METAR and TAF)	26
- Graphical Data: Enroute Wind	34
- Graphical Data: Enroute Temperature	36
- Graphical Data: Enroute Mean Sea Level Pressure	39
- Graphical Data (Icing, Turbulence)	41
- Graphical Data (Clouds and Weather)	43
- Graphical Data (Head/Tail Winds)	45
- NOTAMS	48
Airport CFS Data	78
Foreflight Update	92

(a) flight plan, (b) pressure altitudes and temperatures at desired levels, (c) takeoff and landing distances and distances to clear a 50-ft obstacle for the departure and destination aerodromes, (d) preferred runways for the departure and destination aerodromes, (e) headwind and crosswind components, (f) flight altitudes, (g) weight-and-balance calculations for normal and utility categories, ramp-off, takeoff, landing, and 1.8 hrs into the flight—highlight the data on the loading and moment graphs, (h) fuel requirements for the weight categories identified in part “g”, (i) total time to destination and the via points, (j) GS, CAS, and IAS using a flight sheet or digital computer, (k) true and magnetic tracks and headings, (l) radio frequency information throughout the trip, (m) FBOs, (n) copy of the CFS data along with the en route maps and points of interest, (o) highlight the data chosen on the performance data sheet for the Cessna 172M, and (p) trip range based on the endurance data to be reported to the FSS.

In addition to the previous points, for an actual flight you would also call FSS and let them know about your flight plan intentions, file a day VFR flight plan with them, and ask for a weather debrief. You would also have a debrief session with the CFI (or FIC) and review the en route weather and data, and decide if the trip is a *go-no-go* one.

The flight plan itinerary is presented in Figure 125, Figure 126, and Figure 127. Figure 125 presents the en route data, including the pressure altitude (p_al, top-left), temperature, via points, variations, airspeeds (IAS, CAS, and TAS), wind information (magnitude and direction), headings (true, magnetic, and compass), climb data (time, distance, and fuel consumption), and cruise information (time, distance, ground speed, and rate of fuel consumption). Similar to the previous case studies, the reader may employ the TAS, true track, and wind data in order to calculate the ground speeds and true headings. Given the variations and compass card calibrated data, magnetic and compass headings may be obtained. The fuel required for each segment of the trip can be calculated as the next step. The total fuel and time in between via points is the summation of those of the flight segments. At the bottom of Figure 125, weight-and-balance data related to the normal and utility takeoff and landing on both CYGK and CYHM are presented. Additional weight-and-balance calculations are performed to identify the fuel usage after 1.8 hrs of the flight time as well as the ramp-off fuel data where extra fuel is considered for the aircraft ground handling before taking off.

Figure 126 is the second page of the presented flight itinerary. The left side presents the flight plan (or flight itinerary) that is to be filed with the

Layla S. Mayboudi
The standard temperature is supposed to be 6C at 4500', for 15, (15-6)*9%/10C=9%

Page 2 of 91
BHP: 55%, RPM: 2300

Pre-Flight Planning

P_al	From - To	Alt	Temp (C)	TAS	CAS	IAS	True Track	Wind Dir.	Wind Vel.	True Hdg.	Vr (Kts)	Mag Hdg (Kts)	Compass Hdg	Ground Speed	Dist.	Time	Fuel Calculations (Gallons US)			
																	Start Fuel Used Off	Climb	Cruise	Circuit & Landing
353.0	CYKG	303									13W									
4595	CYKG-TOC	4500	15	100	92	93					13W				011	0:09	-	1.1	1.8	
4595	TOC-SHP	4500	15	100	92	93	250	30019	258	13W	271	272	87	001	0:01	6.5	-	0.1		
4595	SHP-CYTZ	4500	15	100	92	93	256	30019	263	11W	274	275	86	114	1:20	6.5	-	8.6		
4595	CYTZ-CYHM	4500	15	100	92	93	221	30019	231	10W	241	242	95	036	0:23	6.5	-	2.5		
770	CYHM	780								10W										

Pre - 60 30 60 90 120 150 180 210 240
Start 00 31 41 51 02 123 145 166 210 270

Sub Totals

Reserve (Mark 45 or 50 min): 1:53

Other Fuel Req's: +20%

Total Fuel Required: 3.3

Total Fuel Carried: 3.7

Fuel Reserve: 22.2

Fuel Reserve: 47.8

Fuel Reserve: 25.6

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CRUISING ALTITUDES

From - To	True Track	Magnetic Variation	Magnetic Track	Altitude
CYKG-TOC	-	13W	-	-
TOC-SHP	250	13W	263	4500
SHP-CYTZ	256	11W	267	4500
CYTZ-CYHM	221	10W	231	4500

Magnetic Track

VFR - VFR - Odd
Thousand Plus 500
Plus 500'

100° 231°
170° 210°

VFR above 3,000' AGL unless otherwise authorized by ATC

UTILITY WEIGHT AND BALANCE - RAMP OFF - CYKG

Scenario	Full	Take Off	Landing	Weight	Arm	Moment
EMPTY Weight	1515	40.02		1515	40.02	60630.3
Pilot and Front Pass	345	37		345	37	12765
Rear Passenger	0	73		0	73	0
Baggage Area 1	0	95		0	95	0
Baggage Area 2	0			0		0
Sub-Total	1860	39.46		1860	39.46	73395.3
FUEL	158	48		158	48	7584
TOTAL	2018	40.13		2018	40.13	80979.3

1.8 HOUR TEST FLIGHT WEIGHT AND BALANCE - CYKG

Scenario	Full	Take Off	Landing	Weight	Arm	Moment
EMPTY Weight	1515	40.02		1515	40.02	60630.3
Pilot and Front Pass	345	37		345	37	12765
Rear Passenger	0	73		0	73	0
Baggage Area 1	0	95		0	95	0
Baggage Area 2	0			0		0
Sub-Total	1860	39.46		1860	39.46	73395.3
FUEL	158	48		158	48	7584
TOTAL	2018	40.13		2018	40.13	80979.3

NORMAL WEIGHT AND BALANCE - Take Off - CYKG

Scenario	Full	Take Off	Landing	Weight	Arm	Moment
EMPTY Weight	1515	40.02		1515	40.02	60630.3
Pilot and Front Pass	345	37		345	37	12765
Rear Passenger	160	73		160	73	11680
Baggage Area 1	0	95		0	95	0
Baggage Area 2	0			0		0
Sub-Total	2020	42.12		2020	42.12	85075.3
FUEL	280	48		280	48	13440
TOTAL	2300	42.83		2300	42.83	98515.3

NORMAL WEIGHT AND BALANCE - Landing - CYHM

Scenario	Full	Take Off	Landing	Weight	Arm	Moment
EMPTY Weight	1515	40.02		1515	40.02	60630.3
Pilot and Front Pass	345	37		345	37	12765
Rear Passenger	160	73		160	73	11680
Baggage Area 1	0	95		0	95	0
Baggage Area 2	0			0		0
Sub-Total	2020	42.12		2020	42.12	85075.3
FUEL	280	48		280	48	13440
TOTAL	2300	42.83		2300	42.83	98515.3

Cross Country Plan CYKG

Scenario	Full	Take Off	Landing	Weight	Arm	Moment
EMPTY Weight	1515	40.02		1515	40.02	60630.3
Pilot and Front Pass	345	37		345	37	12765
Rear Passenger	160	73		160	73	11680
Baggage Area 1	0	95		0	95	0
Baggage Area 2	0			0		0
Sub-Total	2020	42.12		2020	42.12	85075.3
FUEL	280	48		280	48	13440
TOTAL	2300	42.83		2300	42.83	98515.3

CYKG RWY 25 (3909' x 100')

Scenario	Full	Take Off	Landing	Weight	Arm	Moment
EMPTY Weight	1515	40.02		1515	40.02	60630.3
Pilot and Front Pass	345	37		345	37	12765
Rear Passenger	160	73		160	73	11680
Baggage Area 1	0	95		0	95	0
Baggage Area 2	0			0		0
Sub-Total	2020	42.12		2020	42.12	85075.3
FUEL	280	48		280	48	13440
TOTAL	2300	42.83		2300	42.83	98515.3

CYHM RWY 30 (10006' x 200')

Scenario	Full	Take Off	Landing	Weight	Arm	Moment
EMPTY Weight	1515	40.02		1515	40.02	60630.3
Pilot and Front Pass	345	37		345	37	12765
Rear Passenger	160	73		160	73	11680
Baggage Area 1	0	95		0	95	0
Baggage Area 2	0			0		0
Sub-Total	2020	42.12		2020	42.12	85075.3
FUEL	280	48		280	48	13440
TOTAL	2300	42.83		2300	42.83	98515.3

FIGURE 125 Flight plan itinerary and en route points (page 1).

FIGURE 126 Flight plan itinerary and en route points (page 2).

FIGURE 126 Flight plan itinerary and en route points (page 2).

TRENTON TML CYGK									
128.4		122.5		(transfer information from "Pre-Flight Planning")					
From - To	MOCA	Alt.	RPM	IAS	Comp. Hdg.	Dist.	G/S	ETE	ETA
CYGK-SHP	2100	4500	2300	93	272	012	087	0:10	
SHP-CYTZ	3100	4500	2300	93	275	114	086	1:20	
CYTZ-CYHM	3100	4500	2300	93	242	036	095	0:23	
TOTAL Hours 1:53 hr + 2*10 minutes for TO/LA									
Total Fuel 22.2 gal									

In-Flight Log and Groundspeed Calculations				Engine START:		FitPlan OPEN:	
				Engine STOP:		FitPlan CLOSE:	
Time	Comp. Hdg.	Checkpoints	Elapsed Time	Dist.	Actual G/S	Dist. to Dest.	ETE
		Take-Off					
		Set Hdg at:					
	275	Newtonville		08		043	
	275	Bowmanville		09		034	
	275	Oshawa		08		026	
	275	Ajax		07		019	

Time	Comp. Hdg.	*Checkpoints	Elapsed Time	Dist.	Actual G/S	Dist. to Dest.	ETE
275		Take-Off					
275		Set Hdg at: CYTZ					
	242	Mississaga		07		029	
	242	Burlington		14		015	
	242	Edge of Ham.		06		009	

VFR Position Report: Hourly on 126.7						At Set-Heading:			
Aircraft Ident.	Position	Time	Altitude	VFR Flight Plan	Destination	1. Tune EN-ROUTE FREQ	5. Mixture - LEAN		
						2. Establish ALTITUDE	6. Heading Ind - SET		
Airport Information and ATIS Information						3. Set CRUISE POWER	7. Course - SET (Opening Angles)		
						4. Check CARB HEAT	8. Record TIME		
Airport	Tower or Unicom/Freq	Circuit Altitude	Runway	Circuit Direction	ATIS	Wind Direction	Wind Speed		
CYHM	125.0	780	06/24 L	128.1					
12*/30 L *RWY 12 BTWN						23-07 (RGT)			
TORONTO TERMINAL						ARR	DEP:119.7		
TORONTO CENTRE							PAL:119.7		

21/07/2017

FIGURE 127 Flight plan itinerary and en route points (page 3).

FIC or shared with the dispatch or the responsible person (e.g., the FIC or CFI). On this page, information such as name of the aircraft, the pilot, the flight time, takeoff and landing times, flight duration, estimated flight closing time, and fuel endurance are recorded. If the flight crew is to make stops anywhere enroute, the aerodrome information for these stops is to be provided as well. The estimated flight plan closing time is of utmost important since if the aircraft time of return is delayed, for whatever reason, the search and rescue team are to be dispatched for assistance in a timely fashion. The importance of filing a flight plan can never be overemphasized. Make sure at least one responsible and reliable person is aware of your flight itinerary. Crews of all levels of experience are to adhere to this requirement to ensure their safety and those of their passengers.

The right side of Figure 126 presents the list of the en route points in which the segments of the flight are specified along with the distances and time in between the points. As you fly between the points, the time over each point is to be carefully recorded and given the distances specified, the ground speed is to be estimated (distance traveled over the time). The ground speed then is used to predict the time of arrival to the final destination. You may also calculate the remaining fuel if the rate of fuel consumption is known. The latter may be estimated using the POH data.

The top-right section of Figure 126 presents a summary of the flight itinerary along with the en route flight altitudes, Minimum Obstacle Clearance Altitudes (MOCA), as well as IAS and distances between via points. The time and ground speed are then to be recorded and calculated. This page would be placed face up on your knee board where you are able to record the data when required in a timely fashion. The bottom-right section of Figure 126 presents the en route frequencies. You are also to record the pattern (circuit) elevations, along with the applicable procedures (e.g., right versus left patterns), and selected appropriate runways. It is a good idea to record the information of more than one runway in case a runway change is needed. Figure 127 presents the remainder of the via points along with the frequencies of the en route locations.

SPECIAL SCENARIOS

This section is dedicated to the calculations that do not necessarily depend on the performance charts but on your knowledge of physics and mathematics; however, they are part of the flight management, relevant primarily for long and also to some extent for short hauls. Your FIC or the passengers are to be briefed about the flight; for example, when you are going to perform an uphill takeoff or barrel roll moves. You must recognize the limits of your aircraft—even though the sky has no limits, your aircraft does and so do your passengers. Among important math calculations are Point of Safe Return (PSR) and Critical Point (CP). Even though you may not be working on your commercial license, it is still a good idea to know how to calculate them. The following case studies explore these scenarios together with the related examples.

11.1 Drift Correction Methods

There are three methods by which the drift from the desired course may be corrected: (a) the double-error method—to be used before reaching the halfway point of the trip, (b) the opening-closing angle method—used after passing the halfway point, and (c) flying directly to the destination if the bearing can be determined.

In the double-error method (Figure 128), you first estimate your drift angle, $\Delta\beta$, (also known as track error or opening angle). If you find that you are to the right of your desired course, at Point A (as shown in the figure),

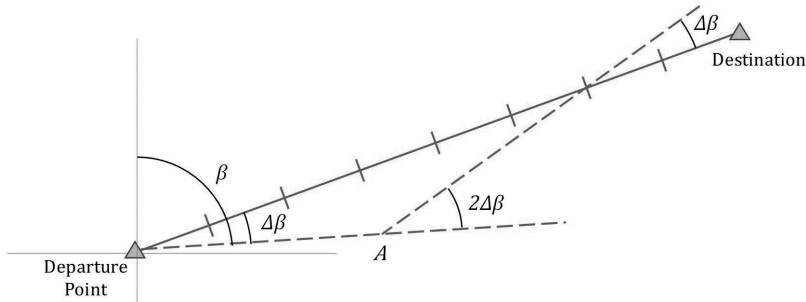


FIGURE 128 Double-error method for drift correction.

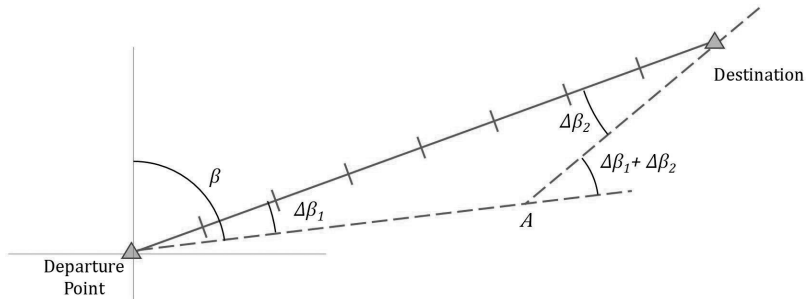


FIGURE 129 Opening-closing angle method for drift correction.

you double the estimated drift angle ($2\Delta\beta$), subtract it from your current bearing (β), $\beta - 2\Delta\beta$, fly on the new bearing for the same time that you have been drifting, and then turn to the original bearing by adding the original drift ($\beta - 2\Delta\beta + \beta = \beta - \Delta\beta$) to return to the track to your destination. Reverse the add (or subtract) instructions if you are to the left of the desired course, so that the new corrected course becomes $\beta + 2\Delta\beta$ and, after returning to the course, the bearing becomes $\beta + \Delta\beta$.

In the opening-closing angle method (Figure 129), for the case of drift to the right or left of the track, you estimate the opening angle (drift angle) from the Departure Point ($\Delta\beta_1$) and the closing angle to the destination ($\Delta\beta_2$), add the two angles, and subtract this sum from your current bearing (β) to get the new bearing $\beta - (\Delta\beta_1 + \Delta\beta_2)$.

11.2 Ground Speed

11.2.1 Ground Speed for the Desired Ground Track

In most cases, you would like to follow a particular course over the ground (track). Ground speed ($v_G = GS$) can be calculated knowing the wind direction (λ) and magnitude ($v_W = W'$) and the TAS ($v_T = TAS$). The

vector diagram used to obtain the ground speed and the crab angle for this case is shown in Figure 130. The wind arrow in the figure represents the velocity vector of the wind, which must be used in vector operations. This angle is 180° offset from the way the wind direction is specified in the weather reports. The angle of the wind (λ) shown in the diagram is relative to the track direction. Wind angles from zero degrees to 90° produce a tailwind component, while angles from 90° to 180° produce a headwind component (Figure 130).

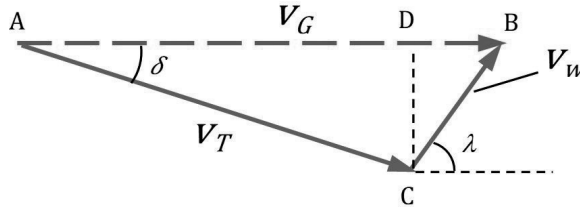


FIGURE 130 Ground speed calculation schematic (desired ground track case).

To calculate the ground speed, using trigonometry, observe that line CD in Figure 130 equals $(v_w \sin \lambda)$ and line DB equals $(v_w \cos \lambda)$. Also, from the Pythagorean relation, you can find line length AD from CD and AC. Adding these two lengths ($|AD| + |DB|$), you get the ground speed, equation (195).

$$v_G = \sqrt{v_T^2 - v_w^2 \sin^2 \lambda} + v_w \cos \lambda \quad (195)$$

The crab angle (δ) can be found from the inverse sine of the ratio of the CD length over v_T (line AC), equation (196).

$$\delta = \arcsin \left(\frac{v_w \sin \lambda}{v_T} \right) \quad (196)$$

The previous two equations can be generalized by introducing the concept of normalized parameters. In this case, all the speeds are normalized by TAS. This allows creation of normalized charts from which specific information can be extracted for any particular value of TAS. Thus, the *normalized wind* is the ratio of the wind speed to TAS ($\Omega_w = v_w/v_T$) and the *normalized ground speed* is the ratio of the ground speed to TAS ($\Omega_G = v_G/v_T$). Using these normalized quantities, one obtains equation (197).

$$\Omega_G = \sqrt{1 - \Omega_w^2 \sin^2 \lambda} + \Omega_w \cos \lambda \quad \text{and} \quad \delta = \arcsin(\Omega_w \sin \lambda) \quad (197)$$

The normalized ground speed is plotted in Figure 131 as a function of the normalized wind for a range of wind vector angles. Thus; for example, with the wind having only the tailwind component ($\lambda = 0$), and with a

normalized wind magnitude of 0.2 ($\Omega_w = 0.2$), you obtain as expected $\Omega_G = 1 + 0.2 = 1.2$, where Ω_G is a linearly increasing function of the normalized wind. Note though what happens when the wind is at 90° to track ($\lambda = 90^\circ$). For relatively small normalized wind values (up to 0.2), the normalized ground speed is at most 2 percent below one ($\Omega_G = 0.98$). Thus, you can say that the crosswind at 90° to track has minimum effect on the ground speed and can be approximated to equal *TAS*. However, at higher normalized wind values, the effect becomes more pronounced. For example, for *TAS* = 150 kt and with a 60 kt crosswind, $\Omega_w = 0.4$ and $\Omega_G = 0.92$ are calculated, and so the ground speed is decreased by 8 percent from *TAS*, giving a 137.5 kt ground speed. The curves in Figure 131 can be used to quickly gauge the effect of the wind on the ground speed.

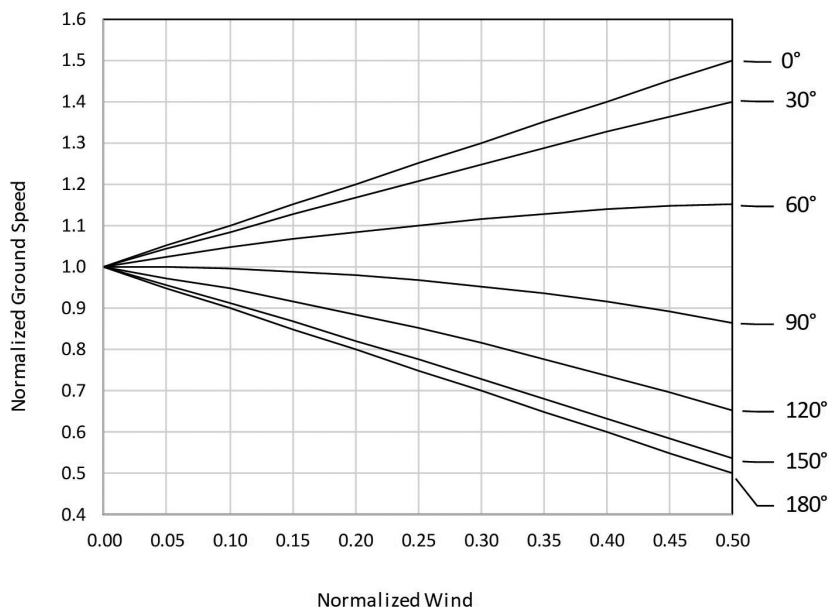


FIGURE 131 Normalized ground speed versus the normalized wind for different wind angles.

Similarly, a crab angle can be plotted as a function of the normalized wind and for different wind directions (Figure 132). Note that only wind angles up to 90° need to be shown. For wind angles (λ) from 90° to 180° , the crab angle will be the same as for the reciprocal of λ ($= 180 - \lambda$), since $\sin \lambda = \sin(180 - \lambda)$. For example, you can observe from the plot that the crab angle will be up to about 10° for normalized wind up to 0.18 with a pure crosswind angle. This means; for example, that for a *TAS* of 150 kt, this angle is produced by a wind speed of 27 kt.

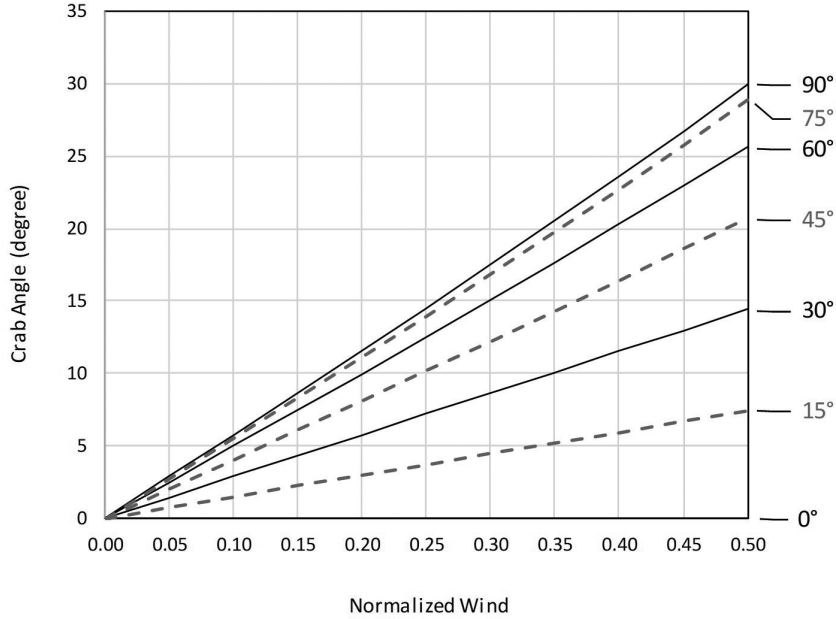


FIGURE 132 Crab angle versus the normalized wind magnitude for different wind angles.

11.2.2 Ground Speed Given a Constant Heading

There may be special circumstances when you may wish to maintain a particular aircraft heading, no matter the wind speed and direction. Perhaps you are monitoring some celestial sight and would like to keep at a constant angle to it. In this case, the ground speed and crab angle calculations are somewhat different than in the previously described case. The vector diagram for this case is shown in Figure 133. As before, the wind direction is referenced here to the true heading (direction of vector v_T). The magnitude of the ground speed (v_G) can be obtained as the hypotenuse of the triangle ABC. The distance AC is calculated by adding AD (v_T) and DC ($v_W \cos \lambda$). The distance BC is obtained as ($v_W \sin \lambda$). Equation (198) is obtained as a result.

$$v_G = \sqrt{(v_T + v_W \cos \lambda)^2 + v_W^2 \sin^2 \lambda} \quad (198)$$

This can be also written as follows using the identity ($\sin^2 \lambda + \cos^2 \lambda = 1$), equation (199).

$$v_G = \sqrt{v_T^2 + 2v_T v_W \cos \lambda + v_W^2} \quad (199)$$

The crab angle (δ) can be found from the inverse tangent of the ratio of the line BC length over the line AC, equation (200).

$$\delta = \arctan\left(\frac{v_W \sin \lambda}{v_T + v_W \cos \lambda}\right) \quad (200)$$

Converting to normalized wind and ground speed, the previous two equations are written as equation (201).

$$\Omega_G = \sqrt{1 + 2\Omega_W \cos \lambda + \Omega_W^2} \quad \text{and} \quad \delta = \arctan\left(\frac{\Omega_W \sin \lambda}{1 + \Omega_W \cos \lambda}\right) \quad (201)$$

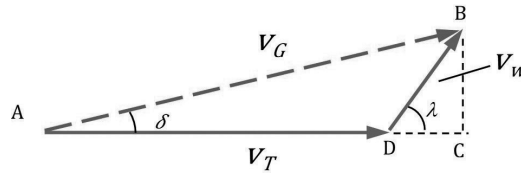


FIGURE 133 Ground speed calculation schematic (desired heading case).

As before, you can plot the previous normalized ground speed (Figure 134) and crab angle (Figure 135) versus the normalized wind for different wind directions. The plots show again that for zero degrees and 180° wind directions, the wind speed is added or subtracted, respectively, from the TAS. For wind at 90° to the heading, the result is different from the previous case of wind at 90° to track. Now, instead of a slight reduction of the ground speed from TAS, you obtain a slight increase. For example, using the same values as before, with $TAS = 150$ kt and with a 60 kt crosswind *relative to heading*,

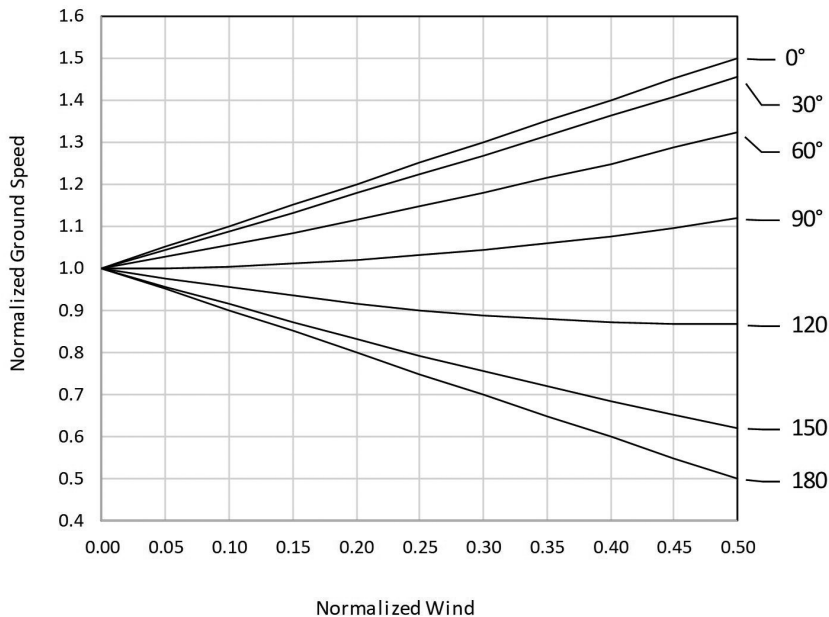


FIGURE 134 Normalized ground speed versus the normalized wind for different wind angles (constant heading case).

you get $\Omega_w = 0.4$ and $\Omega_c = 1.08$, and so the ground speed is *increased* by 8 percent from TAS, to give 161.6 kt.

For the crab angles in this case, there is a difference for the range from zero degrees to 90° and from 90° to 180° since there is a cosine function in the expression for the angle. As the normalized wind increases, the angles increase at a greater rate. For example, compare the curves for 15° and 165° . For 15° , there is mostly a tailwind, and for 165° , there is mostly a headwind, with the same 15° offset from the heading line. However, for the 165° , the crab angle rises to 14° for normalized wind of 0.5, while for 15° , at the same value, the angle is only 5° . As an example, looking again at the case of crosswind at 90° to heading, a 10° crab angle will occur for a normalized wind of 0.178, which for a TAS of 150 kt will be 26.4 kt (Figure 135).

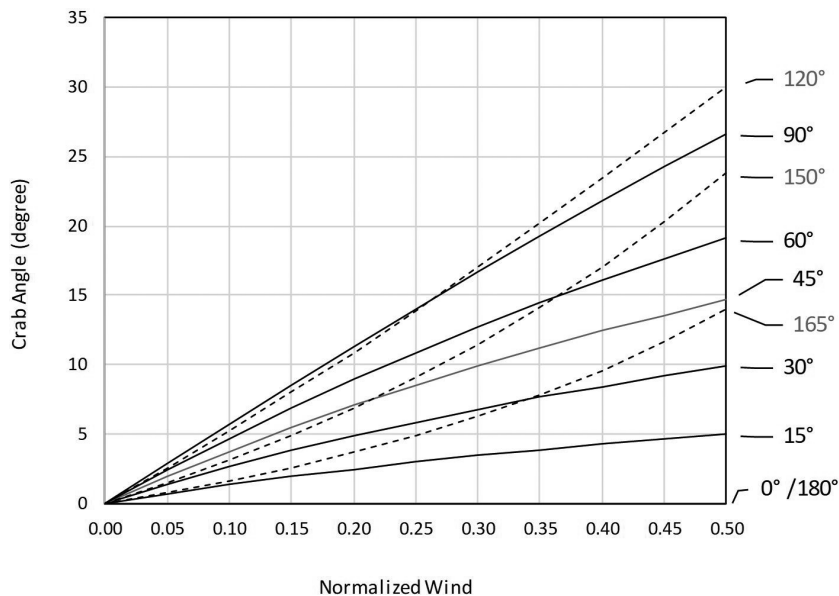


FIGURE 135 Crab angle versus the normalized wind magnitude for different wind angles (constant heading case).

11.3 Point of Safe Return

Point of Safe Return (PSR) is the maximum distance that the aircraft can travel with its available fuel from the departure point (outbound) and still return to the departure point (inbound) in one single haul. This scenario is most likely to apply when flying out over an ocean to a remote island. If the weather at the destination would not allow safe landing or the airport were to be closed for some other reason, the FIC would have no option

but to return to the departure point. PSR is the farthest point away from departure point from which it is still possible to return with the available fuel (while leaving some fuel as a reserve).

The flight out of the departure point (outbound) may be accompanied either by a headwind or a tailwind—and therefore the inbound flight direction will encounter the opposite wind conditions. Thus, to calculate PSR distance, the ground speed needs to be considered. The simplest scenario is when the aircraft is flying in still air; in this situation, the time and also distance to get to PSR are the same for both legs (outbound and inbound). Since the maximum distance depends on the fuel available, the first step is to estimate the safe endurance—the fuel available in the tank to remain in the air without consuming the 30 to 45 min of reserve fuel, discretionary fuel, precautionary fuel, and the fuel needed for the groundwork such as taxiing. Thus, each of these items are to be calculated individually to see how much fuel is available. Given the average fuel consumption for the trip (GPH), the total time possible can be calculated in the air (t_t) given the fuel (F) available ($t_t = F/GPH$).

Given the true airspeed ($v_T = TAS$), the wind component along the track direction ($v'_W = v_W \cos \lambda = w' \cos \lambda$) and total time (t_t) available, you need to calculate the distance from the departure point to PSR (x) and the outbound time ($t_{outbound}$). For the wind, assume a positive value for the case of an outbound tailwind and a negative value for the case of an outbound headwind. To obtain the answer, you need to use two relationships: (a) outbound distance (x) must equal the inbound distance from PSR, and (b) the sum of outbound time and inbound time ($t_{inbound} + t_{outbound}$) must equal the total time (t_t) available in the air.

The time and PSR distance for no-wind conditions can be expressed as shown in equation (202), where subscript “ nw ” indicates the no-wind scenario.

$$t_{nw} = \frac{t_t}{2} \text{ and } x_{nw} = v_T t_{nw} = v_T \frac{t_t}{2} \quad (202)$$

It is also useful to define the difference between the time (Δt) and distance (Δx) with wind versus the no-wind conditions, equations (203) and (204).

$$\Delta t = t_{outbound} - t_{nw} = t_{outbound} - \frac{t_t}{2} \quad (203)$$

$$\Delta x = x - x_{nw} = x - v_T \frac{t_t}{2} \quad (204)$$

Since the distance outbound equals the return distance from PSR (i.e., inbound distance), equation (205) may be obtained.

$$(v_T + v'_W)t_{outbound} = (v_T - v'_W)t_{inbound} \quad (205)$$

In the equation (205), knowing that the total time is the sum of the outbound and inbound times, you can substitute as follows: $t_{inbound} = t_t - t_{outbound}$. Rearranging the terms, equation (206) is obtained.

$$t_{outbound} = \left(\frac{v_T - v'_W}{v_T} \right) \frac{t_t}{2} \quad (206)$$

It is useful to define the ratio of the wind speed component to the TAS, that is, the normalized wind, as $(\mathcal{Q}'_W = v'_W / v_T)$, so that equation (206) can be restated as equation (207).

$$t_{outbound} = \left(1 - \mathcal{Q}'_W \right) \frac{t_t}{2} \quad (207)$$

This also gives $\Delta t = -\mathcal{Q}'_W \frac{t_t}{2}$. You can derive the inbound time by equation (208) as the next step.

$$t_{inbound} = \left(1 + \mathcal{Q}'_W \right) \frac{t_t}{2} \quad (208)$$

The distance to PSR can then be obtained, equation (209).

$$x = (v_T + v'_W)t_{outbound} = (1 + \mathcal{Q}'_W)v_T t_{outbound} \quad (209)$$

Substituting in equation (209) from equation (206), equation (210) is obtained.

$$x = \left(1 - \mathcal{Q}_W'^2 \right) v_T \frac{t_t}{2} \quad (210)$$

From equation (210), it may be deduced that $\Delta x = -\mathcal{Q}_W'^2 v_T \frac{t_t}{2}$. It is instructive to also define normalized quantities for time and distance as equation (211).

$$\mathcal{Q}_t = \Delta t / t_t \text{ and } \mathcal{Q}_x = \Delta x / x_{nu} \quad (211)$$

From previously obtained relationships, equation (211) can be expressed by equation (212).

$$\mathcal{Q}_t = -\mathcal{Q}_W'^2 / 2 \text{ and } \mathcal{Q}_x = -\mathcal{Q}_W'^2 \quad (212)$$

The reader is encouraged to confirm that whatever the outbound or inbound distances are, the expression in (210) is valid.

From the equations obtained, you can conclude that, with an outbound tailwind, the time to PSR is proportionately reduced with increasing wind speed while the return trip time is proportionally extended. You can estimate this difference using equations (207) and (208). Note also that the distance to PSR is always reduced as the wind magnitude increases as shown by equation (210), and this reduction is proportionate to the square of the

wind-to-TAS ratio (Ω_w'). The normalized distance reduction is plotted in Figure 136 and shows an inverse parabolic shape. The hatched area under the fuel consumption (GPH) versus ($1/TAS$) diagram is the fuel usage per distance (GPM) and may also be used to calculate PSR (Figure 137).

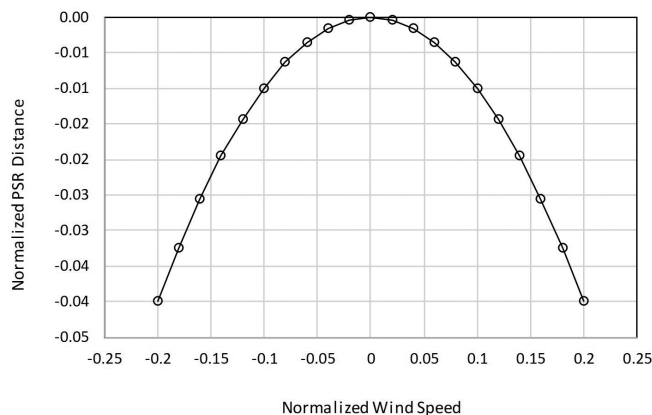


FIGURE 136 Relationship between the normalized point of safe return (PSR) distance reduction and normalized wind speed.

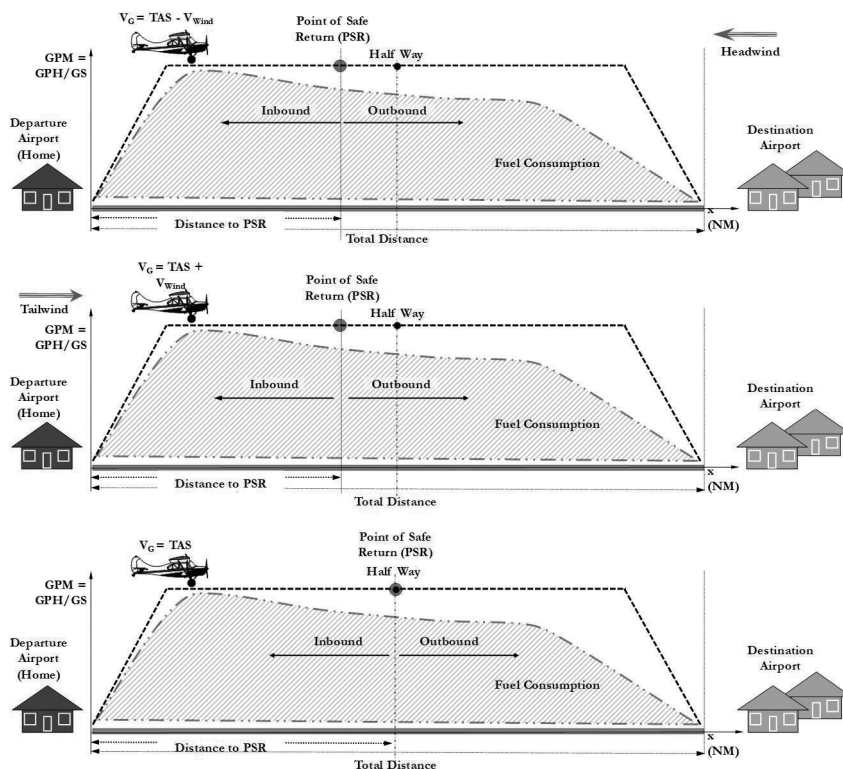


FIGURE 137 Point of safe return (PSR) and fuel consumption per unit distance versus the distance for: (a) headwind, (b) tailwind, and (c) no wind (drawings created using Solid Edge CAD tool).

11.3.1 Approximate versus the Exact Ground Speed

Note that in the previous calculations for PSR, an approximation was made in estimation of the ground speed as the sum of the TAS and the wind component along the track direction. You can examine here under what conditions this approximation is reasonable. The approximate derivation resulted in the estimation of the outbound time as equation (213).

$$\tilde{t}_{outbound} = (1 - \mathcal{Q}_W \cos \lambda) \frac{t_t}{2} \quad (213)$$

If you use the exact expression for the ground speed, equation (214) can be derived:

$$t_{outbound} = \left(1 - \frac{\mathcal{Q}_W \cos \lambda}{\sqrt{1 - \mathcal{Q}_W^2 \sin^2 \lambda}} \right) \frac{t_t}{2} \quad (214)$$

You can estimate the relative effect of the approximation in equation (213) by dividing the difference between the approximate and exact calculations by the total time available in the air (t_t). Carrying out the necessary derivations, equation (215) is obtained:

$$\frac{\tilde{t}_{outbound} - t_{outbound}}{t_t} = \frac{\mathcal{Q}_W \cos \lambda}{2} \left(\frac{1}{\sqrt{1 - \mathcal{Q}_W^2 \sin^2 \lambda}} - 1 \right) \quad (215)$$

Plotting this error versus the normalized wind and for several wind angles results in the curves in Figure 138. The plot indicates that if the normalized wind is not extreme (e.g., below 0.3), the approximation is quite accurate, with the time estimate error below 0.3 percent relative to the total time available in the air.

For the distance from PSR back to the destination, the approximate expression, (derived previously as equation (210)), is restated here as equation (216):

$$\tilde{x} = (1 - \mathcal{Q}_W^2 \cos^2 \lambda) v_T \frac{t_t}{2} \quad (216)$$

Using the exact calculation of the ground speed, the distance can be obtained using equation (217).

$$x = \left[\frac{1 - \mathcal{Q}_W^2 \cos^2 \lambda - \mathcal{Q}_W^2 \sin^2 \lambda}{\sqrt{1 - \mathcal{Q}_W^2 \sin^2 \lambda}} \right] v_T \frac{t_t}{2} \quad (217)$$

The exact distance from PSR can be compared to the approximate estimate by equation (218).

$$PSR \text{ Distance Error} = \frac{\tilde{x} - x}{x_{mv}} = \frac{(\tilde{x} - x)^2}{v_T t_t} \quad (218)$$

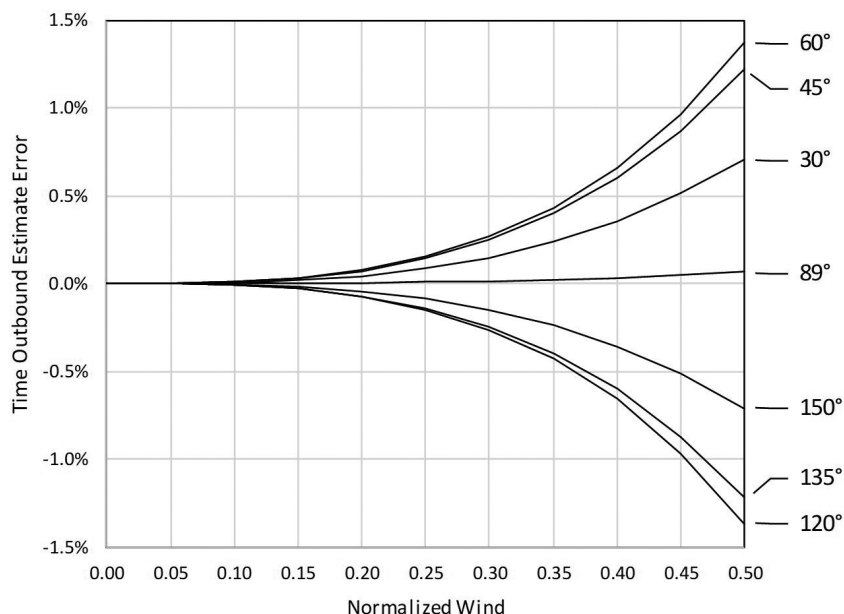


FIGURE 138 Estimated error in calculation of outbound time for point of safe return (PSR).

Plotting the previous distance estimate error, you obtain the curves in Figure 139. For a wind angle of zero degrees, there is no difference between the exact and approximate estimate, as expected. As the angle and the normalized wind increase, the error increases as well. However, as one approaches a 90° wind angle, the wind component along the track approaches zero degrees, but the error (versus angle) then increases to maximum value. One can explain this by considering that the distance is estimated as a product of ground speed and time to PSR. The approximate and exact estimates of the time are the same (i.e., error zero). But the ground speed is reduced (from the TAS) by the crosswind (as was shown in Figure 139). Thus, the total distance covered in the same time will be reduced as well, and there is an error in the approximation. For normalized wind below 0.3, this error reaches about 5 percent, which is of some significance and should be considered when making PSR calculations.

11.4 Critical Point

Critical Point (CP), also known as Point of Equal Time (PET) or Equal Time Point (ETP), is a point between the departure and destination locations where it will take the same time to continue to the destination as it will take to return to the departure point (Figure 140). CP must be particularly

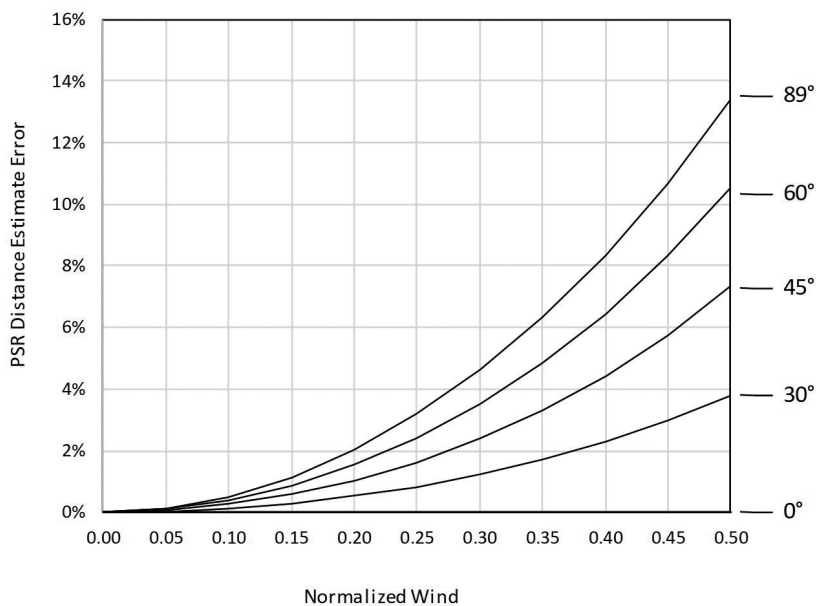


FIGURE 139 Error in calculation of distance from point of safe return (PSR) to the departure point.

carefully considered when flying over an open ocean, a mountainous area, or other areas with limited diversion opportunities. In these situations, the FIC must consider whether the aircraft should go any farther given the current conditions, and the answer depends on the elapsed time and distance from the departure point. If there is no wind, outbound and inbound, the ground speed is the same for both the outbound and inbound directions. However, outbound headwind means an inbound tailwind (and vice versa) that need to be considered when performing the calculations.

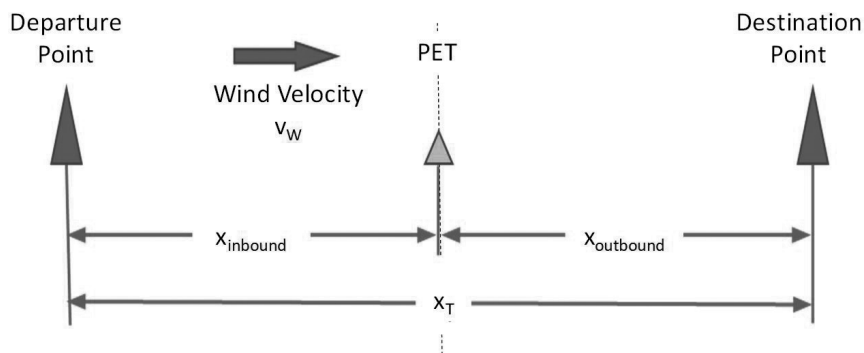


FIGURE 140 Critical point (CP) along with the related distances.

Halfway points are shown in Figure 141 for the headwind, tailwind, and no-wind scenarios. The time and distance to CP may be derived knowing that at CP, the time to return to the departure point ($t_{inbound}$) equals the time to continue to the destination point ($t_{outbound}$). Since the travel time equals distance divided by speed, the ratio of the inbound (return) distance ($x_{inbound}$) to the ground velocity must equal the ratio of the outbound distance ($x_{outbound}$) to the ground velocity, as in Equation (219).

$$\frac{x_{inbound}}{v_T - v_{W-in}} = \frac{x_{outbound}}{v_T + v_{W-out}} \quad (219)$$

The previous equation uses different values for the inbound (v_{W-in}) and outbound (v_{W-out}) winds. This may be the case; for example, when flying over the ocean (e.g., Atlantic Ocean) and the wind direction is different for the Departure-to-CP versus CP-to-Destination segments. The outbound tailwind direction is considered to be positive in the equations here.

Defining the normalized wind as before for inbound and outbound segments, $\Omega_{W-in} = v_{W-in} / v_T$ and $\Omega_{W-out} = v_{W-out} / v_T$, and using equation (219), one can express CP-to-Departure distance by equation (220).

$$x_{inbound} = \frac{x_t (1 - \Omega_{W-in})}{2 + (\Omega_{W-out} - \Omega_{W-in})} \quad (220)$$

The previous equation uses the total distance from departure to destination equal to the sum of the inbound and outbound distances: $x_t = x_{inbound} + x_{outbound}$. If the inbound and outbound wind speeds are assumed equal, equation (220) simplifies to equation (221).

$$x_{inbound} = (1 - \Omega_W) \frac{x_t}{2} \quad (221)$$

Equation (221) shows that the distance from CP to the departure point decreases linearly with the increase in the ratio of the wind speed to the TAS. When wind is blowing from the departure point to CP, CP location shifts toward the direction from which the wind is blowing.

As part of flight planning for flights over the ocean (e.g., Atlantic Ocean), where the diversion opportunities are limited, the crew is to calculate CP based on the single engine scenario that usually results in the loss in altitude and the decrease of TAS. This distance is then employed to calculate the time using the original TAS (before the engine failure or altitude reduction).

Figure 141 shows a diagram of CP for the three possible cases: headwind, tailwind, and no wind. Note that CP shifts toward the direction that the wind is coming from. As you are flying into the headwind, you are traveling slower, but when returning, you will be traveling faster. So, you can go

farther ahead, counting on being able to travel in the opposite direction with a faster ground speed. The hatched area under the fuel consumption ($GPM = GPH/GS$) versus the distance (x) in the diagram represents the total fuel used during the trip.

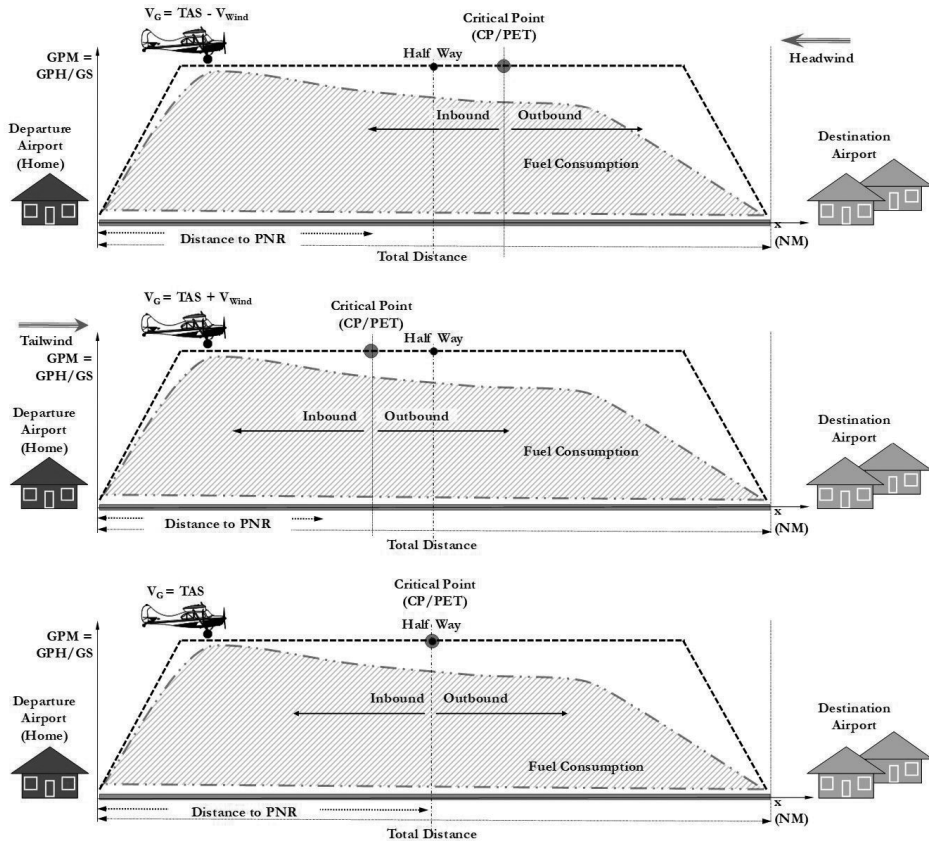


FIGURE 141 Critical point (CP) and fuel consumption per unit distance versus the distance for: (a) headwind, (b) tailwind, and (c) no wind (drawings created using Solid Edge CAD tool).

11.5 Precision Approaches

Precision approaches are instrument-based for approach and landing, which include vertical and lateral guidance and follow certain requirements (minima). These requirements may include restrictions on the Decision Altitude (DA) or Decision Height (DH), Runway Visual Range (RVR), Runway Vertical Visibility (RVV), and overall visibility. The instrument used as a navigation aid may either be located on the ground, inside the cockpit of the aircraft, or displayed on the air-traffic controller's radar screen and communicated to the pilot. There are number of operation categories for

approach and landing within this group, varying from Category 1 (CAT I) to Category 3 (CAT IIIC). For example, for the former case, the minimum DH is 60 m (197 ft) while for the latter case, no restrictions apply. With an increasing category number (i.e., I, II, IIIA, IIIB, IIIC), the limitations reduce (i.e., $DH < 30$ m (98 ft), $RVR > 200$ m (656 ft) for CAT IIIA). In all scenarios, the visibility is not to be less than 800 m (2624 ft) [231].

11.6 Non-Precision Approaches

It is useful to know how to make non-precision approaches and determine the related vertical path. A non-precision approach is a standard instrument approach where only lateral (horizontal) guidance is provided (e.g., Lateral Navigation (LNAV) and Localizer) as opposed to a precision approach where both lateral and vertical guidance are provided (e.g., Instrument Landing System (ILS) and Localizer Performance with Vertical guidance (LPV)). There are standard practices for executing a non-precision approach such as the guidance provided in the Advisory Circular (AC) No. 700-028 by NAV Canada.

Note that the information presented by this advisory and those of a similar nature are mainly suggested proven techniques and are the only methods of compliance with the regulations. The main purpose of such advisories is to improve safety. The main safety concern is related to the Controlled Flight Into Terrain (CFIT) in which an aircraft is flown into terrain, water, or an obstacle without the pilot's prior awareness while maintaining the control of the aircraft.

One of the methods used to fly the final approach segment of a non-precision instrument approach procedure is Constant Descent Final Approach (CDFA) and is in the form of a continuous descent, without leveling-off, from the glide slope intercept until the runway threshold or missed approach point. This method is consistent with stabilized approach procedures and is based on ICAO recommendations. The Final Approach Fix (FAF) is normally at a distance of about 5 NM (9.3 km) from the runway threshold, and the glideslope is at a 3° angle or a gradient of 5.2 percent, providing a descent of 52 m for each km of ground travel. The maximum descent gradient may vary, depending on the category of the aircraft (e.g., 6.5 percent for Category A and B, aircraft, and 10 percent for category H, helicopter). The rate of descent is to be selected so that a continuous descent is achieved to a point that is approximately at 50 ft (15 m) above the landing runway threshold, where the flare maneuver begins [232].

The stabilized approach conditions are achieved when power settings, range of speeds, altitudes, configurations, sink rates, and completion of routine landing checklists specific to the aircraft type comply with the recommended aerodrome approach procedures. The procedures are to be completed and the conditions are to be met at the altitude range recommended, depending on whether flying in the visual or IFR's conditions (e.g., between 500 ft and 1,000 ft above the landing threshold for the IFR conditions) and remain stabilized throughout; otherwise, a go-around or bailed landing is recommended.

The three recommended techniques for vertical flight control for non-precision approaches are: (a) step-down; (b) constant descent angle, also known as Constant Angle of Descent (CAD)—the latter term is used by ICAO; and (c) Stabilized Constant Descent Angle (SCDA), also known as CDFA—the latter term is used by ICAO. Descent gradient, angle, and rate of descent are the three main parameters to be estimated when performing non-precision approaches. Descent angles are normally specified on the approach charts.

The descent gradient is the ratio of the altitude difference to the horizontal distance covered while descending, expressed in ft/NM in equation (222). The descent rate, expressed in fpm (feet per minute) in equation (223), is a product of the descent gradient and ground speed divided by 60. To calculate the constant descent angle, you are to first calculate the elevation difference between the FAF crossing altitudes and step-down or the runway elevation along with the corresponding ground distances. Extract the constant descent angle or descent gradient using the related tables, Table 73 and Table 74. Using the constant descent angle extracted in the previous step, extrapolate or interpolate the descent rate using Table 75. If tables are not available, you may divide the elevation difference between the altitudes of the FAF and the step-down or the runway by the distance between the two locations to find the descent angle by obtaining the inverse tangent of this ratio, equation (224). Note to make sure the conversions are performed properly (1 NM = 6,076.12 ft).

$$\text{Descent Gradient} \left(\frac{\text{ft}}{\text{NM}} \right) = \left(\frac{y_{\text{step-down}} - y_{\text{FAF, Runway}} (\text{ft})}{x_{\text{step-down}} - x_{\text{FAF, Runway}} (\text{NM})} \right) \quad (222)$$

$$\text{Descent Rate} (\text{fpm}) = \text{Descent Gradient} \left(\frac{\text{ft}}{\text{NM}} \right) \frac{\text{GS} (\text{kt})}{60} \quad (223)$$

$$\text{Descent Angle} (^{\circ}) = \frac{180}{\pi} \arctan \left(\text{Descent Gradient} \left(\frac{\text{ft}}{\text{NM}} \right) \frac{1}{6076.12} \right) \quad (224)$$

TABLE 73 Descent angle (°) versus the altitude difference (ft) and horizontal distance difference (NM).

Altitude (ft)	Distance (NM)												
	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
5,000.00	22.36	15.34	11.62	9.35	7.81	6.70	5.87	5.22	4.70	4.28	3.92	3.62	3.36
4,500.00	20.32	13.87	10.49	8.43	7.04	6.04	5.29	4.70	4.24	3.85	3.53	3.26	3.03
4,000.00	18.22	12.38	9.35	7.50	6.26	5.37	4.70	4.18	3.77	3.42	3.14	2.90	2.69
3,500.00	16.07	10.87	8.19	6.57	5.48	4.70	4.12	3.66	3.30	3.00	2.75	2.54	2.36
3,000.00	13.87	9.35	7.04	5.64	4.70	4.03	3.53	3.14	2.83	2.57	2.36	2.18	2.02
2,500.00	11.62	7.81	5.87	4.70	3.92	3.36	2.94	2.62	2.36	2.14	1.96	1.81	1.68
2,000.00	9.35	6.26	4.70	3.77	3.14	2.69	2.36	2.09	1.89	1.71	1.57	1.45	1.35
1,900.00	8.89	5.95	4.47	3.58	2.98	2.56	2.24	1.99	1.79	1.63	1.49	1.38	1.28
1,800.00	8.43	5.64	4.24	3.39	2.83	2.42	2.12	1.89	1.70	1.54	1.41	1.31	1.21
1,700.00	7.96	5.33	4.00	3.20	2.67	2.29	2.00	1.78	1.60	1.46	1.34	1.23	1.14
1,600.00	7.50	5.02	3.77	3.01	2.51	2.15	1.89	1.68	1.51	1.37	1.26	1.16	1.08
1,500.00	7.04	4.70	3.53	2.83	2.36	2.02	1.77	1.57	1.41	1.29	1.18	1.09	1.01
1,400.00	6.57	4.39	3.30	2.64	2.20	1.89	1.65	1.47	1.32	1.20	1.10	1.02	0.94
1,300.00	6.11	4.08	3.06	2.45	2.04	1.75	1.53	1.36	1.23	1.11	1.02	0.94	0.88
1,200.00	5.64	3.77	2.83	2.26	1.89	1.62	1.41	1.26	1.13	1.03	0.94	0.87	0.81
1,100.00	5.17	3.45	2.59	2.07	1.73	1.48	1.30	1.15	1.04	0.94	0.86	0.80	0.74
1,000.00	4.70	3.14	2.36	1.89	1.57	1.35	1.18	1.05	0.94	0.86	0.79	0.73	0.67
900.00	4.24	2.83	2.12	1.70	1.41	1.21	1.06	0.94	0.85	0.77	0.71	0.65	0.61
800.00	3.77	2.51	1.89	1.51	1.26	1.08	0.94	0.84	0.75	0.69	0.63	0.58	0.54
700.00	3.30	2.20	1.65	1.32	1.10	0.94	0.83	0.73	0.66	0.60	0.55	0.51	0.47
600.00	2.83	1.89	1.41	1.13	0.94	0.81	0.71	0.63	0.57	0.51	0.47	0.44	0.40
500.00	2.36	1.57	1.18	0.94	0.79	0.67	0.59	0.52	0.47	0.43	0.39	0.36	0.34
400.00	1.89	1.26	0.94	0.75	0.63	0.54	0.47	0.42	0.38	0.34	0.31	0.29	0.27
300.00	1.41	0.94	0.71	0.57	0.47	0.40	0.35	0.31	0.28	0.26	0.24	0.22	0.20
200.00	0.94	0.63	0.47	0.38	0.31	0.27	0.24	0.21	0.19	0.17	0.16	0.15	0.13
100.00	0.47	0.31	0.24	0.19	0.16	0.13	0.12	0.10	0.09	0.09	0.08	0.07	0.07

TABLE 74 Descent gradient (ft/NM) versus the altitude difference (ft) and horizontal distance difference (NM).

Altitude (ft)	Distance (NM)												
	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
5,000.00	2,500.00	1,666.67	1,250.00	1,000.00	833.33	714.29	625.00	555.56	500.00	454.55	416.67	384.62	357.14
4,500.00	2,250.00	1,500.00	1,125.00	900.00	750.00	642.86	562.50	500.00	450.00	409.09	375.00	346.15	321.43
4,000.00	2,000.00	1,333.33	1,000.00	800.00	666.67	571.43	500.00	444.44	400.00	363.64	333.33	307.69	285.71
3,500.00	1,750.00	1,166.67	875.00	700.00	583.33	500.00	437.50	388.89	350.00	318.18	291.67	269.23	250.00
3,000.00	1,500.00	1,000.00	750.00	600.00	500.00	428.57	375.00	333.33	300.00	272.73	250.00	230.77	214.29
2,500.00	1,250.00	833.33	625.00	500.00	416.67	357.14	312.50	277.78	250.00	227.27	208.33	192.31	178.57
2,400.00	1,200.00	800.00	600.00	480.00	400.00	342.86	300.00	266.67	240.00	218.18	200.00	184.62	171.43
2,300.00	1,150.00	766.67	575.00	460.00	383.33	328.57	287.50	255.56	230.00	209.09	191.67	176.92	164.29
2,200.00	1,100.00	733.33	550.00	440.00	366.67	314.29	275.00	244.44	220.00	200.00	183.33	169.23	157.14
2,100.00	1,050.00	700.00	525.00	420.00	350.00	300.00	262.50	233.33	210.00	190.91	175.00	161.54	150.00
2,000.00	1,000.00	666.67	500.00	400.00	333.33	285.71	250.00	222.22	200.00	181.82	166.67	153.85	142.86
1,900.00	950.00	633.33	475.00	380.00	316.67	271.43	237.50	211.11	190.00	172.73	158.33	146.15	135.71
1,800.00	900.00	600.00	450.00	360.00	300.00	257.14	225.00	200.00	180.00	163.64	150.00	138.46	128.57
1,700.00	850.00	566.67	425.00	340.00	283.33	242.86	212.50	188.89	170.00	154.55	141.67	130.77	121.43
1,600.00	800.00	533.33	400.00	320.00	266.67	228.57	200.00	177.78	160.00	145.45	133.33	123.08	114.29
1,500.00	750.00	500.00	375.00	300.00	250.00	214.29	187.50	166.67	150.00	136.36	125.00	115.38	107.14
1,400.00	700.00	466.67	350.00	280.00	233.33	200.00	175.00	155.56	140.00	127.27	116.67	107.69	100.00
1,300.00	650.00	433.33	325.00	260.00	216.67	185.71	162.50	144.44	130.00	118.18	108.33	100.00	92.86
1,200.00	600.00	400.00	300.00	240.00	200.00	171.43	150.00	133.33	120.00	109.09	100.00	92.31	85.71
1,100.00	550.00	366.67	275.00	220.00	183.33	157.14	137.50	122.22	110.00	100.00	91.67	84.62	78.57
1,000.00	500.00	333.33	250.00	200.00	166.67	142.86	125.00	111.11	100.00	90.91	83.33	76.92	71.43
900.00	450.00	300.00	225.00	180.00	150.00	128.57	112.50	100.00	90.00	81.82	75.00	69.23	64.29
800.00	400.00	266.67	200.00	160.00	133.33	114.29	100.00	88.89	80.00	72.73	66.67	61.54	57.14
700.00	350.00	233.33	175.00	140.00	116.67	100.00	87.50	77.78	70.00	63.64	58.33	53.85	50.00
600.00	300.00	200.00	150.00	120.00	100.00	85.71	75.00	66.67	60.00	54.55	50.00	46.15	42.86
500.00	250.00	166.67	125.00	100.00	83.33	71.43	62.50	55.56	50.00	45.45	41.67	38.46	35.71
400.00	200.00	133.33	100.00	80.00	66.67	57.14	50.00	44.44	40.00	36.36	33.33	30.77	28.57
300.00	150.00	100.00	75.00	60.00	50.00	42.86	37.50	33.33	30.00	27.27	25.00	23.08	21.43
200.00	100.00	66.67	50.00	40.00	33.33	28.57	25.00	22.22	20.00	18.18	16.67	15.38	14.29
100.00	50.00	33.33	25.00	20.00	16.67	14.29	12.50	11.11	10.00	9.09	8.33	7.69	7.14

TABLE 75 Descent rate (fpm) versus the ground speed (kt) and descent angle (°).

CDA (°)	Ground Speed (kt)													
	Gradient (ft/NM)	60.0	80.0	100.0	120.0	140.0	160.0	180.0	200.0	220.0	240.0	260.0	280.0	300.0
1.0	106.0	106.0	141.3	176.7	212.0	247.3	282.7	318.0	353.3	388.7	424.0	459.3	494.7	530.0
1.5	159.0	159.0	212.0	265.0	318.0	371.0	424.0	477.0	530.0	583.0	636.0	689.0	742.0	795.0
2.0	212.0	212.0	282.7	353.3	424.0	494.7	565.3	636.0	706.7	777.3	848.0	918.7	989.3	1,060.0
2.5	265.0	265.0	353.3	441.7	530.0	618.3	706.7	795.0	883.3	971.7	1,060.0	1,148.3	1,236.7	1,325.0
2.7	287.0	287.0	382.7	478.3	574.0	669.7	765.3	861.0	956.7	1,052.3	1,148.0	1,243.7	1,339.3	1,435.0
2.8	297.0	297.0	396.0	495.0	594.0	693.0	792.0	891.0	990.0	1,089.0	1,188.0	1,287.0	1,386.0	1,485.0
2.9	308.0	308.0	410.7	513.3	616.0	718.7	821.3	924.0	1,026.7	1,129.3	1,232.0	1,334.7	1,437.3	1,540.0
3.0	318.0	318.0	424.0	530.0	636.0	742.0	848.0	954.0	1,060.0	1,166.0	1,272.0	1,378.0	1,484.0	1,590.0
3.1	329.0	329.0	438.7	548.3	658.0	767.7	877.3	987.0	1,096.7	1,206.3	1,316.0	1,425.7	1,535.3	1,645.0
3.2	340.0	340.0	453.3	566.7	680.0	793.3	906.7	1,020.0	1,133.3	1,246.7	1,360.0	1,473.3	1,586.7	1,700.0
3.3	350.0	350.0	466.7	583.3	700.0	816.7	933.3	1,050.0	1,166.7	1,283.3	1,400.0	1,516.7	1,633.3	1,750.0
3.4	361.0	361.0	481.3	601.7	722.0	842.3	962.7	1,083.0	1,203.3	1,323.7	1,444.0	1,564.3	1,684.7	1,805.0
3.5	372.0	372.0	496.0	620.0	744.0	868.0	992.0	1,116.0	1,240.0	1,364.0	1,488.0	1,612.0	1,736.0	1,860.0
3.6	382.0	382.0	509.3	636.7	764.0	891.3	1,018.7	1,146.0	1,273.3	1,400.7	1,528.0	1,655.3	1,782.7	1,910.0
3.7	393.0	393.0	524.0	655.0	786.0	917.0	1,048.0	1,179.0	1,310.0	1,441.0	1,572.0	1,703.0	1,834.0	1,965.0
3.8	404.0	404.0	538.7	673.3	808.0	942.7	1,077.3	1,212.0	1,346.7	1,481.3	1,616.0	1,750.7	1,885.3	2,020.0
3.9	414.0	414.0	552.0	690.0	828.0	966.0	1,104.0	1,242.0	1,380.0	1,518.0	1,656.0	1,794.0	1,932.0	2,070.0
4.0	425.0	425.0	566.7	708.3	850.0	991.7	1,133.3	1,275.0	1,416.7	1,558.3	1,700.0	1,841.7	1,983.3	2,125.0
4.5	478.0	478.0	637.3	796.7	956.0	1,115.3	1,274.7	1,434.0	1,593.3	1,752.7	1,912.0	2,071.3	2,230.7	2,390.0
5.0	532.0	532.0	709.3	886.7	1,064.0	1,241.3	1,418.7	1,596.0	1,773.3	1,950.7	2,128.0	2,305.3	2,482.7	2,660.0
5.5	585.0	585.0	780.0	975.0	1,170.0	1,365.0	1,560.0	1,755.0	1,950.0	2,145.0	2,340.0	2,535.0	2,730.0	2,925.0
6.0	639.0	639.0	852.0	1,065.0	1,278.0	1,491.0	1,704.0	1,917.0	2,130.0	2,343.0	2,556.0	2,769.0	2,982.0	3,195.0
6.5	692.0	692.0	922.7	1,153.3	1,384.0	1,614.7	1,845.3	2,076.0	2,306.7	2,537.3	2,768.0	2,998.7	3,229.3	3,460.0
7.0	746.0	746.0	994.7	1,243.3	1,492.0	1,740.7	1,989.3	2,238.0	2,486.7	2,735.3	2,984.0	3,232.7	3,481.3	3,730.0
7.5	800.0	800.0	1,066.7	1,333.3	1,600.0	1,866.7	2,133.3	2,400.0	2,666.7	2,933.3	3,200.0	3,466.7	3,733.3	4,000.0
8.0	854.0	854.0	1,138.7	1,423.3	1,708.0	1,992.7	2,277.3	2,562.0	2,846.7	3,131.3	3,416.0	3,700.7	3,985.3	4,270.0

An additional point to keep in mind is the outside air temperature (OAT) in the vicinity of the aerodrome. When descending on approach and following the nominal 3-degree glide slope using the barometric altimeter, the temperature deviation from ISA will result in the actual angle (the true descent angle) deviating from the intended 3°. This deviation as a function of the air temperature is shown in Figure 142—raw data was taken from [233].

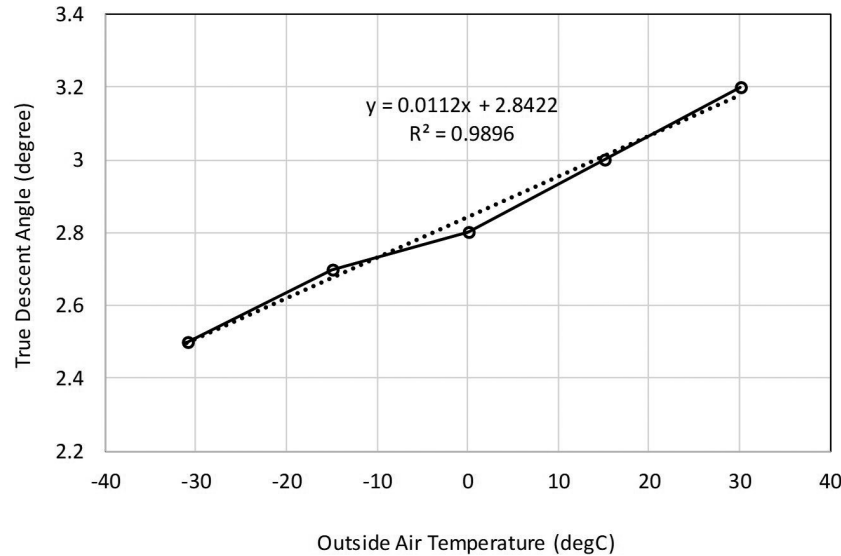


FIGURE 142 True descent angle versus the outside air temperature for a nominal 3-degree descent angle (raw data from [236]).

11.7 Glide Distance for Power-off Descent

As part of flight planning, when flying over a large body of water, you need to know the safe gliding distance for a given altitude and atmospheric conditions in case of an engine failure. Naturally, the gliding distance decreases as altitude decreases (e.g., 1,000 ft altitude gives gliding distance under 2 NM for the Cessna 172M while at a 6,000 ft altitude, the distance is about 9 NM). Figure 143 and Figure 144 represent the linear relationship between the glide distance and altitude for the Cessna 172M. Upper and lower limits are shown along with the equations that predict the glide distance as a function of the altitude and vice versa. The average fitted curve is shown under the label “Average” with their equations enclosed in rectangular boxes so that the data may be easily identified. The conditions under which these relationships apply are as follows: (a) indicated airspeed of 65 kt, (b) windmilling propeller, (c) zero-degree flap setting, and (d) zero-wind magnitude.

An easy way to estimate the altitude required to obtain a particular glide distance is to multiply the glide distance by 700 (e.g., for 10 NM gliding distance, the altitude of 7,000 ft is obtained, which only slightly overpredicts the calculated value of 6,710 ft)—Figure 144. For the inverse problem, an easy way to calculate the glide distance for a given elevation above the ground is to multiply the elevation in thousands of feet by 1.5 (e.g., for an 8,000 ft altitude, the gliding distance of $8 \times 1.5 = 12$ NM is obtained, which is nearly identical to the calculated value of 11.99 NM)—Figure 143. Glide distance tables are normally published in the POH for each aircraft. You can use similar techniques to come up with rule-of-thumb calculations for your aircraft.

✦ Rule of Thumb ✦

$$\text{Glide Distance - C172 (NM)} = 1.5 \times \text{Altitude (ft)} / 1,000$$

Imagine the scenario that your flight instructor sets the throttle to idle to simulate an engine failure and asks you to find a safe place to land. You follow the steps to make a safe emergency landing by: (a) aviating (engine test settings: fuel lever to both tanks, mixture lever to rich, carburetor lever to hot, magnetos switch to two o'clock, master switch to on, primer lever to locked, pressure and temperature gauges within the green zones, and fuel gauge showing plenty of fuel); (b) navigating (search for a proper landing site that fulfills the Civilization, Obstacles, Wind, Landing site, and Surface (COWLS) requirements); (c) resuscitating engine (clearing engine once for every 500 ft altitude reduction); and (d) communicating (transponder set

to 7700, radio set to 121.5 MHz, stating: *May Day [3 times], this is the aircraft's identifier (e.g., GHZJ), the Cessna 172, is to perform a simulated emergency landing due to engine failure over the northwest practice area, current altitude is 3,000 ft, there are 2 people on board.* If you do not have the previous table or diagram on board, although you are supposed to have one included with your POH, you may employ Figure 143 and Figure 144 to determine the speed at the time of landing or the horizontal distance at which the aircraft is to land. Note that if for whatever reason you cannot make the radio connection, you are to set the transponder code to 7600 (in case of radio failure), or 7500 (in case of air piracy) [234].

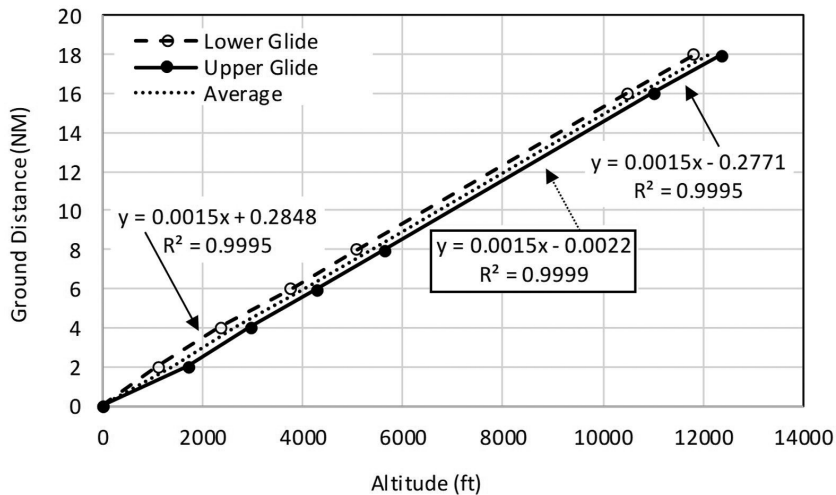


FIGURE 143 Maximum ground distance versus the altitude.

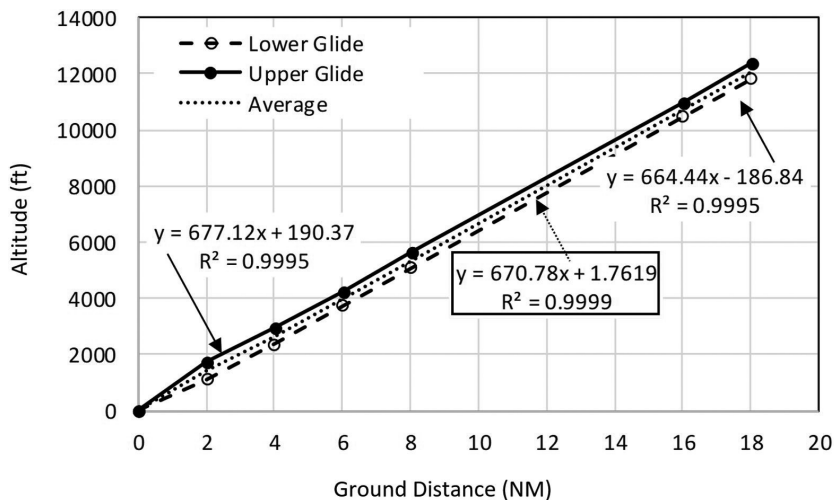


FIGURE 144 Altitude versus the maximum ground distance.

Figure 145 depicts a diagram of an aircraft gliding without power in case of an engine failure. The mass of an aircraft at the beginning of the flight is m_1 and at the time of landing is m_2 —that is to take into consideration all the factors that affect the landing, mass being an important one. Change of mass may occur either due to the change of the fuel or cargo of any kind. One point to keep in mind is that assuming that the aircraft lands at whatever speed physics allows, what impact would the aircraft expect to face? In other words, if it hits the haystack ahead, what reaction force it may expect? Newton's second law of motion takes care of this scenario, equation (225), where impact force (F) is a function of the acceleration (a), mass of the body (m_2), and time (t) derivative of velocity (v). Thus, another advantage of reducing the mass in the aircraft during a Mayday situation is reduction of the impact force.

$$F = m_2 a = m_2 \frac{dv}{dt} \rightarrow F dt = m_2 dv \quad (225)$$

Figure 145 also depicts the relationship between the glide slope and the weight of the aircraft. Note that for the balance of forces to be met, the weight of the aircraft is to equal the vertical component of the lift. Equation (226) calculates the glide distance given the lift force (L), current mass (m'), initial altitude (h), and current altitude (h'). Equation (227) determines the lift based on the acceleration (a), current mass and velocity, gravity acceleration, and coefficient of drag (C_D). The descent angle may be obtained from equation (228). Drag force (F_D) has been added to represent the lift-induced and parasitic drag forces opposing the aircraft's forward motion; however, you may ignore them by reducing the coefficient of drag (C_D) to zero.

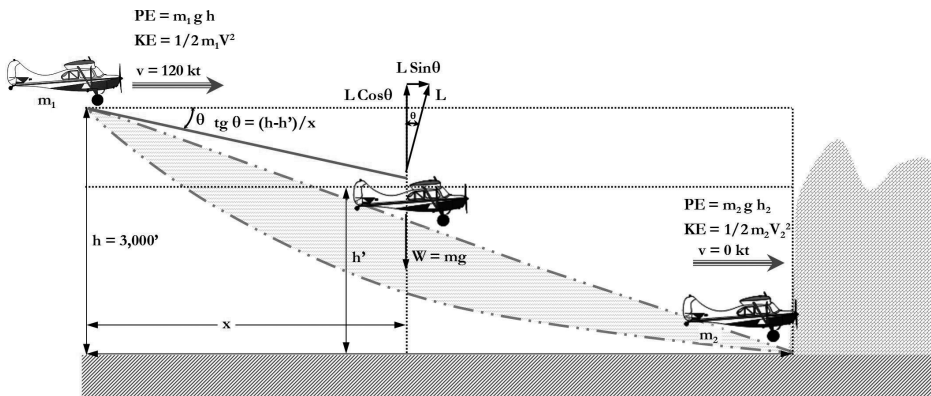


FIGURE 145 Diagram of momentum and forces for the gliding unpowered aircraft (drawings created using Solid Edge CAD tool).

$$x = \frac{(h - h')m'g}{L} / \sqrt{1 - \left(\frac{m'g}{L}\right)^2} \quad (226)$$

$$L = \sqrt{\left(m' \frac{dv}{dt} + C_D \frac{\rho v^2}{2}\right)^2 + (m'g)^2} \quad (227)$$

$$tg\theta = \left(m' \frac{dv}{dt} + C_D \frac{\rho v^2}{2}\right) / m'g \quad (228)$$

11.8 Rules of Thumb for Descent Planning

A reasonable descent angle is 3°—most ILS Glide Slopes are set to 3°. There are two cases when you may need to estimate quickly how far away from your target waypoint you need to be before beginning your descent: (a) for the Top of Descent (TOD) calculation, which identifies the distance from your target waypoint at which you need to begin the descent; and (b) when approaching for landing, where you may want to know at what altitude you should be at a certain distance from the runway.

You need to obtain both the distance and the descent rate, which would give you the desired 3-degree glide slope. To calculate the distance in nautical miles (NM), take your desired change of altitude in thousands of feet and multiply it by 3. To get the descent rate in feet per minute (fpm), take your ground speed in knots (kt) and multiply it by 5.

✎ Rule of Thumb ✎ Distance for 3° Angle (NM) = 3 × Altitude (ft)/1,000

✎ Rule of Thumb ✎ Descent Rate for 3° Angle (fpm) = 5 × GS (kt)
--

For example, assume you are cruising at 12,000 ft and you need to be at 5,000 ft at the Initial Approach Fix (IAF). Calculate how far away you need to be to begin your descent as $12 - 5 = 7$, $7 \times 3 = 21$ NM. Thus, you need to start your descent 21 NM from the fix. Next, what descent rate will get you there? You can obtain your ground speed from GPS or approximately by estimating the TAS and the wind component along your course. For example, let us assume your ground speed is 150 kt. The descent rate of $150 \times 5 = 750$ fpm should give you the desired 3-degree glide slope.

In another scenario, when approaching for landing; for example, at 6 NM from the aerodrome, you are to be at $6 \times 1,000 / 3 = 2,000$ ft and at about 1,000 ft at 3 NM away. As you are on the final approach, your descent rate will need to be decreased as the ground speed decreases. Hence, as the speed decreases to 100 kt, you should descend at $100 \times 5 = 500$ fpm. Slowing to 60 kt, the descent rate should be $60 \times 5 = 300$ fpm.

Here is an explanation of how the previous rule is obtained from the equations. The ratio of the height change to the horizontal distance equals a tangent of 3° : $\tan(3^\circ) = \text{Altitude Change (ft)} / \text{Distance (NM)} \times 6,076.12 \text{ (ft/NM)} = 0.0524$. The preceding equation is then rearranged to give: $\text{Distance (NM)} = \text{Altitude Change (1,000 ft)} \times 3.14$. Thus, multiplying by 3 (versus 3.14) results in a lower than actual estimate by about 5 percent. For the rate of descent calculation, the ratio of the vertical speed to horizontal (ground) speed should equal tangent of 3° :

$$\tan(3^\circ) = \text{Descent Rate (fpm)} / (\text{Ground Speed (NM/hr)} \times 6,076.12 \text{ (ft/NM)}) \times (1/60) \text{ (hr/min)} = 0.0524$$

Rearranging said relation, you obtain $\text{Descent Rate (fpm)} = \text{Ground Speed (NM/hr)} \times 5.307$. In this case, the rule of thumb results in underestimation by about 6 percent (5 versus 5.31).

11.9 Gyro Error

A gyroscope provides a stable reference for the aircraft attitude changes (roll, pitch, and yaw). Gyroscopes are categorized based on: (a) their function (displacement or rate gyros with one degree of freedom); (b) construction (tuned rotor, ring laser, and fiber optic gyros); and (c) power source (pneumatic or electric source gyros). Displacement gyros show the change of the angle—degrees of roll, pitch, and yaw. Displacement gyros may be divided into space gyros (unrestrained by external forces and free to wander) and tied gyros (restrained within their environment or those that return to their original position using a reflexive mechanism against the external forces). The Earth gyros are tied gyros that are oriented by gravity. Rate gyros measure the angular rate (e.g., rate of turn, completing a standard rate turn in 2 min, equivalent to 3° per second) with a single-gimbal system and a spring system that restricts its precession.

In a pneumatic gyro, low pressure from the vacuum source pulls the air through the instrument case, and this air flow drives the gyro rotor. The pressure differential is the driving force and is independent of electrical sources. It may be driven by: (a) a dedicated engine-driven vacuum pump,

(b) low-pressure-air generated in the engine inlet manifold, or (c) a venturi mounted outside the aircraft. The electrically driven gyros are powered by alternating current, with the conversion from DC to Alternating Current (AC) by means of an inverter—if required. Benefits of the electrically-driven gyros include their high spin speed, ability to maintain constant rotor speed, fast increase of RPM, and the seal-ability of the gyro in an airtight unit.

Based on the functionalities related to the aforementioned tasks, they are categorized as: (a) directional gyro indicators (DGI)—a tied gyro also known as a heading indicator showing yaw; (b) artificial horizon—an Earth gyro, also known as an attitude indicator, showing pitch; (c) turn and slip indicator—a rate gyro showing roll; (d) turn coordinator—a rate gyro sensing both roll and yaw used as a guide for a standard-rate 2-min turn; (e) non-magnetic compass—an Earth gyro showing true north direction; and (f) autopilots—equipped with longitudinal and lateral gyros. In more advanced systems, the use of a gyro is expanded to applications such as: (a) Inertial Navigation System (INS), (b) yaw dampers, and (c) radar scanners.

The spinning rotor of the gyro turns around a shaft, a spinning axis, with the orientation identified by that of this axis (horizontal or vertical). Gyroscopes have two main specifications: rigidity and precession. Rigidity is also known as gyroscopic inertia, and it is the ability to maintain the axis orientation despite the rotor axis turning in space. Rigidity is affected by the gyro's mass, effective radius, concentration of the mass around the perimeter, and its speed of rotation. Increasing each of these parameters will increase the rigidity.

Precession is the gyro's resistance against the destabilizing forces. If a force is applied to the rotor to change its direction, the gyro generates force by moving the imposed force 90° and applying it to the rim of the rotor, and along the direction of its rotation compensates for that force. It is inferred that rigidity and precession are opposing characteristics since the greater the rigidity, the more resistant the gyro is to the precession. Balance of momentum will result in the rate of precession obtained from equation (229)—where Ω is the rate of precession, T is torque, I is moment of inertia, $I = 0.5 (a^2 + b^2)$, and ω is angular velocity. A gyro is held in place by being suspended from a frame—also called a gimbal system. Inside the “golden cage,” the gyro is able to rotate freely along a single-axis—one degree of freedom in a single-gimbal system. The direction of the gimbal axis identifies the direction of the aircraft movement (pitch rotation about

the lateral axis and yaw rotation about the vertical axis). To increase the flexibility of movement of the gyro, it is possible to mount the single-gimbal system (inner gimbal with the first degree of freedom) on a second gimbal system (outer gimbal) that is attached to the aircraft by means of a pivot point—the inner gimbal then may move about the second axis (second degree of freedom). This results in two degrees of freedom, excluding the spin axis—equation (229).

$$\Omega = \frac{T}{I\omega} \quad (229)$$

The errors of the DGIs are due to gimbaling error, Real Wander (RW), Earth Rate (ER), Latitude Nut (LN), and Transport Wander (TW). Any deviation of the spin axis from its fixed orientation is called gyro wander (real or apparent wander). Real wander may be further subcategorized into real drift and real topple and apparent wander into apparent drift and apparent topple. Real wander is when the axis of the gyro moves, not being properly fixed, and is mainly due to manufacturing errors or wear such as imbalance of the gimbal system, non-homogeneous mass concentration on the rotor (the simplest form of gyro), and uneven friction between the rotor and its bearing. This is why gyros are to be calibrated annually. Apparent wander though is due to the change of the axis position of a perfect gyro from the perspective of the observer on the Earth due to the Earth's rate of turn or transport wander. Assuming that an aircraft gyro axis is fixed and aligned with that of the Earth's magnetic north (i.e., no transport), with the Earth's rotation, the direction of the true north changes with respect to the magnetic north. This is associated with the negative Earth rate in the Northern Hemisphere and the positive rate in the Southern Hemisphere. Given the convergence of the Earth's meridians, which is minimum at the Equator and maximum at the Poles, apparent drift is a product of the Earth's rotation rate (15° per hour) and sine of latitude, making it 15° per hour at the Poles and zero degrees at the Equator—equation (230). Apparent topple follows an opposite trend—15° per hour at the Equator and zero degrees at the Poles—equation (231). Drift and topple wanders depend on the movement of the gyro axis along the vertical and horizontal axes and are both applicable to horizontal-axis gyros while topple wander is only applicable to vertical axis gyros.

$$\text{Apparent Drift} = \frac{15^\circ}{\text{hr}} \times \text{Sin}(\text{latitude}) \quad (230)$$

$$\text{Apparent Topple} = \frac{15^\circ}{\text{hr}} \times \text{Cos}(\text{latitude}) \quad (231)$$

Assuming that the aircraft is transported—the axis of gyro is not fixed or aligned with that of Earth’s magnetic north—with the Earth’s rotation, the direction of the true north versus the magnetic north changes, and also the axis of the gyro moves. Therefore, apparent drift occurs, which increases with eastward movement—since both the Earth and aircraft move in the same direction (positive drift sign)—while the drift decreases with western movement, since the Earth and the aircraft move in the opposite directions (negative drift sign)—equation (232). The opposite happens in the Southern Hemisphere. The relationship between transport wander and latitude may be expressed by equation (232), where u is the easterly component of the ground speed in knots. As noted, the transport wander may create both drift and topple. Transport wander occurs only when the aircraft makes easterly and westerly flights, and it does not occur on flights along the north-south direction.

$$\text{Transport Wander} = \frac{u}{60} \times \frac{\tan(\text{latitude})}{\text{hour}} \quad (232)$$

The space gyro may drift and topple with negative or neutral stability. The tied gyros can drift but cannot topple.

Although the most common form of the DGI looks like a compass rose, connected to the outer gimbal by gears, it is not a compass and it is to be synchronized with the magnetic compass. In a tied gyro with two degrees of freedom, the rotor axis is the horizontal axis, with the rotor mounted in an inner gimbal and the bearings on the outer gimbal, which can rotate 360° along the vertical axis. The three of them (rotor spin axis, inner gimbal, and outer gimbals) are at right angles with respect to each other.

The airflow passing over the rotor in a pneumatic-driven system is fine-tuned by a wedge-plate affixed to the outer gimbal. The wedge-plate separates the air into two equal-flow air streams so that the reaction force for precession is equally applied to the sides of the rotor and therefore maintains the rotor axis in the yawing plane.

Benefits of having a caged DGI include resynchronization of the DGI with the magnetic compass, preventing topple, and re-erecting the gyro by means of the caging knob. If not caged, the maneuvers need to be limited to keep the roll or pitch in the range of $\pm 55^\circ$ for the pneumatic-driven and $\pm 85^\circ$ for the electrically driven systems.

A latitude nut is attached to the inner gimbal of a spindle. By turning this nut, the DGI drift caused by the opposite error to that of the Earth’s rotation is corrected. The nut may be calibrated at the center on the thread at the

Equator, with the outer-bound position of the thread at the proximity to the North Pole, and the inner-bound position of the thread at the proximity of the South Pole—this is to compensate for the Earth’s rotation. It may only be calibrated at a workshop, to produce real wander, equal and opposite to the apparent wander based on the Earth’s rotation. The latitude at which the gyro is set is the only place where accurate information is provided by the instrument. Therefore, the error associated with the change of location is to be taken into consideration for accurate readings.

To summarize, the errors of the DGIs are due to real wander (*RW*), the Earth rate (*ER*), latitude nut (*LN*) adjustment, and easterly and westerly transport wander (*TW*), as in equation (233) and Table 76.

$$TD = RW + ER + LN + TW$$

(233)

TABLE 76 DGI errors and their signs.

	Error	Northern Hemisphere	Southern Hemisphere
1	ER	Negative	Positive
2	LN	Positive	Negative
3	TW (Eastwards)	Negative	Positive
4	TW (Westwards)	Positive	Negative

11.10 Cabin Pressurization

With the development of aircraft, and especially with the advancement of passenger jets, the aircraft needed to fly at higher altitudes. In the atmosphere, with increasing altitude, the pressure decreases and that affects negatively passenger comfort; hypoxia (lack of oxygen) can lead to dizziness and other symptoms. In order to enhance the passenger comfort, new requirements are introduced or the older requirements are tightened. Setting the appropriate pressure level inside the cabin to meet the minimum passenger comfort is one of these requirements. This makes it possible for a larger demographic (e.g., elderly, children, and passengers with cardiovascular conditions) to be able to use air transportation. There are still circumstances in which the passengers with certain physical conditions are not to use the transportation. Examples include pregnant women after their third trimester, patients with sinusitis conditions, and deep-divers practicing controlled or uncontrolled decompression.

As a scuba diver dives deep down, the increase of water pressure causes faster and greater absorption of the oxygen and nitrogen molecules, but mostly nitrogen molecules, in the blood. The second decompression (e.g., flying at altitudes above the MSL) will result in rapid change of the oxygen and nitrogen levels and may result in symptoms such as extreme muscle pain and conditions that are also known as I DCS or bends. For this reason, dive tables are developed; however, these tables assume that the diving is done somewhere close to the surface of the sea level. This is why it is recommended for divers not to fly 48 hrs after their last deep dive so that the excess nitrogen build-up in the body is safely equalized (unloaded) to that of the environment at the sea level. As a pilot, if you are experiencing a heavy cold with blocked sinuses, do avoid making frequent flights and postpone your training activities until you are recovered. Frequent changes of pressure between the outer and inner ears may cause headaches, worsened cold symptoms, and may even lead to permanent damage to hearing.

Jet fighter pilots may experience similar symptoms to those of the scuba divers due to the fast ascents and descents; therefore, they are to wear G-Suits that are pressurized and wear oxygen masks at all times. The astronauts who are sent to outer space fall within the same category. The United States used to incorporate pure oxygen to reduce the large payload due to the use of the combined oxygen and nitrogen tanks aboard. The space shuttle cabin pressure has been historically maintained at about 25,000 ft (approximately 5.4 psi, 0.37 bar). However, it was found out that the pure oxygen tank is a fire hazard in the atmosphere after the death of Apollo crew members in 1967 during the ground test. Therefore, it was decided to use a 40 percent nitrogen, 60 percent oxygen mix at zero cabin altitude when launching the space shuttle, and pure oxygen in space. The Space Shuttle Orbiter, International Space Station (ISS), and Sky Lab currently use an air-like cabin atmosphere.

It is also required for the space shuttle crew members to wear oxygen masks after the Soyuz 11 (Союз 11) incident in 1971, resulting in the death of Soviet cosmonauts after the cabin air leaked when the cabin vent valve opened accidentally before entering the atmosphere while the crew were not wearing oxygen masks.

Carrying oxygen masks at altitudes above 10,000 ft (3,000 m) above the MSL is required for possible physiological complications that lack of oxygen introduces. At flight altitudes above 12,500 ft, for the journeys that are longer than 30 min, the aircraft is to be equipped with oxygen masks

for all the passengers. At altitudes above 15,000 ft, wearing oxygen masks for an unpressurized cabin is required. Most of the aircraft that travel at high altitudes, such as passenger jets, are generally pressurized. Aircraft pressurization allows the cabin pressure to remain high long after the exterior pressure is decreased to the level below the maximum human tolerance.

The first commercial passenger transport aircraft that had pressurization capability was the Boeing 307 Stratoliner, entering service in 1940 [235]. Pressurization enabled passenger aircraft to fly at higher altitudes (20,000 ft for the 307). The main benefit of this at that time was reducing discomfort due to the unstable atmospheric conditions at lower altitudes. Prior to this; for example, flying a plane such as a popular Douglas DC-3 was quite uncomfortable for the passengers, with significant turbulence, icing, rain, and lightning affecting the flight. Higher altitudes made most flights much less challenging to passengers and so aided significantly in popularizing air travel. Some aircraft are capable of maintaining the cabin pressure at a much higher pressure. The SyberJet SJ30 (with its first flight in 2005) was the first business jet certified for flying at 41,000 ft (12,000 m) altitude and complying with a 12 psi pressurization system, giving pressure in the cabin close to that at sea level.

The pressurized air is provided by the bled-off air from the gas turbine engines or, in spacecraft, it is provided by cryogenic tanks. Depending on the air temperature and humidity level, air is to be cooled and humidified or mixed with auxiliary air (conditioned) before being introduced into the cabin. In the modern turbofan and turboprop planes, the source of pressurized air is the bleed air taken from the compressor stage of a gas turbine engine. The part of the aircraft cabin and cargo area where the pressurization is maintained is referred to as the *pressure vessel*. Inside an aircraft environment, the flight cabin, passenger cabin, and cargo compartment are to be pressurized. The latter is especially important as the decompression of some items may result in damage. Recall your toothpaste, shampoo, or even battery inside your checked-in luggage that leaked out and ruined the adjoining fabric. This is why these items should be sealed inside a zip-bag to safeguard against leakage. Radomes, landing gear bays, and tail and nose cones, on the other hand, do not require pressurization.

Air pressure inside the cabin is set by specifying the *cabin altitude*. This is the pressure at the altitude referenced to the sea level at IAS conditions. Each aircraft, depending on its construction, has a certain maximum pressure differential between the exterior and interior that it can safely tolerate for an extended time. Today's business jets and large commercial

airliners can tolerate pressure differentials up to 9.4 psi, with a typical range of 8 to 9 psi, while an older turboprop, such as the Beechcraft 1900, is only allowed a maximum of a 5.1 psi differential [236,237]. Higher maximum pressure differentials allow planes to fly at higher altitudes and (or) lower cabin altitudes, adding to the passenger comfort.

There are several valves (e.g., outflow, positive and negative pressure, and dump valves) that regulate the cabin pressure. Depending on the pressure level inside the cabin relative to the outside pressure, positive and negative pressure valves are used. The dump valve is manually activated to release some of the inside cabin air to decrease the cabin pressure. Air outflow valves, as part of a pressurization system, let the air leave the aircraft at a controlled rate to maintain the set pressure inside the cabin. If the maximum pressure differential is reached, the valve will open automatically to reduce the pressure to the safe limit. Also, aircraft pressure vessels are not normally designed to tolerate any negative pressure (when the exterior pressure is higher than that of the interior). An aircraft pressurized fuselage can be likened to an aluminum can reinforced with cross-sectional frames tied together with longitudinal stringers. You can see that this structure will tolerate higher loads when pressurized from the inside than when squeezed from the outside (leading to buckling failure).

The planes are normally designed so that they are not pressurized while on the ground, with a sensor in the landing gear monitoring that the weight is applied to it and thus keeping the aircraft unpressurized. Upon takeoff, the sensor output indicates to the pressurization system that the outflow valve can now start operating to pressurize the cabin.

On large airliners, the pressurization control is mostly automated. On smaller planes, manual controls are used. Control is accomplished by the pressurization controller and the rate control selector. The former consists of a fixed disc dial with markers for cabin (CABIN) and aircraft (ACFT) altitudes printed around its circumference (Figure 146). The relationship between the two altitudes is determined by the maximum pressure differential rating for the plane and is fixed—both scales are printed on the same disc. The setting corresponds to the middle of the window in the controller. For example, in the figure, the marker points to about 15,000 ft aircraft altitude and 3,200 ft cabin altitude. The pressure difference may be calculated and compared to the aircraft-rated maximum pressure differential ($13.074 - 8.294 = 4.78$ psi).

As the plane climbs to a higher altitude, the pressure in the cabin needs to be progressively reduced so as not to exceed the maximum differential.

Thus, one can say that the cabin effectively experiences its own rate of climb, distinct from that of the aircraft. This rate is controlled by the small round knob with an arrow printed on it seen in Figure 146, below and to the left of the pressurization controller. For the Beechcraft 1900D, the range of cabin climb rate settings is from 175 to 2,500 fpm.

The state of cabin pressurization is monitored using the cabin altimeter and the cabin climb rate indicator (Figure 147). The cabin altimeter indicates the cabin altitude in thousands of feet on the outer dial and the pressure differential (psi) on the inner dial, with green coloring below 5 psi and red above, with the red color indicating excessive pressure differential. In the figure, the cabin altitude is shown increasing at about 600 fpm. The current cabin altitude is 3,000 ft, with a pressure differential of 2.8 psi.

Before taking off, the pressurization controller is normally set to an aircraft cruise altitude plus a 1,000 ft “buffer” zone. This prevents the

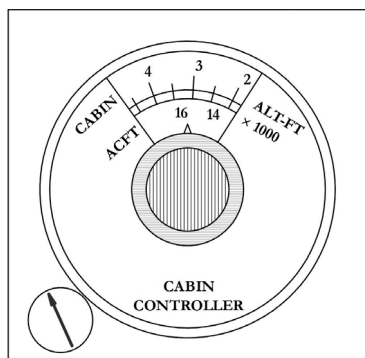


FIGURE 146 Pressurization and cabin climb rate controllers on the Beechcraft 1900D (drawings created using Solid Edge CAD tool).

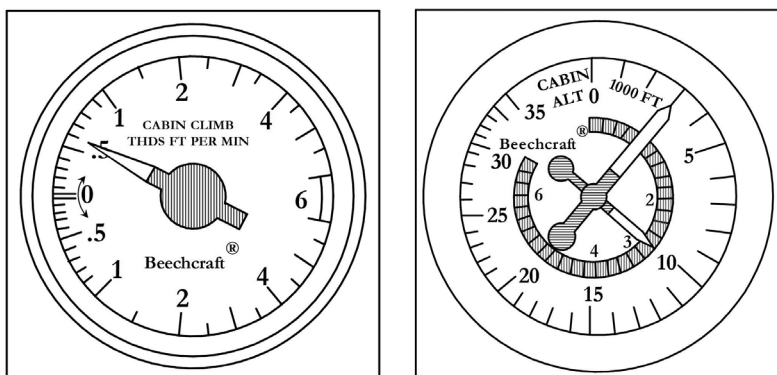


FIGURE 147 Cabin climb rate (left) and cabin altimeter (right) on the Beechcraft 1900D (drawings created using Solid Edge CAD tool).

outflow valve triggering intermittent pressure release and as a result passenger discomfort if the aircraft were to fly at the altitude set on the dial. When preparing for descent to the airport, the pilot sets the pressurization controller, using the outer dial of cabin altitude, typically to an altitude of about 500 ft above the landing field elevation.

11.10.1 Air Pressure Differential and Its Effect on the Aircraft

The aircraft structure must be able to tolerate the mechanical forces due to the pressure difference between the cabin interior and the exterior. Normally, the pressure inside the cabin is higher than outside and the aircraft's cylindrical pressure vessel is forced to expand slightly. The circumferential stress due to interior pressure in any cylindrical vessel is known as *hoop stress* (as it is aligned with hoops holding a barrel together). Stress is an internal force in any material calculated as the applied force divided by the material cross section area over which this force is applied. For example, if you pull on the two ends of a rectangular aluminum bar with a force of 1,000 N and the bar has a $2 \times 5 \text{ mm}^2$ cross section, you can obtain the average stress in the bar along the pull direction as $1,000 \text{ (N)} / 10 \text{ (mm}^2\text{)} = 100 \times 10^6 \text{ Pa} = 100 \text{ MPa}$. The maximum stress that the material can tolerate without breaking is called tensile strength and, for a typical aluminum, may be on the order of 150 MPa. You can thus conclude that the aluminum bar pulled in this example will not break.

As the jet airliners repeatedly climb and descend during their flights, their cabin skin experiences cycles of pressurization and thus hoop stresses. Repeated stress cycling of any material can lead to what is known as material fatigue. As a result of fatigue, depending on the number of loading cycles, stress intensity, and structure shape, the material can fail after being subjected to average stress value well below its tensile strength. In fact, it was the fatigue in the aluminum skin structure that led to several in-flight break-ups for the first commercial jetliner (the de Havilland Comet) that entered service in 1952. The problem was due to square windows which produced higher local stresses in the corners. Also, it is the number of pressurization cycles (i.e., flights), not their age, that sets the lifetime limit for the modern jetliners. For this reason, long-haul airliners, such as the Boeing 747, may be in service for decades as they make perhaps only one flight per day. On the other hand, for budget airlines, their smaller planes (such as the Boeing 737) on regional flights may experience several flights (and pressurization cycles) per day. For this reason, budget airlines (like Ryanair or easyJet) typically have younger fleets than regular airlines (like British Airways or Air Canada).

As described earlier in this section, the jet airliner's service altitude is limited by the maximum allowed difference between interior and exterior air pressures. The interior and exterior pressures are both described in terms of altitude. Thus, it would be instructive to investigate the relationship between air pressure and altitude. There are tables which show the relationship between the altitude above the MSL and atmospheric pressure expressed as an absolute barometer reading (e.g., inches of mercury—inHg) or pressures (e.g., kilo Pascals—kPa) (Figure 148). It is seen that as the altitude increases, the air pressure decreases nonlinearly. The curve-fitted formula presents the fourth order polynomial with the choice of pressure (psi) as the predictor and altitude (ft) as the response variables. Table 77 presents the pressure versus the altitude data for the absolute barometer and atmospheric pressure using equation (234), where P is pressure in Pa, H is altitude in meters, and P_0 is standard pressure at the sea level (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi). b and c are constants ($b = 2.255, 77 \times 10^{-5}$, $c = 5.255, 88$).

Thus; for example, if the cabin altitude is set to 8,000 ft, the corresponding air pressure can be determined as 10.916 psi (Table 77, Case 7). If an aircraft is designed for a maximum differential pressure of 8.5 psi, you can deduct this value from the previous pressure to find the minimum allowed outside pressure equal to 2.416 psi is acceptable, resulting in the maximum flight altitude capability of 42,000 ft (12,802 m). The associated flight altitude can be obtained using the data in Table 77.

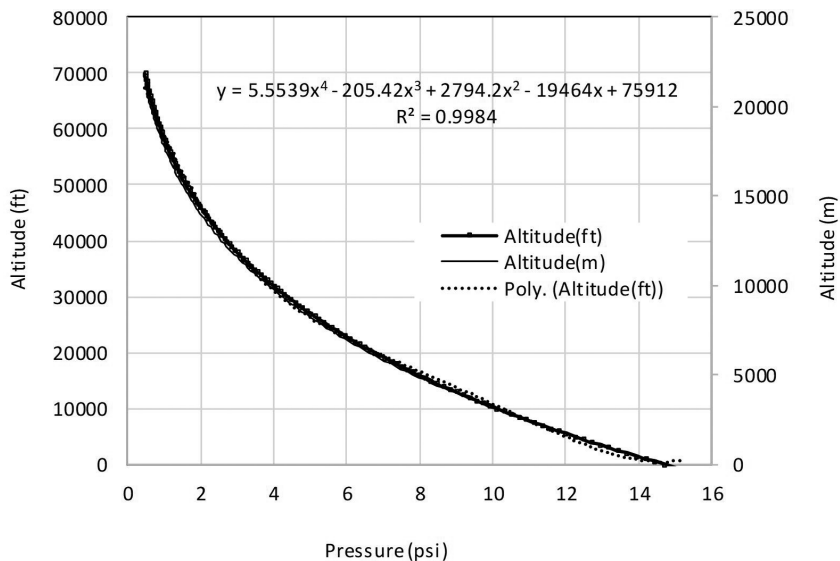


FIGURE 148 Altitude versus the air pressure.

Alternatively, equation (234) may be restated to present the altitude (H) versus the pressure (P), as in equation (235). Equation (236) presents the relationship between the altitude (H) and pressure (P) for different values of the static pressure (P_b), standard temperature (T_b), and lapse rate (Γ_b) for each specific atmospheric layer, where $f = g_0 M / \bar{R} \Gamma_b$, and subscript “ b ” is associated with the specific atmospheric layer. The values for these standard temperatures, static pressures, and lapse rates are presented in Table 78.

$$P = P_0 (1 - bH)^c \quad (234)$$

$$H = \frac{1}{b} \left\{ 1 - \exp \left[\frac{1}{c} \ln \left(\frac{P}{P_0} \right) \right] \right\} \quad (235)$$

$$H = \frac{T_b}{\Gamma_b} \left[\frac{P^{-1/f}}{P_b} - 1 \right] \quad (236)$$

TABLE 77 Pressure as a function of the altitude in various units.

Case	Altitude(ft)	Absolute Barometric Pressure		Absolute Atmospheric Pressure		
		P(inHg)	P(mmHg)	P(psi)	P(kg/cm ²)	P(kPa)
1	0	29.921	760.000	14.696	1.033	101.325
2	2,000	27.821	706.655	13.664	0.961	94.213
3	4,000	25.842	656.383	12.692	0.892	87.511
4	6,000	23.978	609.047	11.777	0.828	81.200
5	8,000	22.225	564.514	10.916	0.767	75.262
6	10,000	20.577	522.655	10.106	0.711	69.682
7	12,000	19.029	483.346	9.346	0.657	64.441
8	14,000	17.577	446.466	8.633	0.607	59.524
9	16,000	16.217	411.898	7.965	0.560	54.915
10	18,000	14.942	379.530	7.339	0.516	50.600
11	20,000	13.750	349.253	6.753	0.475	46.563
12	22,000	12.636	320.962	6.206	0.436	42.791
13	24,000	11.597	294.556	5.696	0.400	39.271
14	26,000	10.628	269.938	5.220	0.367	35.989
15	28,000	9.725	247.013	4.776	0.336	32.932
16	30,000	8.885	225.690	4.364	0.307	30.090
17	32,000	8.106	205.883	3.981	0.280	27.449
18	34,000	7.382	187.508	3.626	0.255	24.999
19	36,000	6.712	170.484	3.297	0.232	22.729
20	38,000	6.092	154.733	2.992	0.210	20.629
21	40,000	5.519	140.182	2.711	0.191	18.750
22	42,000	4.991	126.760	2.451	0.172	17.047
23	44,000	4.504	114.397	2.212	0.156	15.500
24	46,000	4.056	103.030	1.992	0.140	14.098

TABLE 78 Constant variables presented in equation (236) for different atmospheric layers.

<i>b</i>	Altitude (<i>H</i>)		Static Pressure (<i>P_b</i>)		Standard Temperature (<i>T_b</i>)	Lapse Rate (<i>Γ_b</i>)	
	(m)	(ft)	(Pa)	(inHg)	(°C)	(°C/km)	(°C/1,000 ft)
0	0	0	101.33	29.921	15.00	-6.50	-1.981
1	11,000	36,089	22.63	6.683	-56.50	0.00	0.000
2	20,000	65,617	5.47	1.617	-56.50	1.00	0.305
3	32,000	104,987	0.87	0.256	-44.50	2.80	0.853
4	47,000	154,199	0.11	0.033	-2.50	0.00	0.000
5	51,000	167,323	0.07	0.020	-2.50	-2.80	-0.853
6	71,000	232,940	0.00	0.001	-58.50	-2.00	-0.610

One can use the relationship between the atmospheric pressure and altitude shown previously to make the plot in Figure 149 that shows how

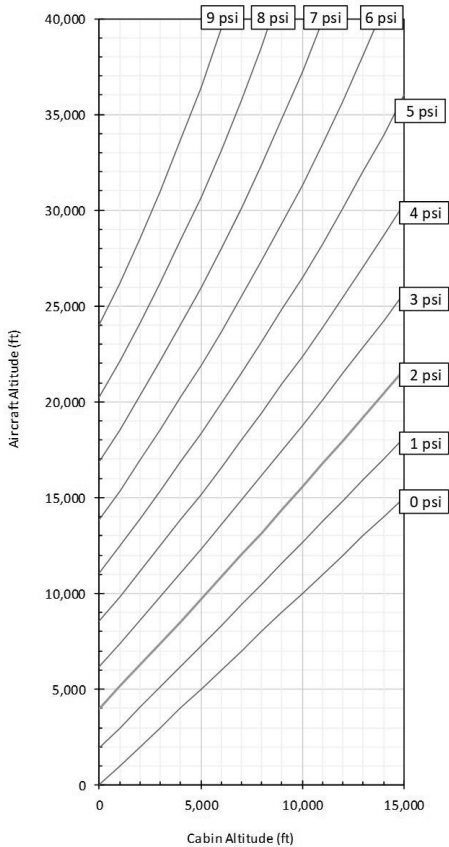


FIGURE 149 Aircraft altitude versus the cabin altitude for pressure differential values from zero to 9 psi.

the aircraft's altitude will vary with the cabin altitude for a range of pressure differentials. For example, for the Beechcraft 1900, which can tolerate up to a 5 psi differential, cruising at 25,000 ft, the lowest possible cabin altitude setting is 9,000 ft. In a large commercial jet; for example, capable of a 9-psi differential, one can be cruising at 36,000 ft and still have only 5,000 ft of cabin altitude.

11.10.2 Setting the Cabin Climb Rate

While the setting of cabin altitude is determined by the desired cruise altitude or the landing aerodrome altitude, it is not clear what one should set the cabin climb rate to. It is possible to set the climb rate too low. If the plane climbs quickly, the maximum pressure differential may be exceeded temporarily if the cabin altitude lags too far behind. On the other hand, setting the climb rate at an excessively high value may cause increased passenger discomfort.

The following presents the calculation for the appropriate cabin climb rate. The variables are defined in Table 79.

TABLE 79 Input variables for the cabin climb rate calculation.

Variable	Units	Terms for Climb Situation
H_{A-D}	ft	aircraft altitude as set on the pressurization controller dial
ΔH_A	ft	buffer between set and actual aircraft altitudes
\dot{H}_A	ft/min (fpm)	aircraft climb rate
H_0	ft	ground elevation at start of climb
H_C	ft	cabin altitude at the TOC
\dot{H}_C	ft/min (fpm)	cabin climb rate
Δt_C	min	time offset for cabin reaching set altitude before aircraft reaching the TOC
t_A^*	min	aircraft time to TOC
t_C^*	min	time for cabin altitude to reach set value

One would like the cabin to reach the set altitude a few minutes before the aircraft reaches the TOC. Thus, for a climb, equation (237) is obtained.

$$t_C^* = t_A^* - \Delta t_C \quad (237)$$

The time it takes to reach the cabin and aircraft altitudes is obtained by equation (238).

$$t_C^* = \frac{H_C - H_0}{\dot{H}_C} \text{ and } t_A^* = \frac{H_{A-D} - \Delta H_A - H_0}{\dot{H}_A} \quad (238)$$

From the previous two relations, equation (239) is obtained.

$$\dot{H}_C = \frac{\dot{H}_A (H_C - H_0)}{H_{A-D} - \Delta H_A - H_0 - \dot{H}_A \Delta t_C} \quad (239)$$

The following presents calculations for the case of descending from the cruise altitude to the field elevation level for landing. The variables are defined in Table 80.

TABLE 80 Input variables for the cabin descent rate calculations.

Variable	Units	Terms for Descent Situation
H_A	Ft	aircraft cruise altitude
\dot{H}_A	ft/min (fpm)	aircraft descent rate
H_0	Ft	ground elevation at field elevation for landing
H_C	Ft	cabin altitude at cruise
ΔH_C	Ft	difference in altitude between setting on the controller and ground elevation
\dot{H}_C	ft/min (fpm)	cabin descent rate
Δt_C	Min	time offset for cabin reaching set altitude before landing
t_A^*	Min	aircraft time to descend
t_C^*	Min	time for cabin altitude to reach set value

Equation (237) is as before. The time it takes for the aircraft and cabin descent to reach the desired altitude are obtained by equation (240).

$$t_C^* = \frac{H_C - (H_0 + \Delta H_C)}{\dot{H}_C} \text{ and } t_A^* = \frac{H_A - H_0}{\dot{H}_A} \quad (240)$$

Substituting the above equation into equation (237), equation (241) is obtained.

$$\dot{H}_C = \frac{\dot{H}_A (H_C - H_0 - \Delta H_C)}{H_A - H_0 - \dot{H}_A \Delta t_C} \quad (241)$$

11.11 Examples

11.11.1 Case Study – Mechanics of Takeoff and Landing Distances

The Boeing 777-300ER, with an MTOM of 775,000 lb (351.534 tons), is powered by two GE90-115B turbofan jet engines, reportedly the most powerful jet engines in the world, with a maximum thrust of 115,300 lbf (513 kN) each [238]. The jet aircraft is to take off from an airport with three runways: 01/19, 07/25, 10/08. Assume the takeoff speed for the current atmospheric conditions and aircraft loading is 190 mph. Ignore drag for all the calculations.

(a) for a day with a southwesterly wind (270°), select the runway and required length of runway for an uphill takeoff from the slightly inclined runway by 0.75° , (b) downhill takeoff from the same inclined runway if the wind becomes north-easterly (50°), (c) select runway and required length if the aircraft is to perform a level takeoff with a southerly wind (180°), (d) acceleration of the aircraft assuming that the initial velocity is zero for three inclination scenarios, (e) time to takeoff based on the calculated acceleration for three inclination scenarios, and (f) part “e” assuming the Runway Friction Index (RFI) is 0.15.

For the calculations presented in Table 81, the takeoff speed is converted to metric units ($190 \text{ mph} = 305 \text{ km/hr}$). The uphill, downhill, and level takeoff diagrams of applicable forces are shown in Figure 150, Figure 151, and Figure 152. The angle of θ , equivalent to 0.75° , is depicted (not to scale). The component of the weight vectors along the runway surface resists the forward motion of the aircraft and also the aircraft is to overcome the friction of the runway as it starts to move forward (threshold drag). The balance of forces is to be performed parallel and perpendicular to the

TABLE 81 Raw data for the Boeing 777-300ER.

Mass (m)	351,534	kg	351.534	ton
Available Thrust (F)	513,000	N	513	kN
Engines	2			
Initial Speed (v_0)	305	km/hr	84.72	m/s
Thrust (F_T)	1,026,000	N	1,026	kN
Weight (W)	3,448,549	N	3,448.549	kN
RFI (μ)	0		RFI (μ)	0.15

runway surface. The balances of forces for the vertical axis as well as uphill, downhill, and level configurations are presented by equations (242), (243), (244), and (245). The takeoffs are to be performed from runways 25, 07, and 19 for uphill, downhill, and level scenarios. The reader is encouraged to analyze the choice of runways.

$$\sum F_y = -mg \cos \theta + N \cos \theta = 0 \rightarrow N = mg \quad (242)$$

$$\sum F_{x_{Uphill}} = Thrust - mg(1 + \mu) \sin \theta = ma \quad (243)$$

$$\sum F_{x_{Downhill}} = Thrust - mg(-1 + \mu) \sin \theta = ma \quad (244)$$

$$\sum F_{x_{Level}} = Thrust - \mu mg = ma \quad (245)$$

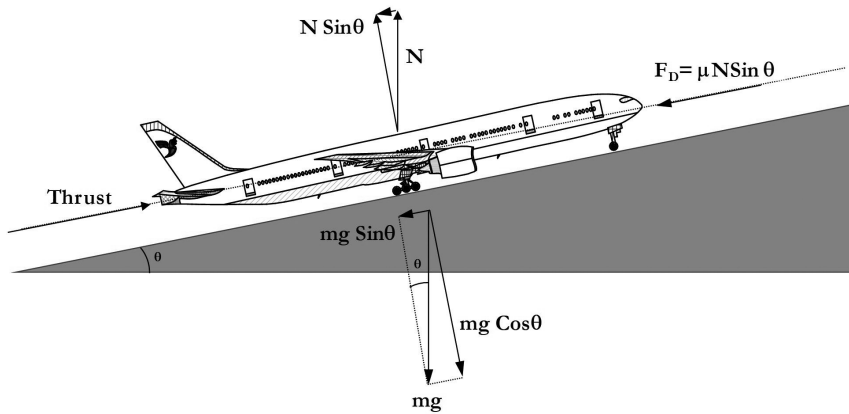


FIGURE 150 The Boeing 777-300ER ready for an uphill takeoff on runway 25 (drawings created using Solid Edge CAD tool).

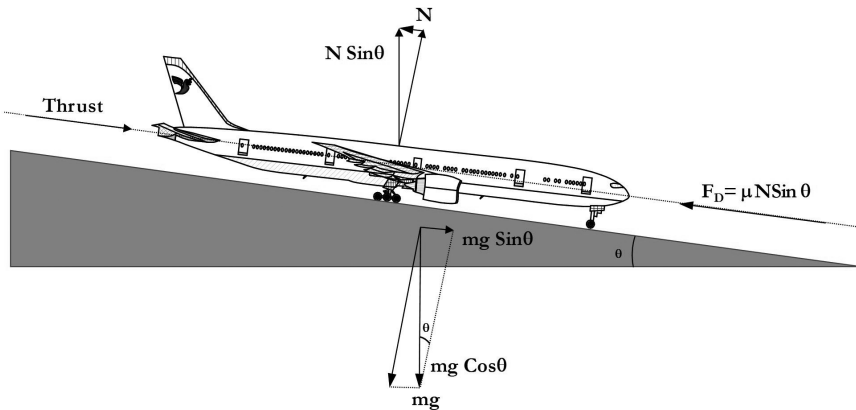


FIGURE 151 The Boeing 777-300ER ready for a downhill takeoff on runway 07 (drawings created using Solid Edge CAD tool).

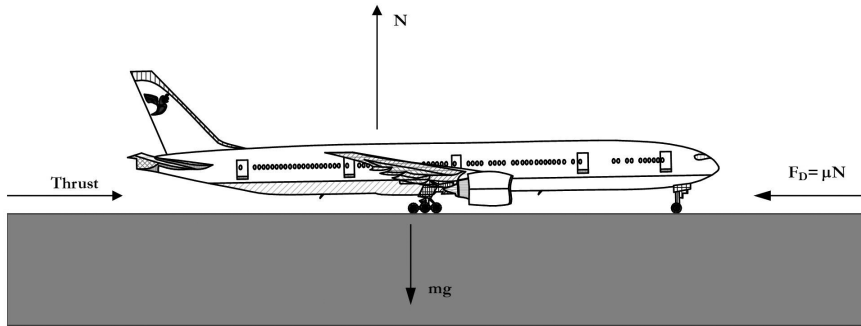


FIGURE 152 The Boeing 777-300ER ready for a level takeoff on runway 19 (drawings created using Solid Edge CAD tool).

Note that in these equations, drag is shown as a percentage of the component of the weight perpendicular to the runway surface, proportionate to a threshold coefficient of friction (μ). The initial force required to move the aircraft is larger than the one required to keep the aircraft moving since the threshold friction coefficient is larger than that of the moving aircraft—Table 81 and Table 82.

11.11.2 Case Study – Barrel Roll Acceleration

The Extra 300S aerobatic airplane located in Florida is performing a barrel roll maneuver as part of the training of a new student pilot. The revered female aerobatic instructor [239] suggests the student should look to the side when flying inverted to minimize the possibility of stomach upset. They are to achieve $3g$ acceleration so that the student can claim the experience on her application for an astronaut position. They achieve $3g$ acceleration at the bottom of their barrel roll maneuver. If the airspeed at this point is 343 km/hr and is increasing at the rate of $22 \text{ km/hr per second}$, calculate:

(a) radius of curvature of the path, (b) repeat part “a” for airspeed of 200 km/hr , and (c) compare the two answers.

A diagram of acceleration vectors is shown in Figure 153. Calculated data are presented in Table 83. The total acceleration is the resultant acceleration due to the tangent and normal acceleration component vectors to the barrel circumferential route, which is the vector sum of the two given vectors ($a_{total} = 3g$). The tangent component is the given airspeed acceleration ($a_{tangent}$). It needs to be expressed in (m/s^2) . The normal acceleration (a_{normal}) can then be obtained using the Pythagorean theorem ($a_{normal} = \sqrt{a_{total}^2 - a_{tangent}^2}$).

TABLE 82 Calculated acceleration, time, and distances for the Boeing 777-300ER.

UPHILL ($RFI = 0$)					UPHILL ($RFI = 0.15$)				
Runway Angle (θ)	0.75	°	0.013	rad	Runway Angle (θ)	0.75	°	0.013	rad
$\sin \theta$	0.013				$\sin \theta$	0.013			
Weight ($W_T = mg \sin \theta$)	45,140	N	45.14	kN	Weight ($W_T = mg \sin \theta$)	51,911	N	51.91	kN
$F_{net} =$ $F_T - mg(1 + \mu) \sin \theta$	980,860	N	980.86	kN	$F_{net} =$ $F_T - mg(1 + \mu) \sin \theta$	974,089	N	974.09	kN
Acceleration ($a = F / m$)	2.79		m/s ²		Acceleration ($a = F / m$)	2.77		m/s ²	
Time ($t = (v_0 - v) / a$)	30.36		s		Time ($t = (v_0 - v) / a$)	30.58		s	
Distance ($x = 0.5at^2 + v_0t$)	1,286	m	1.29	km	Distance ($x = 0.5at^2 + v_0t$)	1,295	m	1.30	km
	4,220	ft	0.69	NM		4,249	ft	0.70	NM
DOWNHILL ($RFI = 0$)					DOWNHILL ($RFI = 0.15$)				
Runway Angle (θ)	0.75	°	0.013	Rad	Runway Angle (θ)	0.75	°	0.013	rad
$\sin \theta$	0.013				$\sin \theta$	0.013			
Weight ($W_T = mg \sin \theta$)	45,140	N	45.14	kN	Weight ($W_T = mg \sin \theta$)	38,370	N	38.37	kN
$F_{net} =$ $F_T - mg(-1 + \mu) \sin \theta$	1,071,140	N	1,071.14	kN	$F_{net} =$ $F_T - mg(-1 + \mu) \sin \theta$	1,064,370	N	1,064.37	kN
Acceleration ($a = F / m$)	3.05		m/s ²		Acceleration ($a = F / m$)	3.03		m/s ²	
Time ($t = (v_0 - v) / a$)	27.81		s		Time ($t = (v_0 - v) / a$)	27.98		s	
Distance ($x = 0.5at^2 + v_0t$)	1,178	m	1.18	Km	Distance ($x = 0.5at^2 + v_0t$)	1,185	m	1.19	km
	3,864	ft	0.64	NM		3,889	ft	0.64	NM
LEVEL ($RFI = 0$)					LEVEL ($RFI = 0.15$)				
Runway Angle (θ)	0	°	0.000	Rad	Runway Angle (θ)	0	°	0.000	rad
$\sin \theta$	0.00				$\sin \theta$	0.000			
Weight ($W_T = 0$)	0.00	N	0.00	kN	Weight ($W_T = 0$)	517,282	N	517.28	kN
$F_{net} = F_T - mg$	1,026,000	N	1,026	kN	$F_{net} = F_T - mg$	508,718	N	508.72	kN
Acceleration ($a = F / m$)	2.92		m/s ²		Acceleration ($a = F / m$)	1.45		m/s ²	
Time ($t = (v_0 - v) / a$)	29.03		s		Time ($t = (v_0 - v) / a$)	58.55		s	
Distance ($x = 0.5at^2 + v_0t$)	1,230	m	1.230	Km	Distance ($x = 0.5at^2 + v_0t$)	2,480	m	2.48	km
	3,864	ft	0.64	NM		3,889	ft	0.64	NM

Note that the centripetal force causes the aircraft to turn around the center of the circular path. Therefore, the angular acceleration is to follow the relationship ($a_{normal} = r\omega^2$)—where ω (v/r) is the angular velocity and r is the radius of the barrel—and therefore $r = v^2/a_{normal}$. Note that for the given acceleration, the radius associated with the higher velocity is greater than that for the smaller velocity. This is expected since the angular acceleration is proportional to the square of the airspeed. Since the speed is decreased, the radius also needs to decrease to maintain the same acceleration.

TABLE 83 Calculated radius and centripetal acceleration for a barrel roll maneuver for the Extra 300S.

High Speed					Low Speed				
V_1	343.00	km/hr	95.28	m/s	V_2	200.00	km/hr	55.56	m/s
$a_{tangent}$	22.00	km/hr/s	6.11	m/s ²	$a_{tangent}$	22.00	km/hr/s	6.11	m/s ²
a_{total}			29.43	m/s ²	a_{total}			29.43	m/s ²
a_{normal}	28.79	m/s ²			a_{normal}	28.79	m/s ²		
R	315.33	m			R	107.21	M		

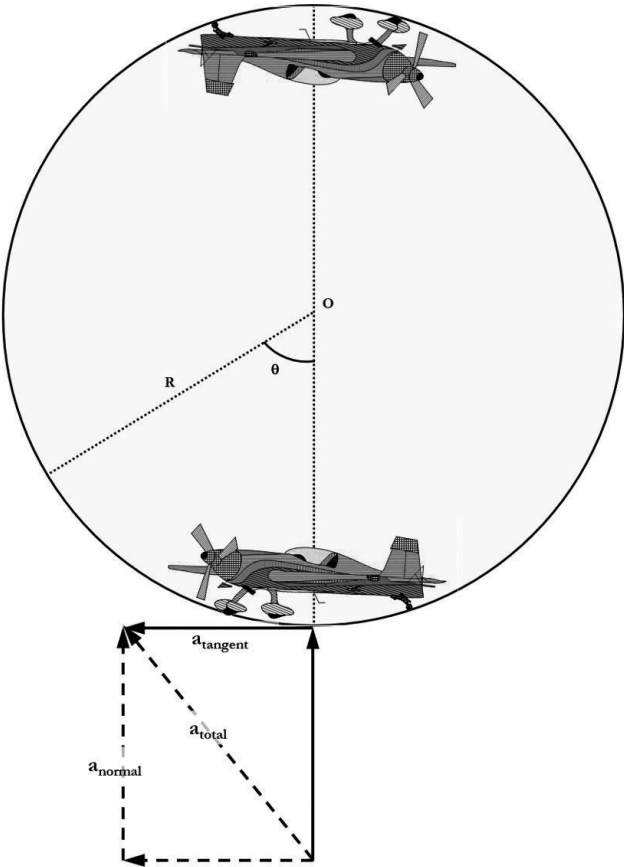


FIGURE 153 Flight path of the Extra 300S when performing a barrel roll maneuver (drawings created using Solid Edge CAD tool).

11.11.3 Case Study – Distance and Time to Point of Safe Return (PSR) Using Fuel Endurance

A very fuel-efficient aircraft is to take off from an aerodrome and fly at a cruise TAS of 200 kt to a destination 400 NM away. The outbound headwind is 20 kt. Tank capacity is 45 gal. The average fuel consumption for the start, taxi, takeoff, and climb is 7.8 GPH; the average consumption for the remainder of the trip, given 65 percent BHP and 2,300 RPM, is 6.5 GPH. The total time from the moment that aircraft Hobbs meter starts ticking to reaching the TOC is 15 min. A total reserve fuel for 45 min is required. In addition, the FIC requests 15 gal of discretionary fuel and also 15 percent of the total fuel tank precautionary requirement given the challenging terrain ahead. The fuel tank is full. Only the available cruise fuel is to be considered for PSR planning. Calculate:

(a) outbound time to PSR, (b) inbound time from PSR, (c) PSR distance, and (d) PSR distance for a no-wind scenario.

The first step is to estimate the cruise endurance (time and fuel) considering having a 45-min fuel reserve, discretionary fuel, precautionary fuel, and fuel needed for taxi and takeoff. Given the data presented in Table 84, 16.43 gal of cruise endurance fuel is obtained. Since the average fuel consumption for the trip cruise is 6.5 GPH, the cruise endurance time of 2.53 hrs (151.6 min) is calculated. This is the maximum time the aircraft can fly while cruising. Next the normalized wind speed is calculated as $-20/200 = -0.1$. The negative sign is applied since this is the outbound headwind case. To apply PSR calculations, the distance to climb to cruise altitude will be ignored. From the total endurance time (t_t) and the normalized wind (\mathcal{Q}_w), the outbound time to PSR ($t_{outbound}$) is obtained as 1.39 hrs and the inbound time ($t_{inbound}$) as 1.14 hrs. Using the given TAS, the distance from PSR to the departure point is obtained as 250.17 NM. If there is no wind, the normalized wind becomes zero and the distance from PSR is 252.7 NM, which simply equals the total maximum distance in the air divided by two ($v_T \frac{t_t}{2}$). Note that the distance to the destination given in the question is not relevant for PSR calculations.

$$t_{outbound} = (1 - \mathcal{Q}_w) \frac{t_t}{2} \quad t_{inbound} = (1 + \mathcal{Q}_w) \frac{t_t}{2} \quad (246)$$

$$t_{outbound} = (1 - \mathcal{Q}_w^2) v_T \frac{t_t}{2} \quad (247)$$

TABLE 84 Calculation of times and distances for point of safe return (PSR) scenario.

Tank Capacity (gal)	45	TAS (kt)	200
Time—Start to TOC (s)	900	Outbound Headwind (kt)	20
Fuel—Start to TOC (GPH)	7.8	Normalized Wind	-0.1
Fuel—Start to TOC (gal)	1.95	Time Outbound (hr)	1.39
Reserve Time (s)	2,700	Time Inbound (hr)	1.14
Fuel—TOC to Remaining (GPH)	6.5	Distance Inbound from PSR (NM)	250.17
Reserve Fuel (gal)	4.88		
Discretionary Fuel (gal)	15		
Variable Reserve (%)	15	For no-wind scenario	
Precautionary Fuel (gal)	6.75	Time Outbound (hr)	1.26
Cruise Fuel (gal)	16.43	Time Inbound (hr)	1.26
Cruise Endurance Time (hr)	2.53	Distance Inbound from PSR (NM)	252.7
Cruise Endurance Time (min)	151.62		

11.11.4 Case Study – Distance and Time to Point of Safe Return (PSR) Using Fuel Consumption per Distance

The Boeing 777-300ER capable of carrying total fuel of 145,538 kg is flying outbound to PSR at the altitude of FL300 with a fuel consumption rate of 7,350 kg/hr. It flies inbound from PSR at FL340 with a fuel consumption rate that follows a linear trend similar (and parallel) to the ATPL aircraft as plotted in Figure 154. The True Airspeed (TAS) is 481.5 kt for the Boeing and 310 kt for the ATPL aircraft. For the Boeing 777-300ER, consider Case 1 of outbound tailwind of 38 kt and Case 2 of outbound headwind of 20 kt; for the ATPL aircraft (ATPL-AC), consider the case of 30 kt outbound tailwind. For the data presented in Table 85, calculate:

(a) outbound and inbound ground speeds for the headwind and tailwind values in Table 85, (b) fuel consumption per distance outbound, (c) fuel consumption per distance inbound, (d) total fuel consumption per distance, (e) distance to PSR, (f) time to PSR in minutes and hours, and (g) total distance outbound in case of no PSR.

Figure 154 shows the fuel consumption versus the altitude for the ATPL aircraft. The fuel rate of consumption for the Boeing 777-300ER follows a parallel line (the same slope) with that of the ATPL and is calibrated, considering its rate of fuel consumption at FL300. In other words, the rate of fuel consumption at FL300 should result in 7,350 kg/hr. Using the new predicted curve with the intercept of 14,475 kg/hr, at FL340, the rate of

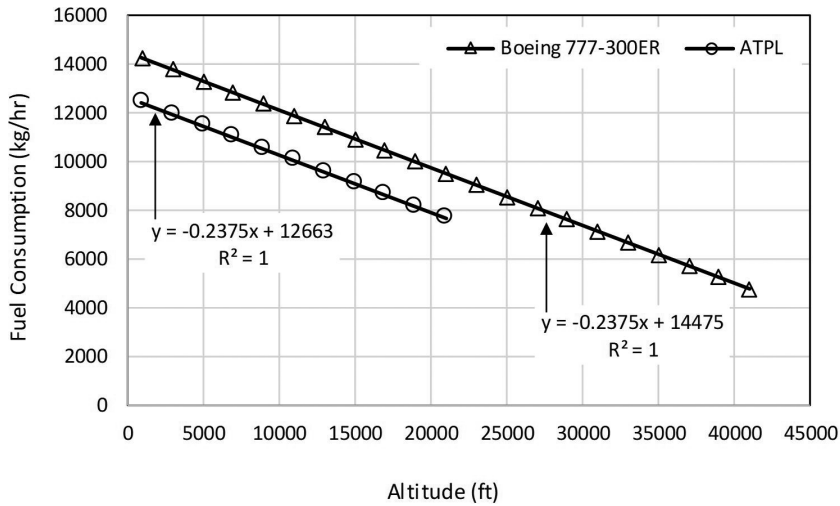


FIGURE 154 Fuel consumption versus the altitude for the ATPL and the Boeing 777-300ER aircraft.

TABLE 85 Calculated time and distance to point of safe return (PSR) based on the fuel consumption.

Item	ATPL AC	Boeing 777-300ER (Case 1)	Boeing 777-300ER (Case 2)
Total Fuel (kg)	39,500	145,538	145,538
Altitude Outbound (ft)	27,000	30,000	34,000
Fuel Outbound (kg/hr)	6,250	7,350	6,400
Altitude Inbound (ft)	31,000	34,000	34,000
Fuel Inbound (kg/hr)	5,300	4,588	4,588
TAS (kt)	310	481.46	481.46
Outbound Tailwind (kt)	30	38	-20
Outbound Ground Speed (kt)	340	519.46	461.46
Inbound Headwind (kt)	30	38	-20
Inbound Ground Speed (kt)	280	443.46	501.46
CO—Fuel Consumption Outbound (kg/NM)	18.38	14.15	30.85
CH—Fuel Consumption Inbound (kg/NM)	22.32	16.57	28.39
Fuel (Total: CO + CH) (kg/NM)	40.70	30.72	59.25
Distance to PSR (NM)	485.21	2,368.51	1,228.27
Time to PSR (hr)	1 hr 26 min	4 hrs 34 min	2 hrs 40 min
Time to PSR (min)	85.63	273.57	159.70
Time from PSR Inbound (hr)	1 hr 26 min	4 hrs 34 min	2 hrs 40 min
Time from PSR Inbound (min)	103.97	320.46	146.96
Total Time (min)	189.60	594.03	306.66
Total Time (hr)	3 hrs 10 min	9 hrs 54 min	5 hrs 7 min
Total Outbound Distance (NM)	1,074.40	5,142.94	2,358.56
Total Outbound-Inbound Distance (NM)	970.43	4,737.02	2,456.53

fuel consumption of 6,400 kg/hr is obtained. The rate of fuel consumption is then used along with the inbound and outbound ground speeds to obtain the inbound and outbound fuel consumption per distance. The summation of the two values are the total fuel consumption per length. The total distance that aircraft may fly given the total fuel consumption per length is the ratio of the total fuel on board to the latter summation. The distance to PSR is half of this total distance. Knowing the distance to PSR, inbound, and outbound ground speeds, the inbound and outbound times to PSR are then calculated. The total distance outbound is the distance that the aircraft would be able to fly if it did not have to return (i.e., constant outbound ground speed from the departure point).

11.11.5 Case Study – Distance and Time to Critical Point (CP)

Consider Case Study 3 “Distance and Time to PSR Using Fuel Consumption per Distance”. For the data given in Table 86, for cases of no wind, tailwind, headwind, wind at 45°, change of the headwind in halfway, and one-engine failure, calculate:

(a) distance to CP, (b) time to CP in minutes and in hours, (c) remaining distance to the destination, (d) remaining time to the destination, and (e) distance and time to CP while also accounting for the case of one engine failure in a two-engine aircraft.

Calculations based on equation (219) are presented in Table 86. Note that to verify the technique used, you may calculate the time inbound and outbound based on the predicted CP distance and ground speeds. The time to return from CP back to the departure point and time to proceed from CP to the destination are to be equal. Another point to keep in mind is that the wind component parallel to your bearing is to be included in the calculations, meaning that if the wind is directed at an angle with respect to the magnetic track direction, its component parallel to this direction (either headwind or tailwind) is to be taken into consideration. If the sine component of wind is used for crosswind calculations, the cosine component is to be used to calculate the headwind (or tailwind) components. In other words, choice of the sine or cosine depends on the relative angle between the wind and the track.

The case of two-engine flight (2-E) is investigated in which the tailwind changes to the headwind halfway along the route. In this situation, CP in a two-engine flight scenario is 985 NM, with the arrival time of under 114 min. Assuming that the aircraft loses one engine and as a result the altitude and TAS decrease, the critical distance will reduce to under

975 NM. Note that the TAS of the aircraft, after being reduced for the loss of altitude after engine failure, is not considered for the time to get to CP in this scenario. You are to use the original TAS for the case of a two-engine operation (475 kt) to calculate the time to get to CP, which is just under 113 min. The reason for this small variation in time between the two scenarios is that the distance to CP is proportional to the ratio of the inbound and outbound relative velocities, and the ratio between the two scenarios is approximately 1.08 versus 1.10, resulting in about a 2.13 percent variation in ground velocity ratio of the two scenarios.

TABLE 86 Calculated time and distance to critical point (CP) (1-E: One Engine, 2-E: Two Engines).

Item	No Wind Direction Change				Change to Headwind Halfway		
	No Wind 2-E	Tailwind 2-E (38 kt)	Headwind 2-E (20 kt)	Headwind 2-E (15 kt@45°)	Tailwind 2-E (45 kt) Headwind (10 kt)	Tailwind 1-E (45 kt) Headwind (10 kt)	Tailwind 2-E (45 kt) Headwind (10 kt)
Total Distance (NM)	400	600	800	1,000	2,050	2,050	2,050
TAS (kt)	481.46	481.46	481.46	481.46	475.00	380.00	475.00
Outbound Tailwind (kt)	0.00	38.00	-20.00	-10.61	-10.00	-10.00	-10.00
Outbound Ground Speed (kt)	481.46	519.46	461.46	470.85	465.00	370.00	465.00
Inbound Headwind (kt)	0.00	38.00	-20.00	-10.61	45.00	45.00	45.00
Inbound Ground Speed (kt)	481.46	443.46	501.46	492.07	430.00	335.00	430.00
Distance to CP (NM)	200.00	276.32	416.62	511.02	984.92	974.11	984.92
Time to CP (hr)	0.415	0.532	0.903	1.085	2.117	1.867	1.900
Time to CP (min)	24.9	31.9	54.2	65.1	127.09	112.40	113.64
Distance from CP to Destination (NM)	200.00	323.68	383.38	488.98	1,065.08	1,075.89	1,065.08
Time from CP to Departure (hr)	0.415	0.623	0.831	1.039	2.283	2.267	2.283
Time from CP to Departure (min)	24.9	37.4	49.8	62.3	137.43	135.92	137.43
Time from CP to Destination (hr)	0.415	0.623	0.831	1.039	2.283	2.317	2.283
Time from CP to Destination (min)	24.9	37.4	49.8	62.3	137.43	138.82	137.43

11.11.6 Case Study – Descent Planning

For the flight levels and altitudes identified in Table 87, using the methodology presented for finding the descent decision point while assuming that the heading is zero degrees, calculate:

(a) horizontal distance from the destination aerodrome where the descent is to start, (b) descent rate to give a descent slope of 3°, (c) headwind given the relative angle of the wind to the aircraft heading, and (d) ground speeds.

TABLE 87 Distance to destination aerodrome and descent rate calculations.

TAS (kt)	Wind (kt@° to course)	Headwind (kt)	Ground Speed (kt)	Altitude	Altitude (ft)	Alt/1,000	3×(ALT/1,000)	Descent Rate (fpm)
475	090°@35kt	0.00	475.00	FL410	41,000	41	123	2,375
350	20°@20kt	18.79	331.21	FL80	8,000	8	24	1,656
300	210°@25kt	-21.65	321.65	5,000	5,000	5	15	1,608
110	075°@15kt	3.88	106.12	3,500	3,500	3.5	10.5	531
100	190°@10kt	-9.85	109.85	2,000	2,000	2	6	549
70	060°@15kt	7.50	62.50	1,000	1,000	1	3	313

11.11.7 Case Study – Engine Failure Physics

For the conditions specified in Table 88, using the physics of engine failure and conservation of energy, taking into consideration the wind magnitude and direction, and assuming that the heading is zero degrees, calculate:

(a) speed in m/s, (b) mass in kg, (c) altitude in m, (d) gravitational acceleration in m/s^2 , (e) kinetic energy in MJ, (f) potential energy in MJ, (g) total energy in MJ, (h) drag in MJ between the cases “n” and “n-1”, (i) maximum touchdown speed based on the final energy, (j) touchdown speed based on the total initial energy—obtained in Case 1, 2,500 lb and 3,500 ft altitude, (k) impact forces given the speeds obtained based on the initial energy after 0.5 s, and (l) impact forces given the speeds obtained based on the total final energy after 0.5 s.

Table 88 and Table 89 present the data on which calculations are performed. Note that 1 lb = 0.453 kg, 1 ft = 0.305 m, and 1 kt = 0.514 m/s. The kinetic energy (K_E) is a function of the mass (m) of the object and the square of velocity (i.e., ground speed, $v^2 = GS^2$). The potential energy (P_E) is a function of the mass of the object and its difference in height (h) from a reference point (mgh). The gravity acceleration (g) is related to the gravitational constant (G), mass of the Earth (m_{Earth}), and the inverse second power of the distance from the center of the Earth (radius of the Earth, r , plus the distance from the references point, h). To calculate the maximum touchdown speed based on the initial energy, use the total energy for Case 1 (2,500 lb and 3,500 ft altitude)— $T_E = K_E + P_E$, consisting of the kinetic and potential energies. To calculate the impact force at the end of the journey based on the initial energy, use data related to Case 1; otherwise, the data for each scenario is used. Note that the cases indicate the progression of the aircraft as it glides to a safe landing; in other words,

TABLE 88 Engine failure and impact data (part 1).

Case	Wind (+Tailwind, -Headwind)	Mass (lb)	g (m/s ²)	Total Energy E (MJ)	Drag (MJ)	Speed (m/s)-Final Energy	Speed (m/s)-Initial Energy	Force (KN)-Final Energy	Force (KN)-Initial Energy
1	0	2,500	9.82	14.41	Base Point	159.43	159.43	797.13	361.57
2	0	2,400	9.82	11.51	2.90	145.45	162.71	698.15	354.27
3	0	2,300	9.82	8.92	2.59	130.79	166.21	601.62	346.81
4	0	2,200	9.82	6.62	2.30	115.18	169.95	506.79	339.18
5	T20kt@ 100°	2,100	9.82	4.89	1.73	101.31	173.95	425.49	331.39
6	20	2,000	9.82	3.22	1.67	84.28	178.24	337.13	323.40
7	H10kt@ 022°	1,900	9.82	1.44	1.79	57.71	182.87	219.31	315.21
8	-20	1,800	9.82	0.56	0.88	36.91	187.89	132.89	306.80
9	0	2,100	9.82	8.15	-7.59	130.79	173.95	549.30	331.39
10	0	1,900	9.82	7.37	0.78	130.79	182.87	496.99	315.21

TABLE 89 Engine failure and impact data (part 2).

Case	Wind (+Tailwind, -Headwind)	Mass (lb)	Mass (kg)	Speed (kt)	Ground Speed (kt)	Speed (m/s)	Altitude (ft)	Altitude (m)	P_E (MJ)	K_E (MJ)
1	0	2,500	1,133.98	130.00	130.00	66.88	3,500	1,066.80	11.88	2.54
2	0	2,400	1,088.62	110.00	110.00	56.59	3,000	914.40	9.77	1.74
3	0	2,300	1,043.26	90.00	90.00	46.30	2,500	762.00	7.80	1.12
4	0	2,200	997.90	70.00	70.00	36.01	2,000	609.60	5.97	0.65
5	T20kt@ 0100°	2,100	952.54	50.00	69.70	35.85	1,500	457.20	4.28	0.61
6	20	2,000	907.18	45.00	65.00	33.44	1,000	304.80	2.71	0.51
7	H10kt@ 022°	1,900	861.82	45.00	35.73	18.38	500	152.40	1.29	0.15
8	H20kt@ 0°	1,800	816.47	45.00	25.00	12.86	200	60.96	0.49	0.07
9	0	2,100	952.54	90.00	90.00	46.30	2,500	762.00	7.13	1.02
10	0	1,900	861.82	90.00	90.00	46.30	2,500	762.00	6.45	0.92

the elevations are decreasing with the case numbers increasing from 1 to 10. Additionally, the weight of the aircraft is decreasing; assume that this is due to the loss of fuel, discharging the luggage, or parachuting the flight crew members.

Note that the impact force is the force applied from the object (e.g., haystack) to the aircraft at the time of a gentle hit (or collision)—where the aircraft stops, meaning that the final velocity is zero, and therefore the time derivative of the velocity is negative as the aircraft decelerates. The data presented in Table 88 and Table 89 concern the magnitude of this impact; the direction of this impact is opposite to that of the aircraft movement.

Following are the steps to be taken when completing Table 88 and Table 89:

1. Obtain the total energy, consisting of potential and kinetic energy at the beginning of the glide path.
2. Using the total energy obtained in step 1, and related mass, calculate the effective velocity of the aircraft.
3. Repeat steps one and two for each elevation, and at the end of the calculation, either use the mass of the same step or the original mass to calculate the speed based on the initial or current energy levels.
4. Acceleration after the landing may be obtained by $a = (v - v_0)/t = F/m$. In this relation, the ultimate speed is zero since the haystack will bring the aircraft to a stop. The initial velocity is then calculated using the total energy.
5. Energy wasted due to the drag is the difference between the total energy for the current case and the previous case.
6. Time is the elapsed time given (0.5 s).
7. Impact force may then be calculated from $F = ma$, where the acceleration (a) is obtained from the previous step and mass (m) is known.
8. Distance after the landing may be calculated by $x = 0.5at^2 + v_0t$.

Calculations are represented in Table 88 and Table 89.

11.11.8 Case Study – Gyro Errors and Readings

For the scenarios presented in Table 90 based on the original perfect DGI readings, latitude, horizontal easterly component of speed, and time, determine:

(a) location latitude in radians, (b) the Earth rotation rate in degrees per hour, (c) transport wander per hour, (d) total wander per hour, (e) total wander given the flight time, (f) gyro reading at the destination based on the total wander, (g) latitude nut, (h) real wander, (i) total drift per hour, (j) total drift, and (k) gyro reading at the destination based on the total drift.

Note that the perfect DGI is at the workshop or the location where its latitude nut is set (i.e., column “Latitude” in Table 90). The track at which the aircraft flies, flight time, and ground speed at which the aircraft is traveling are also presented. Since the DGI is assumed to be perfect, the real wander is zero. A summary of calculations is presented in Table 91.

TABLE 90 Gyro wander and ultimate readings.

Case	Location Latitude (°)	Latitude (rad)	Earth Rate (°/hr)	u (Easterly Component Speed in kt)	Transport Wander (°/hr)	Transport Wander (°/hr)	Total Wander (°/hr)	Time (min)	Total Wander (°)	Gyro Original Reading (°)	Gyro Reading at Destination (°)
1	0	0.00	0.00	540	0.00	0.00	0.00	80	0.00	270	270.00
2	10	0.17	-2.60	520	1.53	1.53	-1.08	90	-1.61	240	238.39
3	20	0.35	-5.13	500	3.03	3.03	-2.10	100	-3.50	210	206.50
4	30	0.52	-7.50	480	4.62	4.62	-2.88	110	-5.28	180	174.72
5	40	0.70	-9.64	460	6.43	-6.43	-16.07	120	-32.15	150	117.85
6	50	0.87	-11.49	350	6.95	6.95	-4.54	105	-7.94	270	262.06
7	50	0.87	-11.49	350	6.95	-6.95	-18.44	105	-32.27	90	57.73
8	60	1.05	-12.99	330	9.53	-9.53	-22.52	115	-43.16	60	16.84
9	70	1.22	-14.10	310	14.20	-14.20	-28.29	125	-58.94	30	-28.94
10	85	1.48	-14.94	290	55.25	-55.25	-70.19	135	-157.92	0	-157.92
Case	Location Latitude (°)	Latitude (rad)	Earth Rate (°/hr)	Transport Wander (°/hr)	Total Wander (°/hr)	Calibration Latitude (°)	Calibration Latitude (rad)	Latitude Nut Error (°/hr)	Real Wander (°/hr)	Total Drift (°/hr)	Total Drift (°)
1	0	0.00	0.00	0.00	0.00	40	0.70	9.64	0	9.64	12.86
2	10	0.17	-2.60	1.53	-1.08	40	0.70	9.64	0	8.57	12.85
3	20	0.35	-5.13	3.03	-2.10	45	0.79	10.61	0	8.51	14.18
4	30	0.52	-7.50	4.62	-2.88	45	0.79	10.61	0	7.73	14.16
5	40	0.70	-9.64	-6.43	-16.07	50	0.87	11.49	0	-4.58	-9.17
6	50	0.87	-11.49	6.95	-4.54	55	0.96	12.29	0	7.75	13.56
7	50	0.87	-11.49	-6.95	-18.44	55	0.96	12.29	0	-6.16	-10.77
8	60	1.05	-12.99	-9.53	-22.52	60	1.05	12.99	0	-9.53	-18.26
9	70	1.22	-14.10	-14.20	-28.29	60	1.05	12.99	0	-15.30	-31.88
10	85	1.48	-14.94	-55.25	-70.19	65	1.13	13.59	0	-56.59	-127.34

TABLE 91 Gyro total wander, total drift, and readings at the destination.

Total Wander (°)	Gyro Reading at Destination Based on “Total Wander” (°)	Total Drift (°)	Gyro Reading at Destination Based on “Total Drift” (°)
0.00	270.00	12.86	282.86
-1.61	238.39	12.85	252.85
-3.50	206.50	14.18	224.18
-5.28	174.72	14.16	194.16
-32.15	117.85	-9.17	140.83
-7.94	262.06	13.56	283.56
-32.27	57.73	-10.77	79.23
-43.16	16.84	-18.26	41.74
-58.94	-28.94	-31.88	-1.88
-157.92	-157.92	-127.34	-127.34

11.11.9 Case Study – Cabin Altitude and Climb Rate

An aircraft is to climb from the sea level to 19,000 ft at a climb rate of 1,600 ft/min (fpm). Use 1,000 ft of difference (buffer) between the set and actual altitudes. The cabin should reach the set altitude 2 min before the time to reach the TOC. The pressurization controller is then set to 20,000 ft and the corresponding cabin altitude is 6,500 ft on the dial. Calculate the required cabin climb rate.

Using the variables presented in Table 92 and using equation (239), the cabin climb rate of 658 fpm is obtained.

TABLE 92 Cabin pressure versus the altitude.

Variable	Value
H_{A-D}	20,000 ft
ΔH_A	1,000 ft
\dot{H}_A	1,600 fpm
H_0	0 ft
H_C	6,500 ft
Δt_C	2 min

11.11.10 Case Study – Cabin Altitude Descent Rate

An aircraft is to descend from a cruise altitude of 19,000 ft to an airport at sea level with a 500 ft difference (buffer) setting for the cabin altitude. The cabin altitude during cruise is 6,500 ft. The aircraft descent rate is 1,600 ft/min (fpm) and set cabin altitude is to be reached 2 min before reaching the aircraft altitude target. Calculate the required cabin descent rate.

Using the variables presented in Table 93 and equation (241), a cabin descent rate of 608 fpm can be obtained.

TABLE 93 Cabin pressure versus the altitude.

Variable	Value
H_A	19,000 ft
ΔH_C	500 ft
\dot{H}_A	1,600 fpm
H_0	0 ft
H_C	6,500 ft
Δt_C	2 min

11.11.11 Case Study – Solar Balloon

A solar balloon was reportedly invented in 1972 by Dominic Michaelis, an Anglo-French inventor and architect. The air inside such a balloon heats up by means of solar thermal radiation, which may be augmented by the heat from a propane burner (as in a regular hot air balloon). The balloon fabric is either covered by a coating that is highly absorptive to the Sun's spectral-directional radiation (given the wavelength and direction) or is made from a material with high absorptivity. The color may be black or any other color (e.g., diffusive gray) with high absorption properties. It is also possible to make the balloon of a material that reflects the thermal radiation. In this case, the thermal radiation is not absorbed by the surface and, although the emissivity of the surface may be close to 100 percent (given the wavelength and direction of the irradiance, also known as the radiant flux), the thermal radiation absorption is minimum. Radiation intensity is the total energy reaching the surface per unit time per unit area. It is possible to either make the entire balloon surface of the absorptive material or have a balloon in which part of the fabric is absorbing and the

rest is reflecting. With this arrangement, by rotating the balloon to the direction of the source of irradiance (the Sun), the exposed exterior heated surface warms the air inside the balloon adjacent to the interior of the heated surface and the convective flow facilitates the heat transfer to the rest of the interior gas molecules. As a result, the density of the air inside the balloon decreases, and lift force is generated due to the buoyancy. On the other hand, when the reflective surface is exposed to the Sun's radiation, the thermal radiation is reflected back to the environment, meaning that the gas molecules adjoining the surface cool down, the temperature of the interior decreases, and the lift force is reduced.

The envelope of a hot-air solar balloon is made of special grade of Nylon, known as Nomex, which is a polymer with special molecular structure of aromatic backbones. The balloon envelope consists of two laser-joined hemispheres. One half is made of Nomex pigmented with absorbing agents, which result in an absorptivity of 0.8. The other half is made of a special grade of Nomex with absorptivity of 0.2. Due to the pigmentation, the color of the absorbing half is darker than the reflective half.

It is assumed that the pilot is capable of controlling the orientation of the balloon so that either the absorptive or the reflective part of the balloon is exposed to the Sun. The minimum value for the solar radiation is $1,000 \text{ W/m}^2$ and the maximum value is $1,361 \text{ W/m}^2$. The balloon is to operate in the temperature range of 20°C to 120°C . Solar radiation is the only source of heat, the envelope transfers heat to the environment by means of convection, and energy storage by the air mixture molecules is time independent (i.e., steady-state analysis) and occurs during a constant volume process.

Air consists of one mole of diatomic oxygen and 3.76 moles of diatomic nitrogen. Heat capacities of the nitrogen and oxygen at constant pressure are temperature-dependent. Thus, heat capacity of the air mixture is also temperature-dependent. Table 94 presents the heat capacities of the two elements at constant pressure and density of air as functions of temperature. The approximate vertical speed of the balloon is 10 m/s ; its radius is 8 m and empty weight is 600 lb . There are two passengers on board (110 lb and 210 lb).

For this balloon, assuming a spherical envelope shape, obtain the following:

(a) diagram of air density versus the temperature, (b) diagram of heat capacity at constant volume versus the temperature, (c) diagram of solar

TABLE 94 Data used for the balloon thermal radiation analysis.

Convection ($\text{W}/\text{m}^2\text{K}$) - h_c	5.000	Sun's Radiation (W/m^2) - P/A	1000.000
Radius (m) - R	8.000	Power by Area (W) - P	314159.265
Area (m^2) - A	314.159	Shape Factor - SF	0.500
Volume (m^3) - V	523.599	Sun's Power by Area (W) - P	157079.633
Ah_c (W/K)	1570.796	Solar Constant (W/m^2) - S_0	1360.800
Average Velocity (m/s) - v	10	Drag Coefficient - C_D	0.38
sigma ($\text{W}/\text{m}^2\text{K}^4$) - σ	5.67E-08	Solar Power Density (W/m^2) - Q	1314.580
Emissivity - e	0.850	Initial Temperature ($^{\circ}\text{C}$) - T_0	20.000
σAe (W/K^4)	1.425E-05	Surrounding Temperature ($^{\circ}\text{C}$) - T_{∞}	25.000
Fitted ρC_V using the C_p and ρ diagrams		$\rho C_V = -0.0029 T_s (\text{degC}) + 0.9201$	
Nitrogen Heat Capacity at Constant Pressure (kJ/kmolK) - $\overline{C_{PN_2}}$		$\overline{C_{PN_2}} = 39.06 - 512.79 \left(\frac{T}{1000} \right)^{-1.5} +$ $1072.7 \left(\frac{T}{1000} \right)^{-2} - 820.4 \left(\frac{T}{1000} \right)^{-3}$	
Oxygen Heat Capacity at Constant Pressure (kJ/kmolK) - $\overline{C_{PO_2}}$		$\overline{C_{PO_2}} = 37.432 + 0.020102 \left(\frac{T}{1000} \right)^{1.5} -$ $178.57 \left(\frac{T}{1000} \right)^{-1.5} + 236.88 \left(\frac{T}{1000} \right)^{-2}$	
Universal Gas Constant (kJ/kmolK) - \overline{R}		$\overline{C_p} - \overline{C_p} = \overline{R}$	
Solar Radiation Power on Day n (W) - P_s		$P_s = F_{1-2} A S_0 \left[1 + 0.034 \cos \left(2\pi \frac{n}{365.25} \right) \right]$	
Rate of Change of Internal Energy (W) - P_i		$P_i = V (\rho C_V)_T \left(\frac{T - T_{\infty}}{dt} \right)$	
Convection Loss (W) - P_c		$P_c = Ah_c (T_s - T_{\infty})$	
Radiation Loss (W) - P_r		$P_r = \sigma Ae (T_s^4 - T_{\infty}^4)$	
Conservation of Energy (W)		$P_s - (P_r + P_c + P_e) = 0$	

power versus the day of the year for different shape factors (0.5, 0.75, and 1), (d) solar power reaching the surface of the balloon assuming a shape factor of 0.75, and emissivity of 0.85, (e) loss of energy due to convection, (f) loss of energy due to radiation, assuming the emissivities of 0.85 and 1, (g) change of air internal energy, (h) balance of energy for the envelope, (i) solving the energy balance equation for the temperature for a steady-state process, where the derivative of the temperature with respect to the time is ignored, (j) steady-state temperature versus the day of the year for the emissivities of 0.85 and 1, (k) effective thermal radiation temperature in degrees Celsius and Kelvin—given the solar radiation and using Stephen Boltzmann radiation law, (l) total weight of the balloon including the passengers assuming that there are two passengers on board (110 lb, and 210 lb) and the mass of the empty balloon is 600 lb, (m) buoyancy force given the temperature difference between the air inside the balloon envelope and the surrounding environment, (n) drag force applied to the vertical speed assuming that the coefficient of drag for a hollow sphere facing the flow is 0.38, (o) buoyancy versus the temperature, (p) determine if the buoyancy is large enough for the balloon to remain afloat, (q) buoyancy versus the day of the year and shape factor, (r) balance of forces, (s) acceleration due to the balance of forces, (t) time required to reach a vertical speed of 10 m, and (u) vertical distance traveled given the acceleration and time.

Table 94 and Table 95 present the data as well the equations used for this study. Figure 155 to Figure 167 show the calculated data versus the parameters requested.

TABLE 95 Data calculated for the balloon thermal radiation analysis.

Density of Air (kg/m ³) - ρ_{air}	$\rho_{air} = 1.295e^{-0.004T(^{\circ}C)}$
Buoyancy Force (N) - F_B	$F_B = V(\rho_o - \rho_{air}) / \rho_{air}$
Weight of Passengers on board (kg) - F_W	$F_W = 110\text{ lb} + 210\text{ lb} = 320\text{ lb} \times 0.453 = 145.15\text{ kg}$
Drag Force (N) - F_D	$F_D = 0.5C_D\rho_{air}v^2$
Lift Force (N) - F_D	$F_L = F_B + F_W + F_D$
Acceleration (m/s ²) - a	$a = F_L / m = F_L / (\rho V)$
Time to Accelerate to reach 10 m/s (s) - t	$t = (v - v_0) / a$
Distance (m) - x	$x = 0.5at^2 + v_0t$

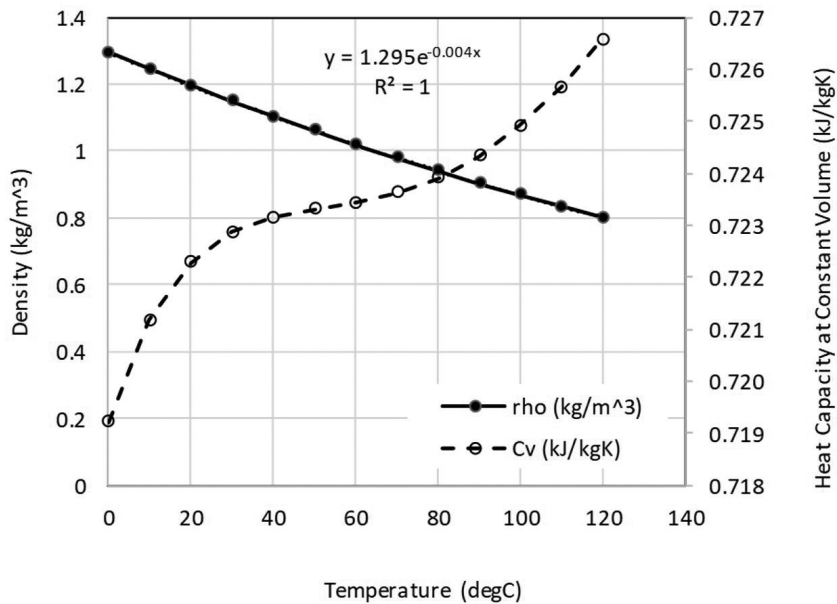


FIGURE 155 Air density and heat capacity at constant volume versus the temperature.

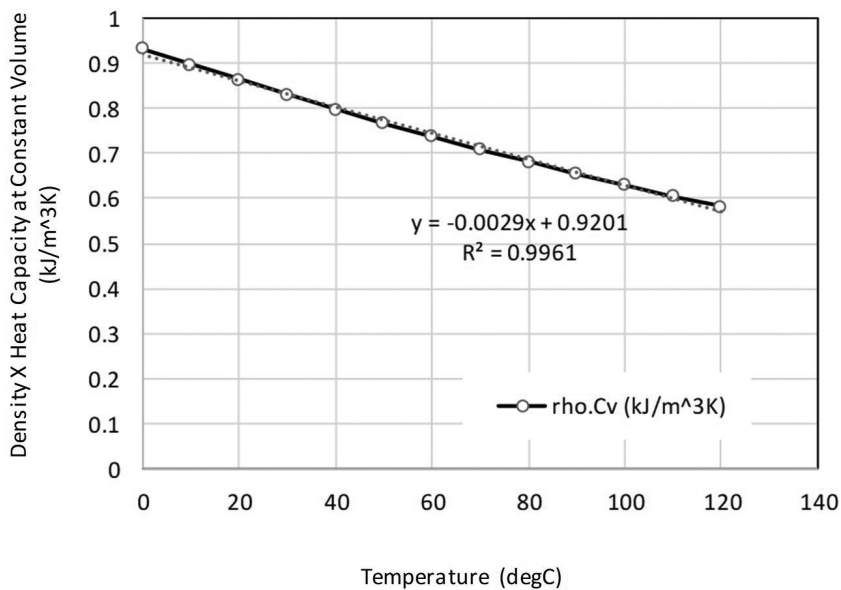


FIGURE 156 Air density and heat capacity product at constant volume versus the temperature.

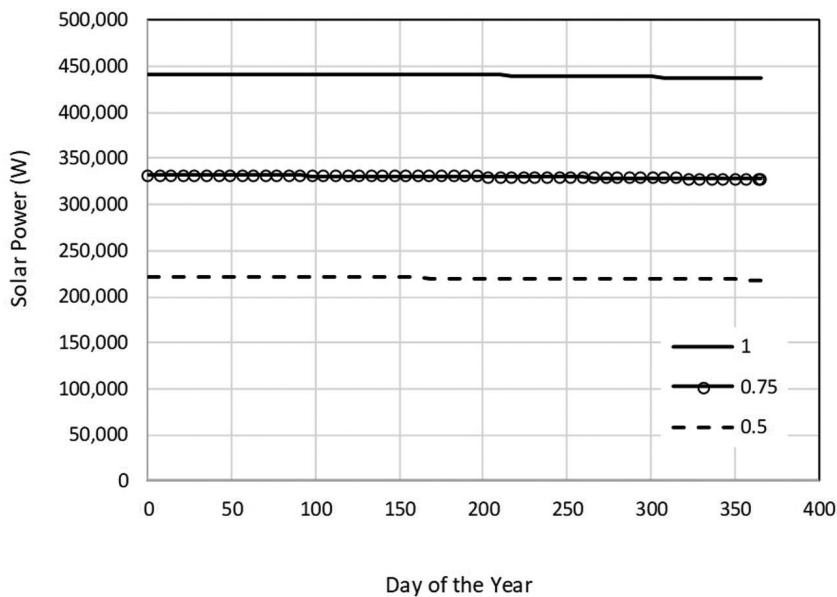


FIGURE 157 Solar power versus the day of the year for different shape factors.

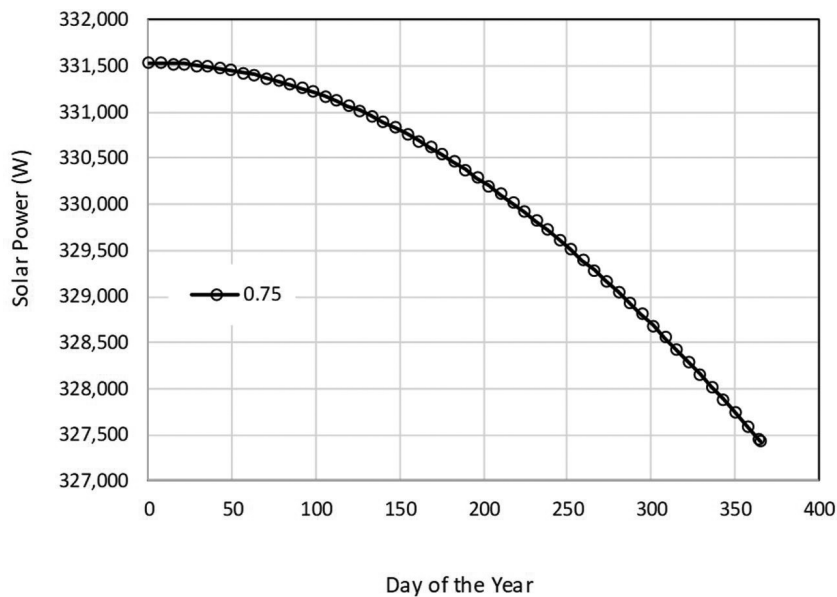


FIGURE 158 Solar power versus the day of the year (SF = 0.75).

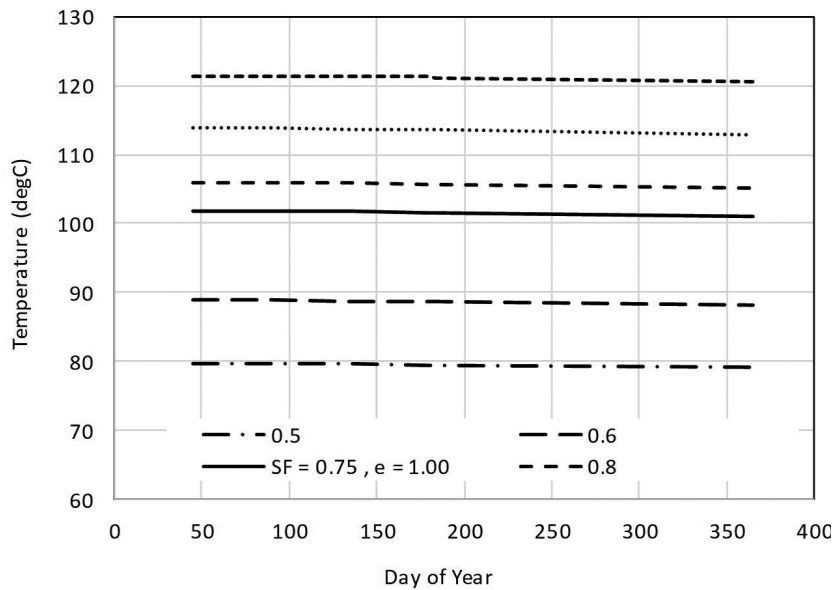


FIGURE 159 Steady-state temperature versus the day of the year for different shape factors ($e = 1$).

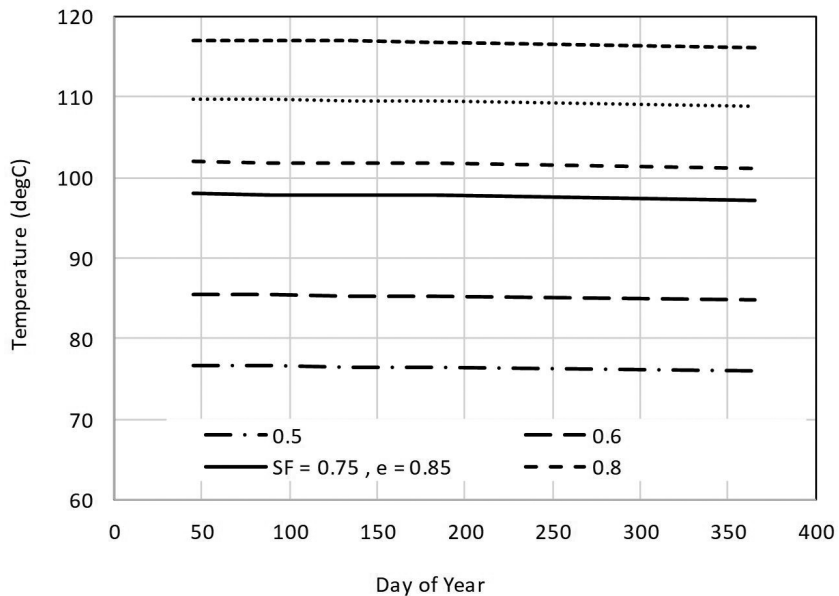


FIGURE 160 Steady-state temperature versus the day of the year for different shape factors ($e = 0.85$).

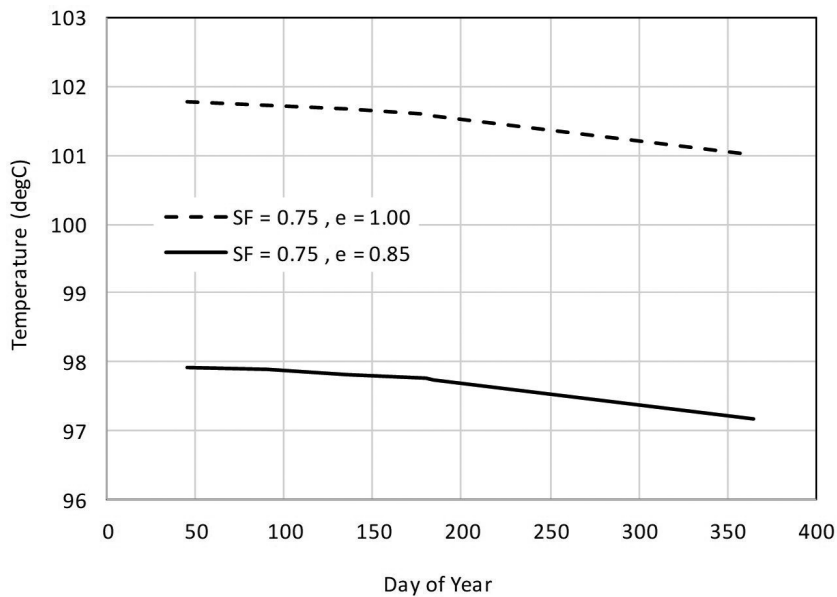


FIGURE 161 Steady-state temperature versus the day of the year (SF = 0.75).

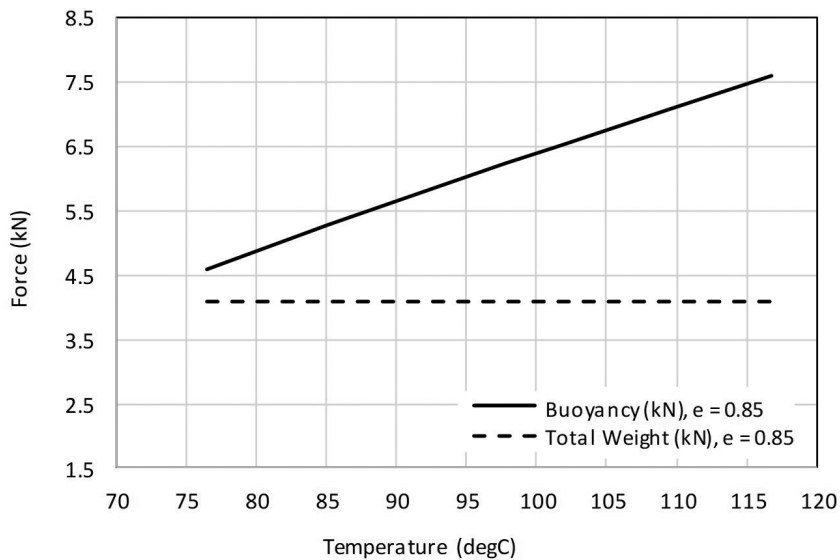


FIGURE 162 Buoyancy and total weight versus the steady-state temperature (e = 0.85).

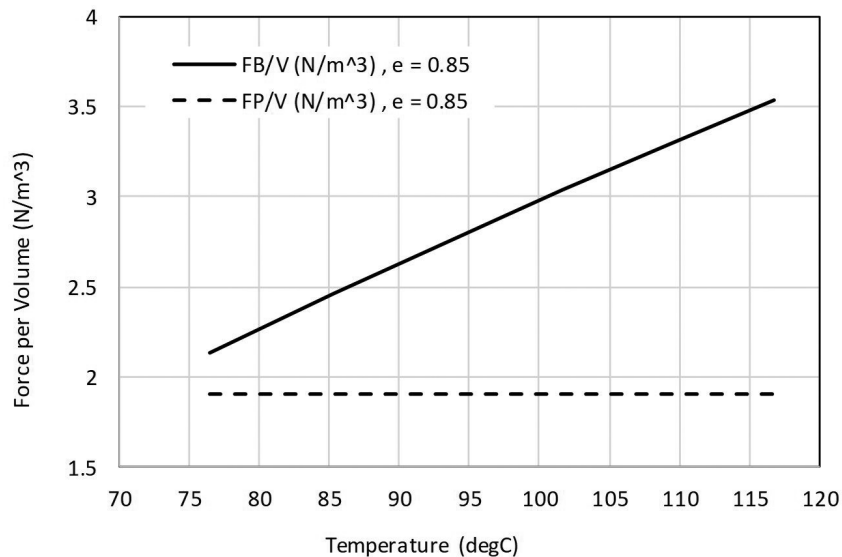


FIGURE 163 Buoyancy and payload per volume versus the steady-state temperature ($e = 0.85$).

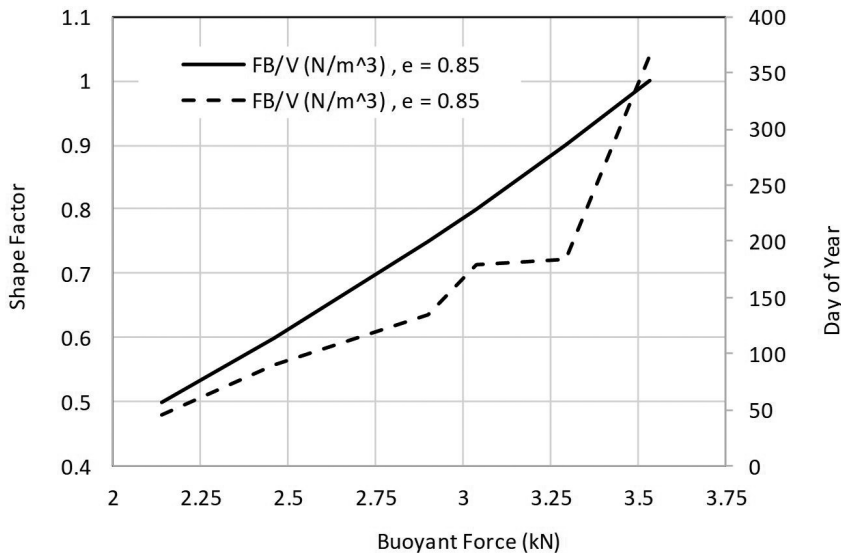


FIGURE 164 Shape factor and day of year versus the lift force ($e = 0.85$).

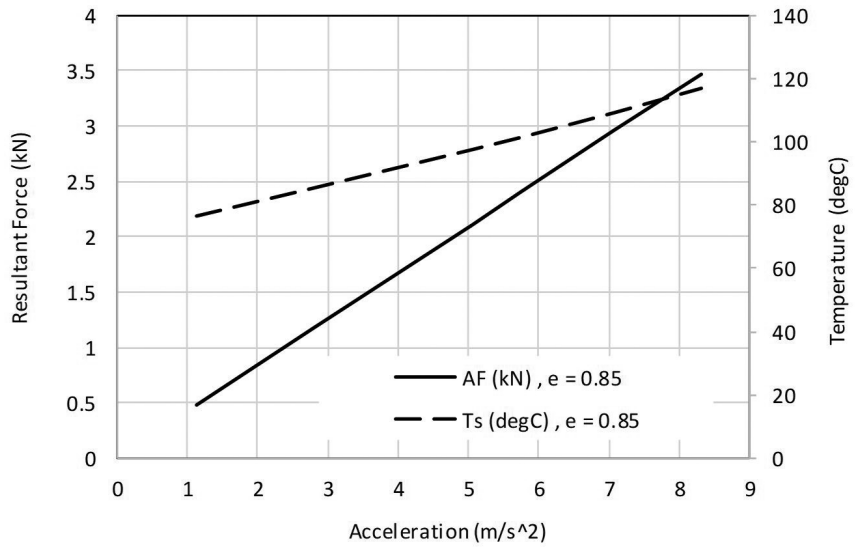


FIGURE 165 Resultant force and steady-state temperature versus the acceleration ($e = 0.85$).

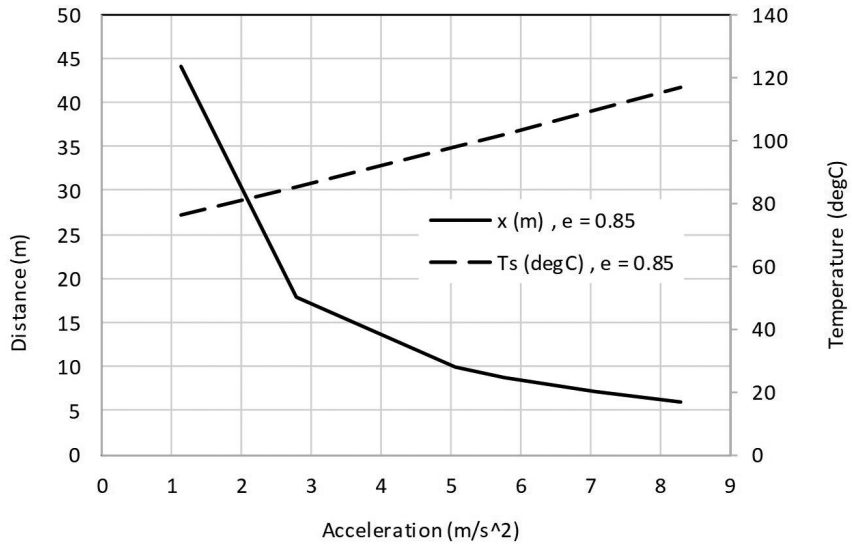


FIGURE 166 Distance and steady-state temperature versus the acceleration ($e = 0.85$).

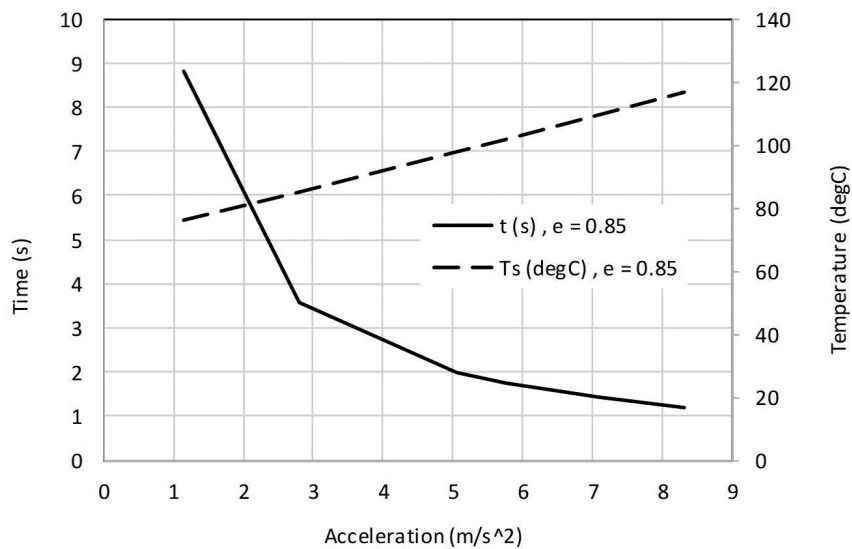


FIGURE 167 Time and steady-state temperature versus the acceleration ($e = 0.85$).

CONVERTING A *PISTON ENGINE* *AIRCRAFT TO A TURBOPROP* *AIRCRAFT*

This chapter describes an aircraft with a regular piston engine that was upgraded with a much more powerful turboprop powerplant. In July 2018, at the EAA Air Venue in Oshkosh, the conversion for a tail-dragger STOL aircraft the PZL-104 Wilga (Figure 168) was presented for the first time [240]. The original 260 hp piston engine (Lycoming 540) was replaced with a PT6-28 Pratt & Whitney Canada turboprop engine rated at 715 hp but limited to 680-hp output due to transmission gearbox limitations. The newly created aircraft was christened the Wilga *DRACO* (Figure 169).

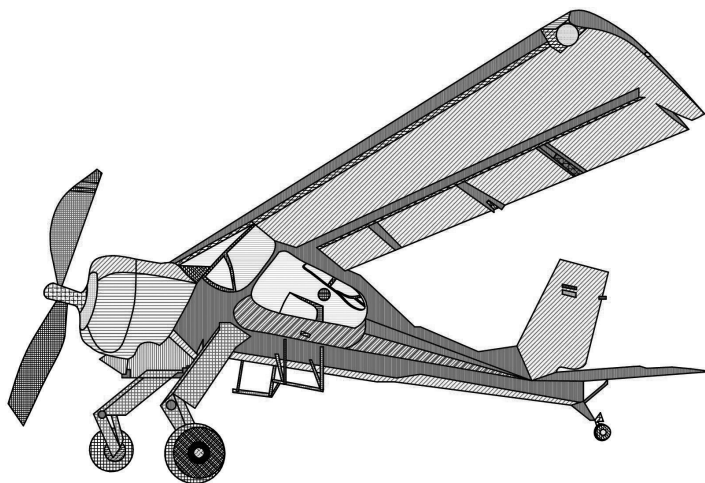


FIGURE 168 The PZL-104 MA Wilga aircraft before the conversion with a piston engine (drawings created using Solid Edge CAD tool).

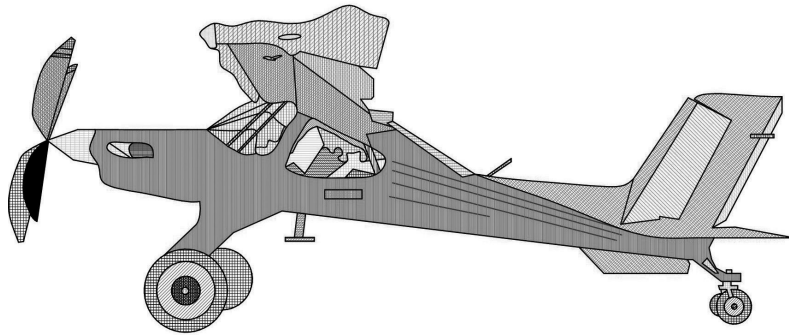


FIGURE 169 The Wilga *DRACO* aircraft after the conversion to a turboprop engine (drawings created using Solid Edge CAD tool).

The prototype of the original aircraft model was first manufactured in Poland, in 1962, with the last plane produced in 2006, with only twenty-four of them ever built. Interestingly, the owner (Mike Patey) owned both the number 1 and 24 serial numbers. The first one he sold but afterward acquired the latter, on which he carried out the conversion. He intended to fly the aircraft in the backcountry of Utah, where the field elevations are typically high, while the weather is hot and humid. Soon after purchase, he found out that the aircraft is not capable of high performance flying in these conditions; this motivated him to carry out the conversion.

This conversion was made in the experimental exhibition category. The converted aircraft has an empty mass of 2,400 lb and is capable of carrying 1,600 lb. The new turboprop engine, while being much more powerful, is actually 120 lb lighter than the piston one it replaced. The engine drives the new 102 inch-MT four-bladed propeller. The new engine's fuel consumption is approximately 28 GPH. The original stall speed is reported as 36 kt [241]. M. Patey claims the stall speed was originally about 57 MPH (50 kt); however, he was able to lower it by means of additional wing modifications (by over 20 MPH = 17 kt), enabling him to reduce the touchdown speed to 37 MPH (32 kt) [242]. Using the added power of the new engine, Average takeoff distances as short as 120 ft and average landing distances down to 117 ft were accomplished. This short landing distance is achieved by application of reverse thrust (up to 300 hp) along with a modified dual-brake mechanism. The new tires are of grade 950, with the expected tail load exceeding 1,000 lb under full load. A climb rate of 4,000 fpm is achievable with this new configuration, and the never-exceed speed is about 180 MPH (156 kt).

12.1 Conversion Process

The following description of the steps taken to carry out the conversion of the piston-engine PLZ-104 Wilga to the turboprop the Wilga *DRACO* is based on the YouTube videos posted by M. Patey, where he describes this process in detail [243,244]:

1. The permissions needed to be obtained first—the governing body (FAA) had to give the go-ahead for the mechanical conversion. The parts were purchased during the wait period and plans were made along with the maintenance manual and paper work.
2. Site preparation—the hangar floor was covered with sheets of plywood to reduce the wear due to the movement of parts and tools.
3. The aircraft needed to be placed in the cruise flight attitude to provide a baseline for the engine and the rest of the cowling items, requiring to lift the aircraft tail off the ground to square off the platform.
4. The plane was jacked up to reduce stress on the parts; additionally, a platform was made to enable easy access to the required locations.
5. A number of additional shell-type components were manufactured on-site from fiber-reinforced composites. To produce these, molds were made from different materials (wood or plastic). Several layers (from 3 up to 8, depending on the component) of carbon fiber fabric (and in some cases glass fiber as well) covered the molds and were impregnated with resin. A plastic sheet covered these and a vacuum was applied to make the sheet squeeze the uncured resin and reinforcements tightly against the mold surface. The part was then inserted inside an oven, which used 500 W halogen heat lamps. The curing process took approximately 1.5 hrs, and sanding was done next.
6. Engine was tested before being mounted inside the cowling to ensure proper engine operation.
7. All parts and fittings were weighed, including the motor, propeller, fittings, and so on.
8. Fuel capacity was increased to 120 gal. The extra capacity was obtained by adding tanks inside cowlings built around the two front gear legs. A mechanical fuel pump was built inside the cowling and two simpler versions of electrical fuel pumps were inserted inside the gear leg for redundancy. The pumps inside the gear leg collect the fuel and send it to the mechanical pump inside the engine cowling.

9. The oil cooler was expanded with additional heat exchangers at the first line, the hottest location.
10. A new instrument panel, manufactured by laser cutting, was installed to house the new set of glass cockpit instruments, to both save weight and add new capabilities.
11. The wires from the throttle quadrant to the engine penetrated through the firewall and then passed through a custom-machined aluminum fitting that aligned them at the correct angle.
12. The new engine is longer than the original one and this results in its COG being more forward of the firewall than the original engine. The extra moment thus produced was addressed by increasing the number of supporting struts from 4 to 7. The new frame was custom made.
13. Ailerons, rudder, and elevator trims, and servo tabs were added or updated. Over 35 ft² equivalent wing area was added by extending ailerons, flaps, wings, and other horizontal control surfaces.
14. The wing leading edge was taken off and replaced. To expand the elevator, the method explored was to insert a C-channel at the wing trailing edge of the aileron and secure it by the side bolts. It is also possible to add a small extension to the end of the trailing edge of the aileron; however, in this scenario the chord cannot be extended too long.
15. The length of the wing remained unchanged since the analyses showed that the load distribution did not support a length extension; however, the chord length was extended both at the leading and trailing edges. At the leading edge this was accomplished by means of slots and slats. Leading edge slats are used to separate the air flow from the front and delay the stall. This also adds the air flow from the bottom side and changes the pitching moment as a result. Stress pull analysis was done to ensure the slotted leading edges are sufficiently strong.
16. The aircraft was laser-scanned to capture all of its dimensions accurately. These dimensional data were imported into a Computer Aided Design (CAD) software (SolidWorks), and analyzed when designing the new aircraft wings with a machining accuracy of within one thousandth of an inch. The scanned data was also used for fluid dynamic modeling.
17. The top and bottom engine cowlings were made and installed in separate steps. They were coated again by the resin to add strength. The cowling thickness was larger at the front, becoming thinner moving backward, creating a clamshell, and making it easier to bend the top and bottom

cowlings toward each other to mate. The overall coating was covered by a 1/4" micro bubble blanket and sanded down by 1/8".

18. Double-entrance ducts were built into the engine cowling as air intakes to equalize the inflow, splitting it into two systems, with an automatic alternate air source from the back bottom of the cowling. If the front air filter is blocked by debris or ice, the intake air source can be changed to the alternate.
19. Up to 90 percent of wiring was redone as existing battery cables did not have the sufficient gauge to accommodate the increased current requirement.
20. The axle of the new landing gear could have been twisted if the connections were not made properly. Thus, for the nose strut tube not to be twisted, it was to be reinforced. To achieve this, the box was twisted slightly diagonally from 1/3 of the way upward and was reinforced with a diagonal plate. This way, the moment is transferred directly to the knuckle versus the tube. That improves the axle and brake systems.
21. Two oxygen tanks are available for flights up to 28,000-ft altitude. Gear legs are equipped with a 45-min fuel reserve. If the aircraft is left with the reserve only, the COG is low and legal on this minimum fuel; the fuel flow is measured by means of special capacitance sensors. The sensor can detect the fuel passing through it without impeding the flow in any way.
22. A 16-in carbon-fiber skid foot (leaf spring) was added below each wing near its tip to absorb the shock if the wing accidentally comes in contact with the ground such as due to a ground loop.
23. Enhanced landing lights (originally designed for the Boeing 737), challenge lights, citation lights, night vision infrared cameras, and autopilot were among the added features.

12.2 Case Study – The Wilga Conversion

Performance data for the converted and original aircraft, as used in this case study, are provided in Table 96. These data include flight mass, typical cruise TAS, wing surface area and span. For this study, the original and the converted aircraft are to follow a given course, at specified altitudes and atmospheric conditions (i.e., temperature, pressure, and wind). Their angle of attack varies depending on their maneuvering position. The aircraft is flying from Salt Lake City (UT) to Oshkosh (WI) to demonstrate its capabilities to the crowd of aviation enthusiasts. It is assumed that the aircraft follows the same direct track to the destination along the path—

TABLE 96 Performance data for the original aircraft (the PZL-104 MA) and converted version (the Wilga DRACO).

Case	Aircraft	Gross Mass (lb)	Gross Mass (kg)	Cruise (kt)	Power (hp)	Power (kW)	Wing Surface Area (ft²)	Wing Surface Area (m²)	Thrust Limit (kN)	Stall Speed (kt)
1	PZL-104 MA Wilga 2,000	3,306	1,500	114	260	224	167.0	15.51	2.86	36
2	Wilga Draco	4,000	1,814	130	680	507	209.5	19.46	5.67	TBD

no diversions are taken. The wind conditions remain the same. This is to facilitate the force comparisons as a function of the angle of attack that varies along the track. You are to compare the drag and lift forces for all cases in this exercise.

Assume the engine efficiencies of 75 and 85 percent, and average cruise speeds of 114 kt and 130 kt, for the unconverted and converted versions, respectively. The lift and drag coefficients presented in Figure 170 (based on a *Clark Y* airfoil) were adjusted to produce sufficient lift to balance the aircraft weight and to balance thrust and drag in level flight with a one-degree angle of attack. After the adjustments, data shown in Figure 171 (for the aircraft before the conversion) and Figure 172 (for the aircraft after the conversion) were obtained. It is to be used here for calculations with different angles of attacks. Assume that the compass card shows a +1.5°

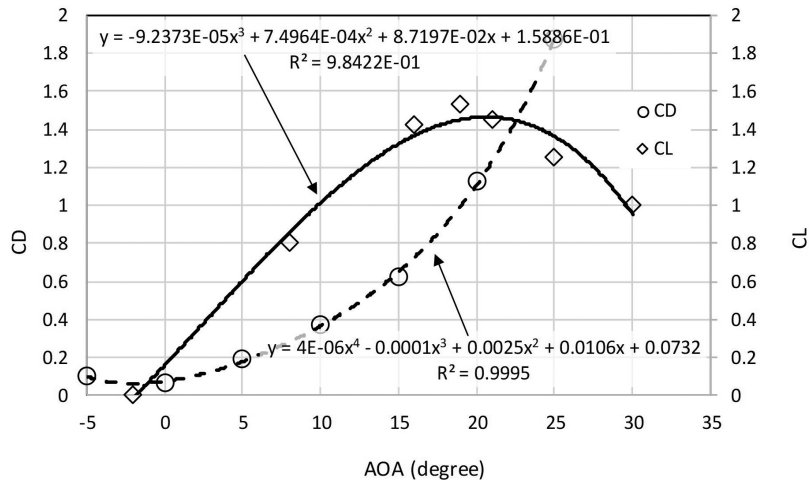


FIGURE 170 Example of drag and lift coefficients versus the angle of 1 attack—CLARK Y airfoil at aspect ratio of 6 (raw data adopted from [131]).

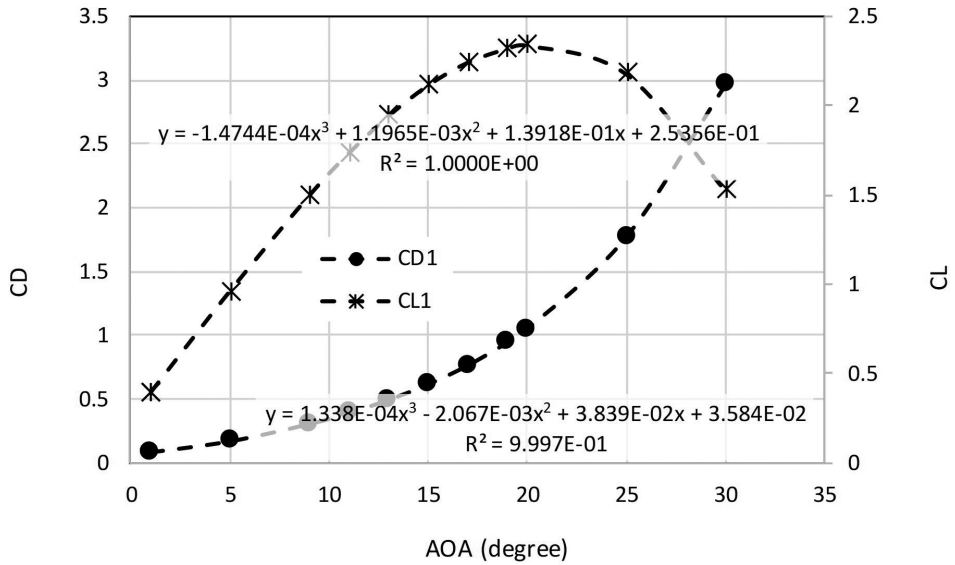


FIGURE 171 Adopted calibrated drag and lift coefficients versus the angle of attack—before the conversion.

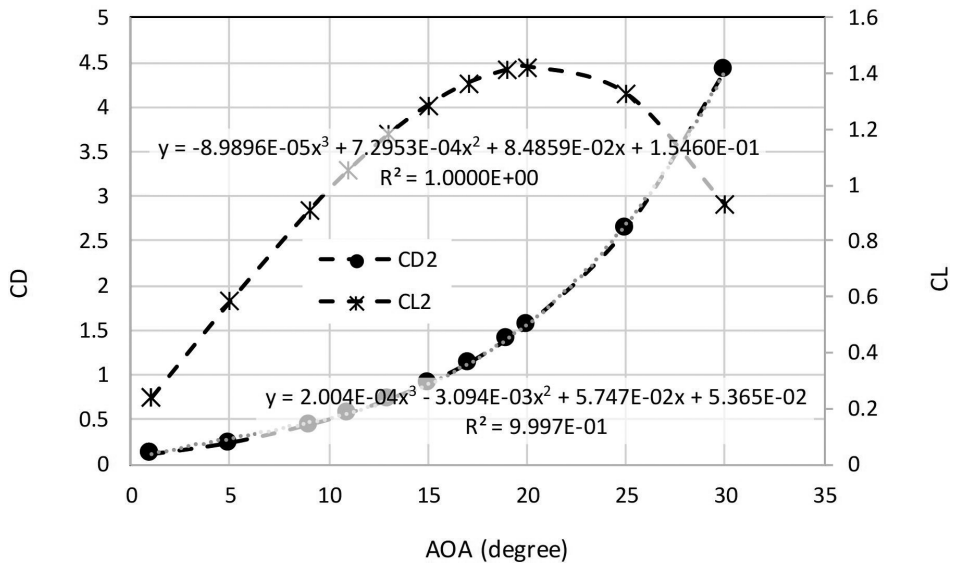


FIGURE 172 Adopted calibrated drag and lift coefficients versus the angle of attack—after the conversion.

TABLE 97 Angle of attack, mass, and wind speed data for the PZL-104 MA and the Wilga *DRACO* (part 1).

Case	Aircraft	AOA α (°)	AOA α (rad)	Mass m (lb)	Mass m (kg)	Wind Speed w (kt)	TAS (kt)	True Course β (°)	Angle of Wind ϕ (°)
1	PZL-104 MA Wilga 2000	1	0.017	3,000.0	1,361	179@7ktG10	146.00	72	179
2	Wilga <i>DRACO</i>	1	0.017				130.35		
3		5	0.087				83.29		
4		9	0.157				66.48		
5		10	0.175				63.83		
6		12	0.209				59.62		
7		14	0.244				56.53		
8		16	0.279				54.28		
9		18	0.314				52.76		
10		20	0.349				51.88		
11	PZL-104 MA Wilga 2000	10	0.175				55.82		
Case	Angle of Wind λ (°)	Angle of Wind λ (rad)	Wind Magnitude w' (kt)	Wind Headwind $w' \cos \lambda$ (kt)	Wind Crosswind $w' \sin \lambda$ (kt)	Crab Angle δ (rad)	Crab Angle δ (°)	THdg (°)	Ground Speed G_S (kt)
1	107.0	1.87	8.50	-2.49	8.13	0.0557	3.19	75.19	148.48
2						0.0624	3.58	75.58	132.83
3						0.0978	5.60	77.60	85.77
4						0.1226	7.02	79.02	68.96
5						0.1277	7.32	79.32	66.31
6						0.1368	7.84	79.84	62.11
7						0.1443	8.27	80.27	59.01
8						0.1503	8.61	80.61	56.77
9						0.1547	8.86	80.86	55.24
10						0.1573	9.01	81.01	54.36
11						0.1461	8.37	80.37	58.31

correction for the westerly headings, and atmospheric temperature is uniform at 15°. Using the data presented in Table 97, calculate:

- (a) angle of attack in radians, (b) angle of wind with respect to the aircraft course in degrees and radians, taking into account the variation, (c) wind magnitude including half of the gust factor, (d) crosswind component of wind with and without gust, (e) headwind component of

wind, (f) aircraft crab angle, (g) true heading, (h) ground speed, (i) pressure in inHg and kPa, (j) assuming that the standard-adiabatic lapse rate is $1.98\text{ }^{\circ}\text{C}/1,000\text{ ft}$, calculate temperature for the given altitude, (k) density, (l) drag coefficients, (m) lift coefficients, (n) balancing speed, (o) lift force, (p) drag force, (q) ratio of the lift to the drag assuming wing surface area as the reference, (r) drag force per unit area, (s) lift force per unit area, (t) total thrust, (u) compare calculated and given thrusts and conclude if the calculated thrust (maximum available thrust) is sufficient for the aircraft to remain aloft given the specific scenarios, (v) distance required to take off and land for the day of arrival for a good asphalt runway surface ($RFI = 1$), (w) recalculate part “v” after assuming landing on a grass surface, causing the runway length requirement to increase by 15 percent. Assume that on the return journey from KOSH to KSLC, weather in Salt Lake City changes to 20010G15KT, and (x) justify the choice of runways and ensure their lengths are sufficient for the aircraft to take off and land.

Note that in this exercise, half of the gust factor— $G_f = (-w + G)$ —is to be added to the wind magnitude ($w' = w + G_f/2$), where G is the gust to obtain the components of the wind along and across the direction of travel in order to find the aircraft crab angle and the true and magnetic headings. The Angle of Attack ($AOA = \alpha$) is the angle between the relative air and the chord line of the airfoil. As the aircraft angle of attack increases, the component of weight parallel to the aircraft axis ($mg\sin\alpha$) resists or aids the forward motion of the aircraft, depending on whether the aircraft nose points above or below the horizon (i.e., positive or negative angle). The crosswind ($w\sin\phi$) and headwind ($w\cos\phi$) components of the wind are the cosine and sine of the relative angle between the true course and the wind multiplied by the wind magnitude, considering the gust factor.

The aircraft crab angle is the arc(sine) of the ratio of the crosswind component to the TAS— $CA = \delta = \arcsin(w\sin\lambda/TAS)$, which can be derived using the vector algebra discussed earlier. Note that the crosswind ($w\sin\phi$) is to be calculated given the relative angle between the true course (β) and the wind (ϕ). The difference between the aircraft true course (β) and wind direction (ϕ) is relative angle: $\lambda = \phi - \beta$.

Also note that ($GS = TAS \pm w\cos\lambda$). Depending on the direction of the wind (blowing from the right side or left side of the aircraft), the aircraft is to crab into the wind; therefore, the aircraft crab angle (δ) is to be deducted or added to the true course, respectively, $THdg = \beta \pm \delta$. The magnetic heading is calculated by adding the westerly or subtracting the easterly variation ($THdg \pm Var = MagHdg$). Salt Lake City International Airport

(KSLC) has a variation of 11°E , so it is to be subtracted from the original wind direction of 179° ($179^\circ - 11^\circ = 168^\circ$). The Wittman Regional Airport (KOSH) has a variation of 3°W , so it is to be added to the original wind direction of 179° ($179^\circ + 3^\circ = 182^\circ$). When calculating the crosswind for takeoff or landing, in either of the geographical locations with different variations, the variation is to be added or subtracted (westerly and easterly cases) to or from the difference between the two angles. Recall that the runway heading is in magnetic degrees. The magnetic heading is still used in order to obtain the magnetic compass reported on the flight plan. Case 10 assumes the variation at the destination airport (3°W) while for the rest of the cases, the variation at the departure airport (11°E) is assumed.

The flight level (FL) may be assumed the same as the pressure altitude (P_{AL}) in this scenario. The flight altitude is odd numbers of thousands plus 500 feet since the aircraft is flying VFR with easterly heading—for this scenario, it is assumed to be 7,500 ft. Given the elevation of the airport above the mean sea level (H) and the standard atmospheric conditions for pressure altitude (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi), you are able to calculate the pressure (P) at the given altitude in inches of mercury (inHg) and then convert it to kPa, $PA = H + 1000(29.92 - P_{inHg})$. Knowing this pressure (P_{inHg}) in Pascal, the outside air temperature ($T = OAT$), and air specific gas constant ($R_d = 287.0 \text{ J/kgK}$) for dry air, you are able to calculate the density of the air ($\rho = P/RT$). This density is then used in combination with the mass (m) and true airspeed ($v_T = TAS$) to find the multiplier for the lift and drag coefficients ($0.5\rho v^2$) to obtain the force per unit area (F/A) for both drag and lift.

Since the wind tunnel tests were conducted and the lift (F_L) and drag (F_D) forces were calculated based on the wing surface area (A) as the reference plane, the lift and drag forces are then calculated. For this problem, the lift and drag coefficients versus the angle of attack are obtained from Figure 171 (for aircraft before the conversion) and Figure 172 (for aircraft after the conversion), which themselves are obtained by calibrating the data in Figure 170. To transform the data presented in Figure 170 in order to match the aircraft in this scenario, the first step was to calculate the lift coefficient (C_L) using the straight-and-level flight conditions (assuming a one-degree angle of attack and cruise performance data, i.e., cruise speed and environmental atmospheric conditions to calculate the air density). This lift coefficient is then implemented in Figure 170 to calculate the drag coefficient (C_D). You are to obtain the lift and drag coefficients for each given angle of attack from Figure 171 (for aircraft before the conversion) and Figure 172 (for aircraft after the conversion).

The steps to follow are: (a) using the extracted lift data, assuming that the lift force equals the component of the weight perpendicular to the direction of motion, calculate the velocity (v_e) required to obtain the force balance: $v_e = \sqrt{2mg \cos \alpha / \rho A C_L}$, where α is the angle of attack, and v_e is equivalent to TAS; (b) lift force must be equal to the component of the weight perpendicular to the movement direction or $F_L = 0.5 A \rho v_e^2 C_L$; (c) given the velocity required to keep the aircraft in the air, and using the extracted drag coefficient, you are to obtain the drag force— $F_D = 0.5 A \rho v_e^2 C_D$; (d) thrust needs to balance the drag force plus the component of the weight parallel to the thrust direction; therefore, it depends on the pitch angle: $Thrust = F_D \pm mg \sin \alpha$; (e) thrust-to-mass ratio is obtained as $Thrust/m$; and (f) limit of the thrust is calculated as the total power divided by TAS, $Thrust_{limit} = \eta Power / Speed_{Cruise}$, where η is the efficiency of the engine. For the aircraft before the conversion, 75 percent efficiency, 114-kt average cruise speed, and 260-hp engine power are assumed. For the aircraft after the conversion, 85 percent efficiency, 130 kt average cruise speed, and 680 hp are assumed.

This example assumes a balance of the forces along the direction of the aircraft motion. You may compare the total required thrust obtained from the calculations with the maximum available thrust for each case. If the required value is below the available thrust for the specific angle of attack, performance parameters, and given atmospheric conditions, the aircraft is able to generate the lift force required to remain in the air.

The comparison between the performance parameters and angles of attack is provided in Table 97 to Table 99 for the original (Case 1 and 11) and converted (Cases 2 and 5) scenarios for a one-degree and ten-degree angle of attack. It is noted that the conditions presented for the original model require a thrust that just meets the required thrust for a one-degree angle of attack while it does not meet the required thrust for a larger angle of attack. A similar trend is seen for the converted aircraft given the conditions.

Wind data are processed to obtain the wind correction factors for the departure and destination aerodromes for gusty conditions (Table 103 and Table 107) and no-gust conditions (Table 104 and Table 108). Given the runway correction factors calculated, the minimum length requirements for the runways are calculated for good asphalt (Table 105 and Table 109) and grass (Table 106 and Table 110) runways.

Table 100 to Table 102 show the effect of temperature on the density, forces of lift and drag, and balance speed for a constant pressure. Note that as density is a function of temperature, the balance speed is also affected by it. The lift and drag forces are affected by both factors and increase

linearly with increase in density and non-linearly (the second-order) with increase in balance speed. The overall trend of these two forces remains unchanged, as temperature changes at constant pressure. Pressure may change, of course, and that will result in a different analysis. Note that with the rise of the outside air temperature, the balance speed (TAS equivalent) also increases, since higher speed is required to generate the same lift when the temperature is higher.

TABLE 98 Angle of attack, mass, and wind speed data for the PZL-104 MA and the Wilga *DRACO* (part 2).

Case	Ground Speed GS (m/s)	Outside Air Temperature (OAT) T (°C)	Pressure Altitude PA (ft)	Altitude H (ft)	Pressure P (inHg)	Pressure P (kPa)	Density ρ (kg/m ³)	Wing Surface Area A (ft ²)	Wing Surface Area A (m ²)
1	76.39	0.15	FL80	7500	29.42	99.63	1.27	167.0	15.51
2	68.33							209.5	19.46
3	44.13							209.5	19.46
4	35.48							209.5	19.46
5	34.11							209.5	19.46
6	31.95							209.5	19.46
7	30.36							209.5	19.46
8	29.20							209.5	19.46
9	28.42							209.5	19.46
10	27.97							209.5	19.46
11	30.00							167.0	15.51
Case	Drag Coefficient C_D	Lift Coefficient C_L	$\rho v_e^2/2$	Lift to Drag $\frac{F_L}{F_D} = \frac{C_L}{C_D}$	Drag Pressure F_D/A (Pa)	Lift Pressure F_L/A (Pa)	Lift F_L (kN)	Drag F_D (kN)	Thrust Limit F_T (kN)
1	0.120	0.240	3,581.88	2.004	429.3	860.0	13.35	6.66	2.71
2	0.120	0.240	2,855.06	2.004	342.2	685.5	13.35	6.66	6.66
3	0.248	0.586	1,165.71	2.358	289.6	683.0	13.29	5.64	6.66
4	0.451	0.912	742.58	2.021	335.0	677.2	13.18	6.52	6.66
5	0.513	0.986	684.59	1.921	351.4	675.2	13.14	6.84	6.66
6	0.654	1.123	597.36	1.716	390.8	670.6	13.05	7.61	6.66
7	0.822	1.239	536.93	1.508	441.1	665.2	12.95	8.59	6.66
8	1.022	1.331	495.18	1.302	506.3	659.0	12.83	9.85	6.66
9	1.266	1.394	467.69	1.101	592.2	652.0	12.69	11.53	6.66
10	1.564	1.424	452.29	0.911	707.6	644.2	12.54	13.77	6.66
11	0.343	1.618	523.65	4.718	179.5	847.1	13.14	2.79	2.71

TABLE 99 Angle of attack, mass, and wind speed data for the PZL-104 MA and the Wilga *DRACO* (part 3).

Case	Thrust F_T (kN)	Thrust/Mass F_{T_x}/m (N/kg)	Weight Horizontal Component $mg\sin\alpha$ (kN)	Weight Vertical Component $mg\cos\alpha$ (kN)	v_e (m/s)	v_e (kt)	v_e (MPH)	Variation (°)	Compass Heading (°)
1	6.66	4.895	0.2	13.3	75.11	146.00	168.01	11E	65.69
2	6.66	4.895	0.2	13.3	67.06	130.35	150.00	11E	66.08
3	6.80	4.998	1.2	13.3	42.85	83.29	95.85	11E	68.10
4	8.61	6.326	2.1	13.2	34.20	66.48	76.50	11E	69.52
5	9.16	6.730	2.3	13.1	32.84	63.83	73.45	11E	69.82
6	10.38	7.629	2.8	13.1	30.67	59.62	68.61	11E	70.34
7	11.81	8.682	3.2	12.9	29.08	56.53	65.05	11E	70.77
8	13.53	9.945	3.7	12.8	27.93	54.28	62.47	11E	71.11
9	15.65	11.501	4.1	12.7	27.14	52.76	60.71	11E	71.36
10	18.34	13.475	4.6	12.5	26.69	51.88	59.70	3W	79.51
11	5.10	3.750	2.3	13.1	28.72	55.82	64.24	11E	70.87

TABLE 100 Effect of outside air temperature on the density and balance speed for the PZL-104 MA and the Wilga DRACO (ISA+20 °C).

ISA + 20 °C								
Case	Thrust F_T (kN)	Lift F_L (kN)	Drag F_D (kN)	v_e (m/s)	v_e (kt)	v_e (MPH)	Outside Air Temperature (OAT) T (°C)	Density ρ (kg/m³)
1	6.661	13.349	6.661	77.807	151.245	174.050	20.15	1.18
2	6.661	13.349	6.661	69.466	135.031	155.391		
3	6.801	13.294	5.637	44.388	86.282	99.292		
4	8.608	13.180	6.520	35.427	68.865	79.249		
5	9.157	13.142	6.840	34.016	66.122	76.091		
6	10.381	13.053	7.607	31.775	61.766	71.079		
7	11.814	12.948	8.586	30.125	58.558	67.388		
8	13.533	12.828	9.854	28.930	56.235	64.715		
9	15.651	12.692	11.527	28.116	54.652	62.893		
10	18.337	12.540	13.773	27.649	53.744	61.848		
11	5.103	13.142	2.785	29.750	57.829	66.549		
ISA								

TABLE 101 Effect of outside air temperature on the density and balance speed for the PZL-104 MA and the Wilga *DRACO* (ISA).

ISA								
Case	Thrust F_T (kN)	Lift F_L (kN)	Drag F_D (kN)	v_e (m/s)	v_e (kt)	v_e (MPH)	Outside Air Temperature (OAT) T (°C)	Density ρ (kg/m ³)
1	6.661	13.349	6.661	75.108	145.998	168.011	0.15	1.27
2	6.661	13.349	6.661	67.056	130.346	150.000		
3	6.801	13.294	5.637	42.848	83.289	95.847		
4	8.608	13.180	6.520	34.198	66.476	76.499		
5	9.157	13.142	6.840	32.836	63.827	73.451		
6	10.381	13.053	7.607	30.673	59.622	68.612		
7	11.814	12.948	8.586	29.080	56.526	65.049		
8	13.533	12.828	9.854	27.926	54.284	62.469		
9	15.651	12.692	11.527	27.140	52.756	60.710		
10	18.337	12.540	13.773	26.689	51.880	59.702		
11	5.103	13.142	2.785	28.718	55.823	64.240		

TABLE 102 Effect of outside air temperature on the density and balance speed for the PZL-104 MA and the Wilga *DRACO* (ISA-20 °C).

ISA - 20 °C								
Case	Thrust F_T (kN)	Lift F_L (kN)	Drag F_D (kN)	v_e (m/s)	v_e (kt)	v_e (MPH)	Outside Air Temperature (OAT) T (°C)	Density ρ (kg/m ³)
1	6.661	13.349	6.661	72.31	140.55	161.75	-19.85	1.37
2	6.661	13.349	6.661	64.56	125.49	144.41		
3	6.801	13.294	5.637	41.25	80.18	92.27		
4	8.608	13.180	6.520	32.92	64.00	73.65		
5	9.157	13.142	6.840	31.61	61.45	70.71		
6	10.381	13.053	7.607	29.53	57.40	66.05		
7	11.814	12.948	8.586	28.00	54.42	62.62		
8	13.533	12.828	9.854	26.89	52.26	60.14		
9	15.651	12.692	11.527	26.13	50.79	58.45		
10	18.337	12.540	13.773	25.69	49.95	57.48		
11	5.103	13.142	2.785	27.65	53.74	61.84		

TABLE 103 Crosswind and headwind components with gust (Outbound: KSLC to KOSH).

Gusting Condition				
		KSLC		
		Wind Direction	179	
		Wind Mag. Gusting (kts)	7	10
Length (ft)	9,597	Runway	17	Runway: 17/ Variation: -11 Wind: 179 @ 7 G 10
Width (ft)	150	Variation	-11	
		degrees difference		2
		kts (headwind)		8.495
		kts (crosswind)		0.297
		Multiplier		0.906
		Head/Tail		Headwind
		KOSH		
		Wind Direction	179	
		Wind Mag. Gusting (kts)	7	10
Length (ft)	8,002	Runway	18	Runway: 18/ Variation: 3 Wind: 179 @ 7 G 10
Width (ft)	150	Variation	3	
		degrees difference		2
		kts (headwind)		8.495
		kts (crosswind)		0.297
		Multiplier		0.906
		Head/Tail		Headwind

TABLE 104 Crosswind and headwind components without gust (Outbound: KSLC to KOSH).

Not Gusting Condition				
		KSLC		
		Wind Direction	179	
		Wind Mag. Gusting (kts)	7	0
Length (ft)	9,597	Runway	17	Runway: 17/ Variation: -11 Wind: 179 @ 7 G 0
Width (ft)	150	Variation	-11	
		degrees difference		2
		kts (headwind)		6.996
		kts (crosswind)		0.244
		Multiplier		0.922
		Head/Tail		Headwind
		KOSH		
		Wind Direction	179	
		Wind Mag. Gusting (kts)	7	0
Length (ft)	8,002	Runway	18	Runway: 18/ Variation: 3 Wind: 179 @ 7 G 0
Width (ft)	150	Variation	3	
		degrees difference		2
		kts (headwind)		6.996
		kts (crosswind)		0.244
		Multiplier		0.922
		Head/Tail		Headwind

TABLE 105 Takeoff and landing distances, without grass
(Outbound: KSLC to KOSH) (POH data from [245]).

KSLC	Temp (°C)	15		0.906 Gust 0.922	
No Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor		
POH	365	800	0.906		
Wind	330.55 Gust 336.63	724.49 Gust 737.82	0.922		
Grass	330.55 Gust 336.63	724.49 Gust 737.82	1.15		
Safety	495.82 Gust 504.94	1086.74 Gust 1106.72	1.5		
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD		
No Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor		
POH	509	850	0.906		
Wind	460.96 Gust 469.44	769.77 Gust 783.93	0.922		
Grass	460.96 Gust 469.44	769.77 Gust 783.93	1.45	0.906 Gust 0.922	
Safety	691.44 Gust 704.15	1154.66 Gust 1175.89	1.5		
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD		
KOSH	Temp (°C)	15			0.906 Gust 0.922
No Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor		
POH	365	800	0.906		
Wind	330.55 Gust 336.63	724.49 Gust 737.82	0.922		
Grass	330.55 Gust 336.63	724.49 Gust 737.82	1.15		
Safety	495.82 Gust 504.94	1086.74 Gust 1106.72	1.5		
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD		
No Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor		
POH	509	850	0.906		
Wind	460.96 Gust 469.44	769.77 Gust 783.93	0.922	0.906 Gust 0.922	
Grass	460.96 Gust 469.44	769.77 Gust 783.93	1.45		
Safety	691.44 Gust 704.15	1154.66 Gust 1175.89	1.5		
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD		

TABLE 106 Takeoff and landing distances, with grass
(Outbound: KSLC to KOSH) (POH data from [248]).

KSLC	Temp (°C)	15	
Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor
POH	365	800	0.906
Wind	330.55 Gust 336.63	724.49 Gust 737.82	0.922
Grass	380.13 Gust 387.12	774.07 Gust 788.31	1.15
Safety	570.2 Gust 580.68	1161.11 Gust 1182.46	1.5
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD
Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor
POH	509	850	0.906
Wind	460.96 Gust 469.44	769.77 Gust 783.93	0.922
Grass	668.39 Gust 680.68	977.2 Gust 995.18	1.45
Safety	1002.58 Gust 1021.02	1465.8 Gust 1492.76	1.5
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD
KOSH	Temp (°C)	15	
Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor
POH	365	800	0.906
Wind	330.55 Gust 336.63	724.49 Gust 737.82	0.922
Grass	380.13 Gust 387.12	774.07 Gust 788.31	1.15
Safety	570.2 Gust 580.68	1161.11 Gust 1182.46	1.5
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD
Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor
POH	509	850	0.906
Wind	460.96 Gust 469.44	769.77 Gust 783.93	0.922
Grass	668.39 Gust 680.68	977.2 Gust 995.18	1.45
Safety	1002.58 Gust 1021.02	1465.8 Gust 1492.76	1.5
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD

TABLE 107 Crosswind and headwind components with gust (Inbound: KOSH to KSLC).

Gusting Condition				
		KSLC		
		Wind Direction	200	
		Wind Mag. Gusting (kts)	10	15
Length (ft)	9,597	Runway	17	Runway: 17/ Variation: -11 Wind: 200 @ 10 G 15
Width (ft)	150	Variation	-11	
		degrees difference		19
		kts (headwind)		11.819
		kts (crosswind)		4.070
			Multiplier	0.869
			Head/Tail	Headwind
		KOSH		
		Wind Direction	200	
		Wind Mag. Gusting (kts)	10	15
Length (ft)	8,002	Runway	18	Runway: 18/ Variation: 3 Wind: 200 @ 10 G 15
Width (ft)	150	Variation	3	
		degrees difference		23
		kts (headwind)		11.506
		kts (crosswind)		4.884
			Multiplier	0.872
			Head/Tail	Headwind

TABLE 108 Crosswind and headwind components without gust (Inbound: KOSH to KSLC).

Not Gusting Condition				
		KSLC		
		Wind Direction	200	
		Wind Mag. Gusting (kts)	10	0
Length (ft)	9,597	Runway	17	Runway: 17/ Variation: -11 Wind: 200 @ 10 G 0
Width (ft)	150	Variation	-11	
		degrees difference		19
		kts (headwind)		9.455
		kts (crosswind)		3.256
			Multiplier	0.895
			Head/Tail	Headwind
		KOSH		
		Wind Direction	200	
		Wind Mag. Gusting (kts)	10	0
Length (ft)	8,002	Runway	18	Runway: 18/ Variation: 3 Wind: 200 @ 10 G 0
Width (ft)	150	Variation	3	
		degrees difference		23
		kts (headwind)		9.205
		kts (crosswind)		3.907
			Multiplier	0.898
			Head/Tail	Headwind

TABLE 109 Takeoff and landing distances, *without* grass
(Inbound: KOSH to KSLC) (POH data from [248]).

KSLC	Temp (°C)	15		0.869 Gust 0.895
No Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	365	800	0.869	
Wind	317.07 Gust 326.65	694.94 Gust 715.95	0.895	
Grass	317.07 Gust 326.65	694.94 Gust 715.95	1.15	
Safety	475.6 Gust 489.98	1042.41 Gust 1073.93	1.5	
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD	
No Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	509	850	0.869	
Wind	442.16 Gust 455.53	738.38 Gust 760.7	0.895	
Grass	442.16 Gust 455.53	738.38 Gust 760.7	1.45	
Safety	663.24 Gust 683.29	1107.56 Gust 1141.05	1.5	
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD	
KOSH	Temp (°C)	15		0.872 Gust 0.898
No Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	365	800	0.872	
Wind	318.34 Gust 327.67	697.72 Gust 718.18	0.898	
Grass	318.34 Gust 327.67	697.72 Gust 718.18	1.15	
Safety	477.5 Gust 491.5	1046.58 Gust 1077.27	1.5	
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD	
No Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	509	850	0.872	
Wind	443.93 Gust 456.94	741.33 Gust 763.06	0.898	
Grass	443.93 Gust 456.94	741.33 Gust 763.06	1.45	
Safety	665.89 Gust 685.41	1111.99 Gust 1144.6	1.5	
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD	

TABLE 110 Takeoff and landing distances, with grass
(Inbound: KOSH to KSLC) (POH data from [248]).

KSLC	Temp (°C)	15		0.869 Gust 0.895
Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	365	800	0.869	
Wind	317.07 Gust 326.65	694.94 Gust 715.95	0.895	
Grass	364.63 Gust 375.65	742.5 Gust 764.95	1.15	
Safety	546.94 Gust 563.48	1113.75 Gust 1147.43	1.5	
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD	
Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	509	850	0.869	
Wind	442.16 Gust 455.53	738.38 Gust 760.7	0.895	
Grass	641.13 Gust 660.51	937.35 Gust 965.69	1.45	
Safety	961.69 Gust 990.77	1406.02 Gust 1448.53	1.5	
Runway Length (ft)	9597 , 9597	9597 , 9597	GOOD	
KOSH	Temp (°C)	15		0.872 Gust 0.898
Grass	Takeoff Distance (ft)	Takeoff Distance to clear 50' (ft)	Correction Factor	
POH	365	800	0.872	
Wind	318.34 Gust 327.67	697.72 Gust 718.18	0.898	
Grass	366.09 Gust 376.82	745.47 Gust 767.33	1.15	
Safety	549.13 Gust 565.23	1118.21 Gust 1150.99	1.5	
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD	
Grass	Landing Distance (ft)	Landing Distance to clear 50' (ft)	Correction Factor	
POH	509	850	0.872	
Wind	443.93 Gust 456.94	741.33 Gust 763.06	0.898	
Grass	643.69 Gust 662.56	941.1 Gust 968.69	1.45	
Safety	965.54 Gust 993.85	1411.64 Gust 1453.03	1.5	
Runway Length (ft)	8002 , 8002	8002 , 8002	GOOD	

UPPER AIR WORK

Preparation for the flight in general and test day in particular involves a number of steps to ensure a smooth experience. The extra effort should be worth it, given the time and cost the student would have invested by this time. The following are suggested steps to be followed when preparing for your flight lesson or the big day.

You may get lucky and see the rainbow over the practice area once or twice when you fly either solo or duo. You do all the upper air work well, 5S maneuvers are great, the crosswinds have been among the best ones ever, and you attribute that partially to the attitude of the patient instructor sitting by your side who creates a calm learning environment and wants you to succeed—as they should. It should not be just “a random good flight day,” as some may say.

13.1 On the Test Day

A flight test normally involves the ground portion in addition to the check ride (test of flying ability) and requires completion of a number of practical flight hours (duo and single) prior to the test and an instructor's recommendation. You should dedicate a number of hours to prepare for the flight test, reviewing your notes and the resources available. Here are the recommended steps to be taken before the flight test:

1. Review the test syllabus of the ground school again and again before the test day.
2. Prepare a checklist, which notes all you need to do, from your morning routine to your detailed flight test tasks.
3. Contact the flight examiner the night before the test to be provided the route and weight information to do the route planning and weight-and-balance calculations the night or hours before.
4. Have a light but healthy meal, such as salad and fruits; a banana may help keep you full, and have organic ginger tablets available in case you need them to reduce motion sickness. You may also use blood-pressure controlling bracelets on your wrists, which by pressing on your pressure points, may help prevent nausea.
5. Check the weather forecast and update your flight plan accordingly.
6. Check the FIR advisories to see if any special maneuvers are happening around your flight test path—for example, north to the Algonquin in the flight plan example if you were to travel from CYGK to CYHM in Case Study 10.8.11 “Flight Plan and Itinerary Plan.”
7. Dress as you please in comfortable professional clothing.
8. Leave well ahead of your scheduled test time so that you can settle at the site, prepare the paperwork, and organize the folders which you are to present to the examiner demonstrating the airworthiness of your aircraft.
9. Do the walkaround beforehand, even if you are required to do it again with the examiner. Make sure a reliable person is available onsite in case you have questions or come across any discrepancies such as snags in the documents.
10. Make sure to have communicated to the local office the quantity of fuel required to make the aircraft fly in the utility category. You may use the ramp-off data and have some extra gallons on board to account for the start-up and taxi, especially if your aircraft is equipped with a short-range tank.
11. Hopefully, you have picked the day that the weather is cooperating; if not, make sure the conditions are not too challenging for you. If you do end up with an upset stomach, you may either continue the test or request to reschedule it. But if the motion sickness does get the better of

you, don't be too disconcerted—you are likely not the first person the examiner has seen experiencing this problem.

12. The examiner is to make you at ease during the test; the questions should remain within the test syllabus and be among those commonly asked on such occasions. It may happen that you may be asked to perform a maneuver that you have not learned or practiced in your flight training. Try to do your best and do not let it affect the rest of your test.
13. Some examiners are quick to provide positive feedback by saying that your forward slip or your landing were among the better ones they have seen—you would know then that you did something good even if you did not successfully complete the test.
14. When you arrive on the ground, you have either passed or failed. Marginally passing maneuvers and not meeting the total points (under 60 percent) is as much a fail as obtaining high points on all maneuvers and failing marginally on three maneuvers—yes, it is possible to fail with 85 percent.
15. Be prepared to have shaky knees when you put your feet on the ground after your test or laugh to ease up your tension even if there is a whirlwind inside your stomach.
16. It is beneficial to make prior arrangements with your flight instructor to attend the debrief with you, especially if this is your first flight test. It is really hard if they are not around, especially if you were asked during the test to perform maneuvers that were never practiced in your training. Hopefully, if things did not go well, your instructor will be responsible enough to speak with you the day after and encourage you to stay on course and to not lose heart. Remember that you are not the only one who has experienced this; rest assured that it has happened before, and more than once to some [246]. But let's hope for the best and that you successfully pass your test.

13.2 On Other Days

1. Carry a professional attire and demeanor; follow the golden ethical rule: "Do unto others as you would have others do unto you."
2. Keep away from the propeller area while other students are going through the start-up procedure; safety should not be compromised at any time and the student does not need the extra distraction.

3. Wear breathable clothing, possibly cotton if Flame Resistant Clothing (FRC) is not available; secure the sleeves and ankles; avoid wearing flip-flops.
4. Practice good cockpit management; secure your kneeboard.
5. Prepare a care bag to have with you in the aircraft that consists of the following:
 - (a) Ginger tablets to reduce stomach upset to be used after experiencing motion sickness or before each flight—organic ginger tablets are natural and do not contain sleep-inducing ingredients;
 - (b) Water bottle for a drink and mouthwash if needed;
 - (c) Chewing gum, mint-scented possibly, to use in case you feel stomach upset;
 - (d) A banana or two to eat before each flight or after;
 - (e) A couple of nutrition bars with nuts for when you feel hungry or have low blood sugar;
 - (f) Aircraft bag for motion sickness if needed;
 - (g) Extra batteries for your headset, for both you and the FIC (or SIC);
 - (h) Wallet-size tissue box to wipe off your sweat, clean your face, or dry your tears when makeup gets into your eyes;
 - (i) One or two pairs of plastic gloves suitable for both male and female aircraft handlers to be available especially for the long-haul flights;
 - (j) Basic first aid kit such as band aids and an antibiotic ointment;
 - (k) A timer for measuring field lengths. It may be wearable or also attachable to the dashboard and it should be equipped with a forward and backward chronometer;
 - (l) Flashlight for walkaround, inspecting the aircraft, finding items fallen in between the harness and belt attachments or on the cabin floor;
 - (m) For winter months, warm and non-slipping gloves to avoid cold or wet palms when touching the controls—you can put them under your hips or inside your jacket to warm up if required;
 - (n) Sunglasses that reduce glare; and
 - (o) Magnifying glass to read the maps or prescription glasses.

GOOD PRACTICES AND LEAN SIX SIGMA IMPLEMENTATION

The term *best practices* is a well-known expression in a variety of engineering disciplines in which the product CDIO (Conceive, Design, Implement, and Operate) lifecycle concept is used. When working on a model, analysis, or process of any kind, a variety of techniques may be employed, revised, and expanded upon. Hence, the author does not necessarily agree with the expression *best practices*; instead there are *good practices*, those which are more likely to lead to a useful outcome. Nevertheless, the challenge endures to make them better. This is to ensure improving performance and eliminating waste (i.e., Muda; a Japanese word for Futility, Uselessness, and Wastefulness), focusing on critical-to-quality characteristics [247].

There is no preferred approach to flight since anything about it fits in two categories: (a) technique and (b) approach. The techniques and science of flight improve or get revised; as science develops, new technologies are presented, and new measuring and sensing tools are introduced or old ones are improved. For the purpose of standardization, these techniques are usually based on the recommendations made by a body (e.g., Flight Safety Board—FSB) and are to be followed by the members that are part of a bigger community (e.g., International Civil Aviation Organization—ICAO). These are the responsibilities that come with being part of something bigger. These regulations are tested, proven, and accurate to the dots of the “i’s” and crosses of the “t’s.” They are not to be treated casually.

The approaches, on the other hand, may vary from one FIC to another. Recall that FIC is the short form for the First-In-Command. They are the persons in command and their judgment supersedes the rest. This is the concept of the *judgment priority*, the purpose of which is to avoid chaos in the cockpit. Thus, at first look, the cockpit environment may appear to be an example of a pure dictatorship, but is it? In his popular book *Outliers*, Malcom Gladwell tells a story of the tragic accident with a Korean airliner [248]. Upon reviewing the black box recordings, the investigators concluded that one of the main causes was lack of engagement of the flight crew due to the *extreme* respect for the FIC authority. It was pointed out that this kind of respect for authority was part of the culture. However, it could also have happened in any other environment where one is used to saying “yes, sir” or “yes, ma’am” blindly.

A similar story (of a 1978 United Flight 173) was discussed in Matthew Syed’s *Black Box Thinking* [249]. In that incident, a gear indicator light was not lighting up to confirm that the landing gear was indeed down; the gear was actually down and only the indicator light was faulty. The FIC was so focused on dealing with that light that he ignored repeated (gentle) warnings from another flight crew member about the fuel running low. Eventually, the plane did run out of fuel, short of the runway. After that 1978 accident, new training in Crew Resource Management (CRM) was developed and specific techniques implemented to improve assertiveness of the crew members. They were trained to think in terms of PACE (Probe, Alert, Challenge, and Emergency) when communicating with the FIC regarding any issue. Thus, while the FIC may still be a “king” or a “queen,” it is more like a parliamentary monarchy, where they are supposed to listen to and address the concerns of their flight crew.

Although it may appear that independent thinking is discouraged in a strictly controlled environment such as that experienced in flight, you need to remember that in the end, the FIC (and SIC) are responsible for the aircraft control. For example, the dispatch may be advising you to divert to the Northwest during your cross-country flight while you are flying inbound from the East, but at the same time you see the dark gathering clouds in that area. Based on the conditions, you decide to continue the route inbound and reduce the elevation since it is clearer and safer. Would you expect to be blamed later on for not following the instructions? You should not be, since the dispatch might not have been aware of the local conditions.

A good practice is to use the resources that are available to you. Sometimes, you will make use of the resources and experiences that on the surface are only very remotely connected to your present activity. Some years ago, you may have been investigating the lifecycle data for an aircraft hot section turbine disk. After hours of struggling to find what you needed, you took the initiative, and decided there must have been a better way to extract this information. As a result, a better database organization was developed that reduced the number of steps required to retrieve information from eight to one, reducing the time from over 2 hrs to under 5 min with minimum training required.

The last note is that you should always try to think ahead of the aircraft. Ask yourself: What is happening next and one more step after that? Will I need to make a radio call? On what frequency? I may set that up now. What is happening after that? Should I check the weather at the next waypoint? Thus, always think at least two steps ahead.

So, every time you make any response or choice, whether positive, negative, or a maybe, remember that you may be called upon to defend it later.

CONCLUSION

Your flight experience is not very different from other learning experiences if you do it professionally or as a “hobby.” You are to make sure that the difference between the potential and actual capability, which is to reflect the entitlement capability, is minimized to the extent possible. In other words, the difference between the entitlement and actual capabilities is the determining factor in defining the quality. Despite the common perception that the world is getting more complex as time passes due to technological enhancements, people remain the driving force—they are the ones who use their faculties to create a better world. When it comes to flying, direct and clear communication is a must.

Implement the 5S methodology (Sort, Set in order, Shine, Sustain, and Standardize) in your flight-related activities. The following is quoted from the Author’s work *Using COMSOL® in Heat Transfer Modelling from Slab to Radial Fin*. It is an example that shows the importance of this technique.

Recall that first solo (your first solo flight) party; the evening you made fresh home-made cookies—with love and respect, carefully packed party supplies such as tea-cups, plates, cutleries, serviettes, sugar bowl, and electric kettle in a blue cooler bag and drove to your evening [flight ground school] class. You made the freshly-brewed tea and let everybody help themselves with the goodies. When the party was over, you were left with cleaning up the party chaos. You started with: 1) sorting the items into their proper places—the extra cups were transferred to the blue bag, 2) setting items in order by

inserting smaller cups in larger ones, 3) shining the glass bowl after washing it, 4) standardizing the steps by taking mental note of the observed pros and cons of organizing such party, and 5) sustaining your [faint] smile after a long day. Step 2 helps you to organize the thoughts. Sorting steps can be organized more systematically by assigning the items to 5M baskets (i.e., material, machine, method, measurement, man and money).

Standardization, step 4, is particularly important. The challenge of any activity (e.g., the basics of learning to fly) is how to standardize the steps and sustain the developed good standards.

The FAA has initiated the Airman Certification Standard (ACS) program and has been in the process of implementing it since 2011, which is an improvement upon the well-known Practical Test Standards (PTS). The purpose is to associate points to the task-related knowledge (e.g., ground and upper air work), and to evaluate the task ensuring the activity meets the standard that is shared by all. It is all about implementing the good practices in knowing, planning, executing, and reporting the flight-related tasks to obtain aircraft ratings. This is a systematic approach to all the knowledge tests as well as ground and upper air work—the *know*, *consider*, and *do*.

For example, when taxiing, there are a number of steps to be performed. You are to debrief the FIC on the taxi diagrams; options, provisions, and routine checklists are to be followed; radio communications are to be established; clearance to taxi is to be obtained; and initial movement such as following the centerline and curves as well as obstacle avoidance are to be observed. The FIC is to monitor and record each task and provide a point system so that you can learn about the areas that need improvement and those where you are doing well, so you will keep doing the right things by reinforcing them. If you are to perfect execution of some task, you may be advised to employ a simple mock up (e.g., a cardboard) of the dashboard; for example, so that you can find location of switches in the future even blindfolded—it becomes your second nature. They call it *learning the flow*.

Think ahead of the airplane. Have you done your GUMPS [250] (Gas, Undercarriage, Mixture, Propeller, and Seat Belts) check to overcome the distraction and preoccupation during landing while respecting the sterile period? Recall the laws of learning—Intensity, Primacy, Readiness, Effect, Freedom, Exercise, Requirement, and Recency (IPREFERR) [251]. Note that the author chooses to place the *Effect* before the *Exercise*, for the principle of effect is directly related to the emotional reaction of the student

and the motivation. It states that learning is reinforced when accompanied by a pleasant or satisfying feeling. Whatever the learning situation, it must include the basics that influence the students positively and give them a feeling of satisfaction. Additionally, learning is weakened when associated with an unpleasant feeling (e.g., repetitive demeaning and teasing). The student will endeavor to continue doing whatever provides an enjoyable outcome to continue learning. Positive reinforcement is more apt to result in success and motivate the learner, so the instructor should recognize and acclaim the improvement if they care for students' success. This also affects *Readiness*, in other words, the degree of focus and eagerness—physically, mentally, and emotionally [252,253].

The author believes that Chris Austin Hadfield, a Canadian engineer, pilot, and astronaut, used this technique that combines knowledge with repetition to accomplish the most challenging tasks and to even overcome his fear of heights. In his memoirs, he tells a story that happened to him while doing one-on-one combat training in the CF-18. He found that his G-Suit's hose was accidentally disconnected during maneuvers by his elbow [254]. A G-Suit is worn when pilots experience high G-loads (acceleration) due to the fast maneuvers that may cause sudden rush of blood to or away from the brain—also known as red-out or black-out [255]. The G-Suit is supposed to help adjust the blood pressure to prevent blood's excessive accumulation or lack of it by pressing against the body parts, to provide resistance to the blood flow and therefore delay these adverse effects. As a result of the G-Suit's malfunction, C. A. Hadfield became unconscious for 16 s while his friend was trying to communicate with him; he called upon his operational awareness and got back on the ground first, before trying to find what happened in the air. His experience resulted in modifications to the G-Suit connection in the CF-18 to improve safety. He recommends not to focus on the bee in the helmet but instead on the task at hand. He is referring to the unavoidable circumstances arising under unknown conditions. Self-reportedly, he is also scared of heights, a sensation that he experienced from a young age when he used to fly in his father's biplane. Knowledge and experience have helped him realize that he is not going to fall and that he is not helpless, and in fact he does have some control. You will feel the nerves and stress in a "high-stakes situation"—as he puts it—but never be terrified. Were you aware of an astronaut who was scared of heights until now? Apparently, they are not the superheroes the author thought them to be, but only human beings, just like the author, with a range of emotions, capabilities, and occasional insecurities.

Technology and capabilities in the aviation and aerospace fields are being continuously enhanced. The Aeolus Satellite, as part of the European Space Agency (ESA) operating in Toulouse, France, continues to overcome technical challenges associated with the mechanical and thermal issues in the operation of a pioneering laser technology used for predicting atmospheric climate wind conditions from space as part of their unique initiative started in 2002. This sixteen-year effort has borne fruit. The powerful tool is like a human-made Sun, the laser sending pulses to the atmosphere whose reflections return after 3 ms, carrying only a minute fraction of the original energy. The received data, such as wavelength, frequency, and time, are employed to obtain the required atmospheric condition measures. Imagine laser pulses sent out 50 times per second, with a temperature between 20 °C to 1,700 °C. The fatigue generated in the optical elements through which the light passes from this thermal cycle-induced UV light stresses the coating due to the heat exposure. The two types of damage—laser induced damage of the coating and contamination due to the organic material that carbonizes and darkens the optics—would normally cause the laser to lose 50 percent of its initial energy within the first 6 min of operation. Although the vacuum is clean compared to the test cell on the Earth, the system is equipped with two small liquid oxygen tanks that run continuously for the duration of its lifetime and help it to combust the potential debris created and deposited on the compartments. Aeolus Project has been able to extend the lifecycle of this satellite system from the initial 6 min to three years (1,577,880 min based on a Julian year), which is equivalent to a 26,297,900 percent improvement in operation time; that constitutes an average improvement of 16,436 percent per year—a fittingly astronomical improvement [256].

Consider how humanity's knowledge and technology have progressed over the millennia. Ancient Greeks described the Sun as the chariot with fire-breathing horses driven by Helios (also known as Apollo). On August 30, 2018, as the concluding remarks of this work are being written, Parker Solar Probe is 17 days into its mission. This satellite will become the nearest human-made object to the Sun. On its mission, it will be passing through the Sun's corona, where it will remain for 6 years and 321 days, while relaying to the Earth data on the Sun's plasma, magnetic fields, and dust.

On June 6, 2018, the ISS Expedition 56-57 launched, sending three astronauts and cosmonauts (American, German, and Russian) aboard Soyuz MS-09 to the International Space Station. They had been preparing for months for this event, or more accurately for years; they were fitted

for their spacesuits, interviewed, and escorted to the vehicles at Star City in the Moscow Oblast in Russia—the home to Valentina Vladimirovna Tereshkova—a Russian politician, engineer, textile assembly worker, pilot, and cosmonaut, the first woman in space and the only one who completed a solo space mission when she flew the Vostok 6 in 1963—and Yuri Alekseyevich Gagarin—a Russian pilot and cosmonaut, the first human to fly in space in 1961, flying Vostok 1—the author’s childhood heroine and hero. From Star City, they were taken to Baikonur Cosmodrome in Kazakhstan. As they walk up the stairs to their spacecraft, with smiles on their faces, memories of previous launches inundate one’s mind. How quickly the time passed and how much progress has been made! This year, NASA is celebrating its 60th anniversary and revealing the logo commemorating the Apollo Program’s 50th anniversary.

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LIST OF SYMBOLS

C.1 Variables

a	acceleration (m/s^2)
a	fit parameters
a	semi-major axis (m, km, ft, mi, NM, SM)
\arctan	arc tangent operator
A	area (m^2 , ft^2)
A	surface area (m^2 , ft^2)
A_c	constricted area (m^2 , ft^2)
A_i	inflow area (m^2 , ft^2)
A_o	outflow area (m^2 , ft^2)
A_t	theoretical air mass (g, lb) or molecule (mole)
A'	constant value
A/A_t	mass or molecule ratio of the actual air to the theoretical air (kg/kg, lb/lb)
AOB	angle of bank ($^\circ$, rad)
AS	airspeed (kt, m/s, mph)
b	fit parameter
BHP	Brake Horsepower (%)

c	fit parameter
c	speed of sound in air (kt, m/s, mph)
c_p	specific heat capacity at constant pressure (J/kgK, cal/lb°F)
c_s	free stream speed of sound in air (kt, m/s, mph)
c_s	speed of sound in static air (kt, m/s, mph)
c_v	specific heat capacity at constant volume (J/kgK, cal/lb°F)
c_0	reference speed of sound in air (644.0 kt, 331.3 m/s, 741.1 mph)
C_D	drag coefficient
C_L	lift coefficient
C_P	heat capacity at constant pressure (J/K, cal/K)
C_V	heat capacity at constant volume (J/K, cal/K)
$\overline{C_P}$	molar heat capacity at constant pressure (J/molK, cal/mol°F)
$\overline{C_V}$	molar heat capacity at constant volume (J/molK, cal/mol°F)
$ChLong$	change of longitude (°, min)
$CoLat$	colatitude (°, min)
Cos	cosine operator
$Cotan$	cotangent operator
Csc	cosecant operator
CAS	calibrated airspeed (kt, m/s, mph)
CS	climb speed (kt, m/s, mph)
COG	center of gravity (m, km, ft, mi, NM, SM)
d	line of sight (m, km, ft, mi, NM, SM)
d	distance (m, km, ft, mi, NM, SM)
d	distance from the surface of the heavenly body to its parent (m, km, ft, mi, NM, SM)
d_{cal}	calculated distance obtained (m, km, ft, mi, NM, SM)
d_{sv}	distance obtained from Sky Vector (m, km, ft, mi, NM, SM)
dq	differential heat (J/kg, cal/lb)
ds	differential specific entropy (J/kgK, cal/mol°F)
dw	differential specific work (J/kg, cal/lb)
dw_s	differential saturated moisture content (g of water vapor/kg of dry air)

dx	finite difference along the x-coordinate (m, km, ft, mi, NM, SM)
dy	finite difference along the y-coordinate (m, km, ft, mi, NM, SM)
dz	finite difference along the z-coordinate (m, km, ft, mi, NM, SM)
dz	differential elevation (m, km, ft, mi, NM, SM)
dE	differential internal energy (J, cal)
dP	differential pressure (Pa, kPa, atm, inHg, mbar)
dQ	differential heat (J, cal)
dS	differential entropy (J/K, cal/°F)
dT	differential temperature (°C, K, °F)
dW	differential work (J, cal)
DA	density altitude (m, km, ft, mi, NM, SM)
DOM	dry operating mass (kg, lb)
e	error
e	water vapor partial pressure (Pa, kPa, atm, inHg, mbar)
$e_p(t)$	partial error
e_s	saturated water vapor partial pressure (Pa, kPa, atm, inHg, mbar)
e_{s0}	saturated water vapor partial pressure at the reference temperature (Pa, kPa, atm, inHg, mbar)
e'	correction factor for the temperature ratio
\dot{e}	single-derivative of error
\ddot{e}	double-derivative of error
E_{gen}	energy generated (J, cal)
E_{in}	input energy (J, cal)
E_{out}	output energy (J, cal)
E_{st}	energy storage (J, cal)
$Energy$	energy (J, cal)
EAS	equivalent airspeed (kt, m/s, mph)
ETP	equal time point (s, min, hr)
\dot{E}	rate of change of internal energy (W, hp)
\dot{E}_{gen}	rate of energy generated (W, hp)
\dot{E}_{in}	rate of input energy (W, hp)

\dot{E}_{out}	rate of output energy (W, hp)
\dot{E}_{st}	rate of energy storage (W, hp)
\dot{Energy}	rate of energy (W, hp)
f	ratio of the lift to the drag forces or coefficient
f	function
f_v	volume fraction
F	function
F	force (N, lbf)
F	rotor force (N, lbf)
$F_{elevator}$	elevator force (N, lbf)
F_i	force for rotor “ i ” (N, lbf)
F_B	buoyancy force (N, lbf)
F_D	drag force (N, lbf)
F_L	lift force (N, lbf)
F_T	thrust force (N, lbf)
F_w	passenger weight (N, lbf)
$Fuel$	fuel volume (lit, gal)
FP	flap setting ($^\circ$, rad)
$FUEL$	fuel mass (kg, lb)
g	gravity acceleration (m/s^2)
G	gravitational constant ($6.674 \times 10^{-11} \text{ Nkg}^{-2}\text{m}^2$)
GPH	rate of fuel consumption (gph)
GPM	fuel consumption per distance (gpm)
GR	ground roll (m, km, ft, mi, NM, SM)
GS	ground airspeed (kt, m/s, mph)
h_c	Planck constant ($6.62607040 \times 10^{-34} \text{ Js}$)
h_c	convective heat transfer coefficient (W/m^2K)
h_c	specific enthalpy of flow in the constricted area (J/kg, cal/lb)
h_f	molar enthalpy of formation at 25 $^\circ\text{C}$ (J/mol, cal/mol)
h_i	height associated with antenna “ i ” (m, km, ft, mi, NM, SM)

h_i	height of aircraft at location “ i ” (m, km, ft, mi, NM, SM)
h_i	specific enthalpy of reactants (inflow gases) (J/kg, cal/lb)
h_o	specific enthalpy of products (outflow gases) (J/kg, cal/lb)
h_r	radiative heat transfer coefficient (W/m ² K)
H	height (m, km, ft, mi, NM, SM)
H	elevation (m, km, ft, mi, NM, SM)
H_c	inner product of two vectors
H_A	aircraft cruise altitude (m, km, ft, mi, NM, SM)
H_{A-D}	aircraft altitude as set on the pressurization controller dial (m, km, ft, mi, NM, SM)
H_C	cabin altitude at the top of the climb (TOC) (m, km, ft, mi, NM, SM)
H_C	cabin altitude at cruise (m, km, ft, mi, NM, SM)
H_0	ground elevation at start of climb (m, km, ft, mi, NM, SM)
H_0	ground elevation at field elevation for landing (m, km, ft, mi, NM, SM)
\dot{H}_A	aircraft climb rate (ft/min—fpm)
\dot{H}_A	aircraft descent rate (ft/min—fpm)
\dot{H}_C	cabin climb rate (ft/min—fpm)
\dot{H}_C	cabin descent rate (ft/min—fpm)
I_{xx}	moment of inertia (kgm ² , lbin ²)
IAS	indicated airspeed (kt, m/s, mph)
k	dimensionless refractive index of the medium
k	dimensionless refractive index
k	spring-mass constant (N/m, lbf/in)
k	thermal conductivity (W/mK)
k_{air}	dimensionless refractive index of air ($k_{air} = 4/3$)
k_x	thermal conductivity along the x-coordinate (W/mK, cal/in°F)
k_y	thermal conductivity along the y-coordinate (W/mK, cal/in°F)
k_z	thermal conductivity along the z-coordinate (W/mK, cal/in°F)
k_B	Boltzmann’s constant (1.380649×10^{-23} J/K, cal/°F)

k_d	control derivative constant
k_{di}	control derivative constant for rotor “ i ”
K_f	control proportionality constant
K_{fi}	control proportionality constant for rotor “ i ”
K_i	control integral constant
K_{ii}	control integral constant for rotor “ i ”
K_p	control proportional constant
K_{pi}	control proportional constant for rotor “ i ”
K_E	kinetic energy (J, cal)
Kn	dimensionless Knudsen number ($Kn = l/l'$)
$KCAS$	calibrated airspeed (kt, m/s, mph)
$KEAS$	equivalent airspeed (kt, m/s, mph)
$KIAS$	indicated airspeed (kt, m/s, mph)
$KTAS$	true airspeed (kt, m/s, mph)
l	mean free path (m, km, ft, mi, NM)
l'	characteristic length (m, km, ft, mi, NM)
\ln	natural logarithm operator
\log	logarithm operator
L	latent heat (J/kg, cal/lb)
L	latitude ($^\circ$, min)
L_i	latitude at location “ i ” ($^\circ$, min)
L_f	latent heat of melting and freezing (J/kg, cal/lb)
L_s	latent heat of sublimation and deposition (J/kg, cal/lb)
L_v	latent heat of vaporization and condensation (J/kg, cal/lb)
Lat	latitude ($^\circ$, min)
$Long$	longitude ($^\circ$, min)
$Long_i$	longitude at location “ i ” ($^\circ$, min)
LA	Astronomical latitudes ($^\circ$, min)
LG	Geocentric latitude ($^\circ$, min)
LN	Geodetic latitudes ($^\circ$, min)
m	mass (kg, lb)

m	multiplier
m_d	mass of the dry air (kg, lb)
m_h	mass of the heavenly body (kg, lb)
m_{hp}	mass of the heavenly parent (kg, lb)
m_i	mass of the reactants (inflow gases) (kg, lb)
m_i	mass of the aircraft at location “ i ” (kg, lb)
m_i	mass of the component “ i ” (kg, lb)
m_o	mass of the planet (kg, lb)
m_o	mass of the product (outflow gases) (kg, lb)
m_v	mass of the water vapor (kg, lb)
M	dimensionless Mach number ($M = c_s/c_0$)
M	molecular mass (kg/mol)
M_d	molecular mass of the dry air (g/mol)
M_i	moment for rotor “ i ” (Nm, lbfin)
M_v	molecular mass of the water vapor (g/mol)
MLM	maximum landing mass (kg, lb)
$MTOM$	maximum take-off mass (kg, lb)
$MZFM$	maximum zero-fuel mass (kg, lb)
n	multiplier
n	day of the year
n	convergence factor
n	number of molecules (mol)
n_i	number of reactant (inflow gases) molecules (mol)
n_o	number of product (outflow gases) molecules (mol)
p	multiplier
p	response function
P	power (W, hp)
P	pressure (Pa, kPa, atm, inHg, mbar)
P_c	convection loss (W, hp)
P_c	constricted flow pressure (Pa, kPa, atm, inHg, mbar)
P_d	partial pressure of dry air (Pa, kPa, atm, inHg, mbar)

P_i	inflow pressure (Pa, kPa, atm, inHg, mbar)
P_{inHg}	pressure (inch of mercury)
P_{inHg0}	mean sea level pressure (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi)
P_o	outflow pressure (Pa, kPa, atm, inHg, mbar)
P_r	radiative loss (W, hp)
P_s	solar radiation power (W, hp)
P_s	static pressure (Pa, kPa, atm, inHg, mbar)
P_t	total (stagnation) pressure (Pa, kPa, atm, inHg, mbar)
P_t	free stream pressure (Pa, kPa, atm, inHg, mbar)
P_v	partial pressure of moist air (Pa, kPa, atm, inHg, mbar)
P_v	partial pressure of vapor (Pa, kPa, atm, inHg, mbar)
P_E	potential energy (J, cal)
P_{ISA}	international standard atmospheric pressure (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi)
P_{MSL}	mean sea level pressure (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi)
P_0	reference pressure (Pa, kPa, atm, inHg, mbar)
P_0	standard pressure (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi)
P_0	mean sea level pressure (101,325 Pa, 1 atm, 29.92 inHg, 14.67 psi)
PA	pressure altitude (m, km, ft, mi, NM, SM)
PET	point of equal time (s, min, hr)
PSR	point of safe return (km, mi, NM)
q	response function
q_x	heat flux along the x-coordinate (W/m ²)
q_y	heat flux along the y-coordinate (W/m ²)
q_z	heat flux along the z-coordinate (W/m ²)
\dot{q}_{gen}	volumetric heat generation (W/m ³)
\dot{q}_0	initial volumetric heat generation (W/m ³)
$Q_{c.v.}$	molar heat release (J/mol, cal/mol)

r	multiplier
r	fit parameter
r	compression ratio, ratio of the bottom dead centre (BDC) to the top dead centre (TDC) ($r = V_1/V_2$)
r	radius (m, km, ft, mi, NM, SM)
r	distance from the center of the heavenly body to its moon (m, km, ft, mi, NM, SM)
$r = R + d$	distance from the center of the heavenly body to the center of its parent (m, km, ft, mi, NM, SM)
r_a	apogee radius (m, km, ft, mi, NM, SM)
r_i	internal radius (m, km, ft, mi, NM, SM)
r_m	ratio of the mass components
r_p	perigee radius (m, km, ft, mi, NM, SM)
r_0	external radius (m, km, ft, mi, NM, SM)
R	rotational matrix
R	specific gas constant (kJ/kgK, cal/lb°F)
R	radius of the heavenly body (m, km, ft, mi, NM, SM)
R_d	dry air specific gas constant (287.0 J/kgK)
R_v	moist air specific gas constant (461.5 J/kgK)
R_{Earth}	radius of the Earth (m, km, ft, mi, NM, SM)
R^T	transpose of the rotational matrix
\bar{R}	universal gas constant (8.314 J/molK)
RH	relative humidity
ROC	rate of climb (fpm)
RPM	revolution per minute (rpm)
RPM	propeller angular speed (rpm)
s	specific entropy (J/kgK, cal/lb°F)
S	entropy (J/K, cal/°F)
$Scale$	scale
Sec	secant operator
Sin	sine operator

SF	shape factor
t	fit parameter
t	time (s, min, hr)
t	difference longitude ($^{\circ}$, min)
t_{nw}	time, no-wind scenario (s, min, hr)
t_t	total time, no-wind scenario (s, min, hr)
t_A^*	aircraft time to TOC (s, min, hr)
t_A^*	aircraft time to descend (s, min, hr)
t_C^*	time for cabin altitude to reach set value (s, min, hr)
t_i'	time for the portion “ i ” of the trip (s, min, hr)
\tilde{t}	approximate time (s, min, hr)
\tan	tangent operator
T	torque (Nm, lbf·in)
T	temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T	dry-bulb temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_{amb}	ambient temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_b	fluid-bulk temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_c	constricted area temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_c	isentropic-saturation temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_d	dew-point temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_f	freezing temperature of water (0 $^{\circ}\text{C}$, 273 K, 32 $^{\circ}\text{F}$)
T_i	initial temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_i	inflow temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_m	melting temperature of water (0 $^{\circ}\text{C}$, 273 K, 32 $^{\circ}\text{F}$)
T_o	outflow temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_s	element surface temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_s	static temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_s	free stream temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_t	total temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_v	virtual temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
T_w	wet-bulb temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)

T_E	total energy (J, cal)
T_{ISA}	international standard atmospheric temperature (15 °C, 288 K, 59°F)
T_{MSL}	mean sea level temperature (15 °C, 288 K, 59°F)
T_0	reference temperature (°C, K, °F)
T_0	standard temperature (15 °C, 288 K, 59°F)
T_0	mean sea level temperature (15 °C, 288 K, 59°F)
T_∞	surroundings temperature (°C, K, °F)
T'	temperature corrected for expansion coefficient (°C, K, °F)
$\overline{T_v}$	average virtual temperature (°C, K, °F)
TAS	true airspeed (kt, m/s, mph)
$TC50$	total distance to clear 50-ft obstacle (m, km, ft, mi, NM, SM)
TL	traffic load (kg, lb)
u	crosswind component (kt, m/s, mph)
u	dependent variable
u	response function
u	axis along the x-coordinate
u_i	dependent variable component along the x-coordinate
u_x	change of dependent variable along the x-coordinate
u_{xx}	derivative of change of dependent variable along the x-coordinate
u_ψ	angular change of dependent variable about the x-coordinate
v	velocity (kt, m/s, mph)
v	headwind component (kt, m/s, mph)
v	dependent variable
v	response function
v	axis along the y-coordinate
v_c	velocity of flow in constricted area (kt, m/s, mph)
v_e	escape velocity (kt, m/s, mph)
v_e	equivalent, balance velocity (kt, m/s, mph)
v_{ea}	escape velocity at apogee (kt, m/s, mph)

v_{ep}	escape velocity at perigee (kt, m/s, mph)
v_i	dependent variable component along the y-coordinate
v_i	velocity of reactants (inflow gases) (kt, m/s, mph)
v_o	velocity of products (outflow gases) (kt, m/s, mph)
v_{oc}	circular orbital velocity (kt, m/s, mph)
v_{oe}	elliptical orbital velocity (kt, m/s, mph)
v_{rms}	root mean square velocity (m^2/s^2)
v_{W-in}	inbound wind
v_{W-out}	outbound winds
v_x	change of dependent variable along the y-coordinate
v_{xx}	derivative of change of dependent variable along the y-coordinate
v_G	ground speed (kt, m/s, mph)
v_T	true airspeed (kt, m/s, mph)
v_W	wind speed magnitude (kt, m/s, mph)
v_W'	wind component (kt, m/s, mph)
v_0	initial velocity (kt, m/s, mph)
v_ϕ	angular change of dependent variable about the y-coordinate
V	volume (m^3 , ft^3)
V_i	volume of cylinder under the piston (m^3 , ft^3)
Var	Variation ($^\circ$)
w	moisture content (g of water vapor/kg of dry air, grains of water vapor per pound of dry air)
w	wind magnitude (kt, m/s, mph)
w	dependent variable
w	response function
w	axis along the z-coordinate
w_i	dependent variable component along the z-coordinate
w_i	weight fraction for content “i” (kg of content/kg of the total, grains per grains)
w_x	change of dependent variable along the z-coordinate

w_{xx}	derivative of change of dependent variable along the z-coordinate
w_θ	angular change of dependent variable about the z-coordinate
w'	wind speed magnitude (kt, m/s, mph)
x	spring displacement (m, km, ft, mi, NM, SM)
x	coordinate along the x-axis
x	distance along the x-coordinate (m, km, ft, mi, NM, SM)
x_i	dependent variable component along the x-coordinate
x_i	length along the x-coordinate (m, km, ft, mi, NM, SM)
x_{nw}	distance, no-wind scenario (m, km, ft, mi, NM, SM)
\tilde{x}	approximate distance (m, km, ft, mi, NM, SM)
y	multiplier
y	fit parameter
y	coordinate along the y-axis
y	distance along the y-coordinate (m, km, ft, mi, NM, SM)
y_i	dependent variable component along the y-coordinate
y_i	depth along the y-coordinate (m, km, ft, mi, NM, SM)
y_i	depth for the reactants (inflow gases) (m, km, ft, mi, NM, SM)
y_o	depth for the products (outflow gases) (m, km, ft, mi, NM, SM)
y_0	reference depth (m, km, ft, mi, NM, SM)
Y	latitude (°, min)
z	coordinate along the z-axis
z	distance along the z-coordinate (m, km, ft, mi, NM, SM)
z_i	dependent variable component along the z-coordinate
z_i	height along the z-coordinate (m, km, ft, mi, NM, SM)
z_i	height for the reactants (inflow gases) (m, km, ft, mi, NM, SM)
z_o	height for the products (outflow gases) (m, km, ft, mi, NM, SM)
z_0	reference height (m, km, ft, mi, NM, SM)
z_0	elevation (m, km, ft, mi, NM, SM)

C.2 Greek Symbols

α	absorptivity
α	thermal expansion coefficient (1/°C, 1/K)
α	thermal diffusivity (m²/s)
α	angle of attack (°, rad)
α	angular acceleration (rad/s²)
α_p	thermal expansion coefficient at constant pressure (1/°C)
β	coefficient of thermal expansion (1/°C, 1/K)
β	aircraft heading (°, min)
β_t	constant
γ	heat capacity ratio ($\gamma = c_p/c_v$)
γ_{air}	heat capacity ratio of the air ($\gamma_{air} = c_{p,air}/c_{v,air} = 1.40$)
γ_{vapor}	heat capacity ratio of the vapor ($\gamma_{vapor} = c_{p,vapor}/c_{v,vapor} = 1.34$)
Γ	lapse rate (°C/m, °C/ft)
Γ_d	dry-adiabatic lapse rate (°C/m, °C/ft)
Γ_w	moist (saturated)-adiabatic lapse rate (°C/m, °C/ft)
Γ_{ELR}	environmental (standard)-adiabatic lapse rate (°C/m, °C/ft)
Γ_{MSL}	mean sea level lapse rate (°C/m, °C/ft)
δ	constant length (m, km, ft, mi, NM, SM)
δ	pressure ratio, ratio of the static pressure to the reference pressure ($\delta = P_s/P_0$)
δ	crab angle (°, rad)
Δ	difference operator
$\Delta \bar{h}$	molar difference of enthalpy of formation from 25 °C (J/mol, cal/mol)
ΔH_A	difference between set and actual aircraft altitudes (m, km, ft, mi, NM, SM)
ΔH_C	difference in altitude between setting on the controller and ground elevation (m, km, ft, mi, NM, SM)
Δt	time step (s, min, hr)
Δt	time difference (s, min, hr)

Δt_c	time difference (offset) for cabin reaching set altitude before landing (s, min, hr)
Δt_c	time difference (offset) for cabin reaching set altitude before aircraft reaching the TOC (s, min, hr)
ΔT	temperature difference ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
Δx	step size along the x-coordinate (m, km, ft, mi, NM, SM)
Δx	distance difference along the x-coordinate (m, km, ft, mi, NM, SM)
Δy	step size along the y-coordinate (m, km, ft, mi, NM, SM)
Δy	distance difference along the y-coordinate (m, km, ft, mi, NM, SM)
Δz	step size along the z-coordinate (m, km, ft, mi, NM, SM)
Δz	distance difference along the z-coordinate (m, km, ft, mi, NM, SM)
ϵ	ratio of the gas constant for the dry air to that of the water vapor ($\epsilon = R_d/R_v = 0.622$)
ϵ	emissivity
η	efficiency
η_f	fin efficiency
θ	longitude ($^{\circ}$, min)
θ	potential temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
θ	temperature ratio, ratio of the static temperature to the total temperature ($\theta = T_s/T_0$)
θ	dimensionless fluid-bulk temperature ($^{\circ}\text{C}$, K, $^{\circ}\text{F}$)
θ	orbital angle ($^{\circ}$, rad)
θ	pitch angle ($^{\circ}$, rad)
θ	rotation angle about the vertical axis, z-coordinate ($^{\circ}$, rad)
θ_{amb}	dimensionless ambient temperature
θ_b	dimensionless fluid-bulk temperature
θ_i	dimensionless initial temperature
θ_m	dimensionless melting temperature
θ_s	dimensionless element surface temperature

θ_{∞}	dimensionless surroundings temperature
$\dot{\theta}$	angular velocity about the vertical axis, z-coordinate (rad/s)
$\ddot{\theta}$	angular acceleration about the vertical axis, z-coordinate (rad/s ²)
λ	longitude (°, rad)
λ	wavelength (nm, mm, m)
λ	wind relative angle to the aircraft path (°, rad)
λ_i	longitude at location “i” (°, min)
μ	coefficient of friction
ν	kinematic viscosity (m ² /s, in ² /s)
ρ	reflectivity
ρ	density (kg/m ³ , lb/in ³)
ρ_i	inflow density (kg/m ³ , lb/in ³)
ρ_o	outflow density (kg/m ³ , lb/in ³)
ρ_w	density of water (kg/m ³ , lb/in ³)
ρ_0	reference density (kg/m ³ , lb/in ³)
σ	Stefan Boltzmann constant (W/m ² K ⁴)
σ	standard deviation
τ	time (s, min, hr)
τ	transmissivity
ϕ	entropy (J/K, cal/°F)
ϕ	latitude (°, min)
ϕ	colatitude (°, min)
ϕ	rotation angle about the lateral axis, y-coordinate (°, rad)
$\dot{\phi}$	angular velocity about the lateral axis, y-coordinate (rad/s ²)
$\ddot{\phi}$	angular acceleration about the lateral axis, y-coordinate (rad/s ²)
ψ	rotation angle about the longitudinal axis, x-coordinate (°, rad)
$\dot{\psi}$	angular velocity about the longitudinal axis, x-coordinate (rad/s)
$\ddot{\psi}$	angular acceleration about the longitudinal axis, x-coordinate (rad/s ²)
ω	angular velocity (rad/s)
ω_i	angular velocity for rotor “i” (rad/s)

Ω	gyro rate of precision
Ω_t	normalized time ratio, ratio of the time difference to the total time ($\Omega_t = \Delta t/t_t$)
Ω_x	normalized distance ratio, ratio of the distance difference to that of the no-wind ($\Omega_x = \Delta x/x_{nw}$)
Ω_G	normalized aircraft speed, ratio of the ground speed to the true airspeed ($\Omega_G = v_G/v_T$)
Ω_W	normalized wind speed, ratio of the wind speed to the true airspeed ($\Omega_W = v_W/v_T$)
Ω_W'	normalized wind speed, ratio of the wind speed component to the true airspeed ($\Omega_W' = v_W'/v_T$)
Ω_{W-in}	normalized wind speed inbound, ratio of the inbound wind speed to the true airspeed ($\Omega_{W-in} = v_{inbound}/v_T$)
Ω_{W-out}	normalized wind speed outbound, ratio of the outbound wind speed to the true airspeed ($\Omega_{W-out} = v_{outbound}/v_T$)

C.3 Subscripts

<i>a</i>	apogee
<i>c</i>	constricted
<i>d</i>	dry air
<i>Equator</i>	Equator
<i>generated</i>	generated
<i>i</i>	inflow
<i>i</i>	item number (e.g., rotor, subsystem)
<i>in</i>	input
<i>inbound</i>	inbound
<i>MSL</i>	mean sea level
<i>nw</i>	no-wind
<i>o</i>	outflow
<i>out</i>	output
<i>outbound</i>	outbound
<i>p</i>	perigee

P	constant pressure
P	products
<i>Pole</i>	Pole
R	reactants
R	radius of the Earth (m, km, ft, mi, NM, SM)
<i>storage</i>	storage
T	constant temperature
v	water vapor
V	constant volume
0	international standard atmosphere (ISA)
∞	surroundings

GLOSSARY

AC	Advisory Circular
AC	Alternate Current
ACS	Airman Certification Standard
AD	Anno Domini
ADF	Automatic Direction Finder
ADS-B	Automatic Dependent Surveillance Broadcast
AGL	Above Ground Level
AIM	Aeronautical Information Manual
AIRMET	Airmen's Meteorological Information
ALAR	Approach and Landing Accident Reduction
AME	Aircraft Maintenance Engineer
APV	Approach with Vertical Guidance
AROWJILI	Airworthiness, Registration, Pilot's Operating handbook, Weight-and-balance, Journey logbook, Insurance, Licenses, and Interception procedures
AS	Airspeed
ASBU	Aviation System Block Upgrades
ASDA	Accelerated Stop Distance Available

ATM	Air Traffic Management
ATS	Air Traffic Service
AVGAS	Aviation Gasoline
BC	Before Christ
BCE	Before Common Era
BDC	Bottom Dead Center
BDS	BeiDou navigation Satellite system
BHP	Brake Horsepower
CAD	Computer Aided Design
CAD	Constant Angle of Descent
CAFE	Contingency, Alternate, Final Reserve, and Extra
CAP	Civil Air Patrol
CAPE	Convective Available Potential Energy
CAPECON	Civil UAV Applications and Economic Effectivity of Potential Configuration Solutions
CAS	Calibrated Airspeed
CDFA	Constant Descent Final Approach
CDIO	Conceive, Design, Implement, and Operate
CFD	Computational Fluid Dynamics
CFI	Chief Flight Instructor
CFIT	Controlled Flight Into Terrain
CFS	Canada Flight Supplement
CME	Coronal Mass Ejection
COG	Center of Gravity
COP	Center of Pressure
COWLS	Civilization, Obstacles, Wind, Landing site, and Surface
CP	Critical Point
CRFI	Canadian Runway Friction Index
CRM	Crew Resource Management
CVFR	Controlled Visual Flight Rules
DA	Decision Altitude

DAQ	Data Acquisition
DC	Direct Current
DGI	Directional Gyro Indicator
DH	Decision Height
DME	Distance Measuring Equipment
DOF	Degree of Freedom
DOM	Dry Operating Mass
DR	Dead Reckoning
DST	Daylight Saving Time
DT	Daylight Time
EAS	Equivalent Airspeed
EAS	Experimental Aircraft Association
EASA	European Aviation Safety Agency
EDM	Electronic Distance Measurement
ELT	Emergency Locator Transmitter
EM	Electro-Magnetic
ER	Earth Rate
ESA	European Space Agency
ETP	Equal Time Point
FAA	Federal Aviation Administration
FAF	Final Approach Fix
FAR	Federal Aviation Regulations
FBO	Fixed-Base Operator
FEM	Finite Element Method
FIC	First-In-Command
FIC	Flight Information Center
FIR	Flight Information Region
FIS	Flight Information Service
FL	Flight Level
FMS	Flight Management Systems
FO	First Officer

fpm	Feet Per Minute
FRC	Flame Resistant Clothing
FSB	Flight Safety Board
FSS	Flight Service Station
GAF	Australian Government Factories
GEOREF	World Geographic Reference System
GIS	Geographic Information System
GL	Ground Level
GLONASS	Global Navigation Satellite System used in Russia
GML	Geographical Markup Language
GMT	Greenwich Mean Time
GNSS	Global Navigation Satellite System
GNSS Galileo	Global Navigation Satellite System used in Europe
GPS	Global Positioning System
GUMPS	Gas, Undercarriage, Mixture, Propeller, and Seat belts
HASEL	Height, Area, Security, Engine, and Lookout
HF	High Frequency
HG	Heat Generation
HR	Humidity Ratio
IAF	Initial Approach Fix
IAPC	Instrument Approach Procedure Chart
IAS	Indicated Airspeed
ICAO	International Civil Aviation Organization
IFR	Instrument Flight Rules
IHP	Indicated Horsepower
ILS	Instrument Landing Systems
INC	Instrument Flight Rule Navigation
INS	Inertial Navigation Systems
INSPIRE	Infrastructure for Spatial Information in the European Community
IPREFERR	Intensity, Primacy, Readiness, Effect, Freedom, Exercise, Requirement, and Recency

IRNSS	Indian Regional Navigation Satellite System
ISA	International Standard Atmosphere
ISS	International Space Station
IVAO	International Virtual Aviation Organization
IWXXM	ICAO Meteorological Information Exchange Model
KISS	Keep It Simple, Stupid
LASER	Light Amplification by Stimulated Emission of Radiation
LCC	Lambert Conformal Conic
LDA	Landing Distance Available
LFR	Low Frequency Radio
LN	Latitude Nut
LNAV	Lateral Navigation
LOP	Line of Position
LOS	Line of Sight
LPV	Localizer Performance with Vertical guidance
MAP	Missed Approach Point
MAV	Micro-Aerial Vehicle
MC	Mercator Conformal
MDA	Minimum Descent Altitude
METAR	Aviation Routine Weather Report
MFD	Multi-Function Display
MLM	Maximum Landing Mass
MoGas	Motor Gasoline
MOCA	Minimum Obstacle Clearance Altitude
MOS	Model Output Statistics
MSL	Mean Sea Level
MTOM	Maximum Take-off Mass
MTOW	Maximum Take-off Weight
MZFM	Maximum Zero-Fuel Mass
NACA	National Advisory Committee for Aeronautics
NASA	National Aeronautics and Space Administration

NDB	Non-Directional Beacon
NexGen	Next Generation
NHP	Nominal Horsepower
NIST	National Institute of Standards and Technology
NM	Nautical Mile
NNSS	Navy Navigation Satellite System
NORDO	No-Radio
NPA	Non-Precision Approach
NPA	Notices of Proposed Amendment
NREL	National Renewable Energy Laboratory
OSR	Optical Solar Reflectors
PA	Polyamide
PACE	Probe, Alert, Challenge, and Emergency
PDT	Pacific Daylight Time
PET	Point of Equal Time
PFD	Primary Flight Display
PID	Proportional, Integral, and Differential
PIREP	Pilot Report
POH	Pilot's Operating Handbook
PPL	Private Pilot License
PSR	Point of Safe Return
PTS	Practical Test Standards
QNE	29.92 inHg (101,325 Pa, 1 atm, 14.67 psi) altimeter subscale setting for International Standard Atmosphere (ISA)
QNH	Quasi Non-Hydrostatic
QZSS	Quasi-Zenith Satellite System
RCAM	Runway Condition Assessment Matrix
RDF	Radio Direction Finding
RFI	Runway Friction Index
RH	Relative Humidity
RMK	Remark

RORSAT	Radar Ocean Reconnaissance Satellite
RPM	Revolutions Per Minute
RPV	Remoted Pilot Vehicle
RRDF	Reverse Radio Direction Finding (RDF)
RTK	Real Time Kinematic
RVR	Runway Visual Range
RVV	Runway Vertical Visibility
RwyCC	Runway Condition Codes
RW	Real Wander
SCDA	Stabilized Constant Descent Angle
SESAR	Single European Sky Air Traffic Management (ATM) Research
SHP	Set Heading Point
SHP	Shaft Horsepower
SIC	Second-In-Command
SIGINT	Signals Intelligence
SIGMET	Significant Meteorological Report
SMART	Specific, Measurable, Achievable, Relevant, and Timely
SOP	Standard Operating Procedures
SPECI	Special Weather Report
SRC	System Resource Controller
ST	Standard Time
STZ	Sierra, Tango, and Zulu
SVFR	Special Visual Flight Rules
TAC	Terminal Area Charts
TACAN	Tactical Air Navigation
TAF	Terminal Aerodrome Forecast
TAI	International Atomic Time
TAS	True Airspeed
TBO	Time in Between Overhaul
TCA	Tropical Cyclone Advisory

TCCA	Transport Canada Civil Aviation
TDC	Top Dead Center
TEL	Tetraethyl Lead
TIMWOODS	Transportation, Inventory, Motion, Waiting, Over-production, Over-processing, Defects, and Skills
TL	Traffic Load
TMS	Thermal Modelling Suite
TOC	Top of Climb
TODA	Take-off Distance Available
TORA	Take-off Runway Available
TTF	Trend Type Forecast
TW	Transport Wander
UAV	Unmanned Aerial Vehicles
UHF	Ultra High Frequency
USAF	United States Air Force
UTC	Universal Time Coordinated
UV	Ultra Violet
VAA	Volcanic Ash Advisory
VFR	Visual Flight Rules
VHF	Very High Frequency
VNC	Visual Flight Rules (VFR) Navigation Chart
VOR	Very high frequency (VHF) Omnidirectional Range
VORTAC	Very high frequency (VHF) Omnidirectional Range (VOR) and Tactical Air Navigation (TACAN)
VSI	Vertical Speed Indicator
VTa	Visual Flight Rules (VFR) Terminal Area Chart
VTOL	Vertical Take-off and Landing
WAAS	Wide Area Augmentation Systems
WAC	World Aeronautical Charts
WHP	Wheel Horsepower
WMO	World Meteorological Organization

XML	Extensible Markup Language
ZFM	Zero-Fuel Mass
5M	Material, Machine, Method, Measurement, Man, and Money
5S	Sort, Simplify, Shine, Standardize, and Sustain
5S	Stall, Spin, Spiral, Steep turn, and Slow flight

INSPIRED THOUGHTS: INSPIRING LIVES

This section presents a collection of those individuals whose lives I found inspiring. For each one, I include a “thought inspired” which is my own reinterpretation of this person’s words.

Austen, Jane

Sky is a canvass of azure brushed by the delicate white, grey, yellow, and orange hues, naturally moving over the regions and adapting to the altitudes.

A British novelist, Austen is well-known for her six major novels, among which are *Sense and Sensibility* (1811) and *Pride and Prejudice* (1813). Her works interpret, critique and comment upon her contemporary British social class, focusing on the hypocrisy and pretensions in the majority of her works, where people of different walks of life travel to the town of Bath to shop, attend balls, and learn about the latest gossip. She depicts the changes that she thought ought to occur or that had commenced at the transition to the nineteenth century, affecting the social classes and interactions among them. The common conceptions of the time that women were to be dependent on men to project a favorable social standing and “approved” economic security, and furthermore, had to make babies to be worthy of living and respect, are implied in her writings. Her works have inspired many other books as well as motion pictures. At the time, married British women were not legally able to sign contracts, and they were to

be presented by a male relative to sign their contract if they wished to be published. Jane Austen's books were published anonymously, similar to other female authors, since the woman's main responsibility was perceived to marry and have children; therefore, other activities such as authoring a book would have been considered a secondary role, a part-time job. Jane Austen's *Sense and Sensibility* was published under "By a Lady," and after its success, all of Jane Austen's subsequent books were published under "By the author of *Sense and Sensibility*." The name of this literary goddess never appeared on her books during her lifetime. Her brother Henry Austen wrote a bibliographical note for Jane Austen's works, *Persuasion* and *Northanger Abbey*, that after her passing were published as a set in collaboration with her sister (Cassandra) and her publisher (Murray) in which her name (Jane Austen) appeared for the first time, identifying her as the author. Her novels have never been out of print. She uses plenty of wit and triste irony to express her feelings along with realism and social commentaries, and for that she is among the most powerful writers of all time [257].

Bach, Richard David

Pilots adhere to the laws of physics and mechanics of flight to remain afloat, similar to the musicians surrendering to their feel for the harmony in scores and dancers surrendering to the choreographed steps attuned to the music.

An American writer, Bach is known as the author of bestsellers such as *Jonathan Livingston Seagull* (1970) and *Illusions: The Adventures of a Reluctant Messiah* (1977). Most of Bach's books are either directly related to flight or, as in *Stranger to the Ground*, employ concepts that are metaphorically flight-related. Bach served in the United States Navy Reserve and New Jersey Air National Guard's 108th Fighter Wing and 141st Fighter Squadron (USAF) as a fighter pilot. Bach also undertook other responsibilities such as working as a technical writer for Douglas Aircraft and a contributing editor for *Flying* magazine [258].

Curie, Marie Skłodowska

The nature, its beauty, and mechanism of its minutest details working together harmoniously have constantly inspired human's awe; one rejoices in them with the child's wonder.

A Polish and naturalized-French physicist and chemist, Marie Curie made pioneering research on radioactivity. Also known as Madame Curie, she was the first woman to win a Nobel Prize and the first person and only woman to win it twice in two different sciences. Along with her husband (Pierre Curie), daughter (Irène Joliot-Curie) and son-in-law (Frédéric Joliot-Curie), they were awarded five Nobel Prizes on different occasions. Marie Curie was the first woman to become a professor at the University of Paris. Along with her husband Pierre, she was entombed in the Panthéon in Paris in 1995 due to her heroic effort of using X-rays to take the pictures of fractured bones during World War I. She was born in Warsaw, Poland, and studied at Warsaw's clandestine Flying University, also known as Floating University, an underground educational organization. It was given this name since the courses were spread throughout the city with the locations often changing to prevent the Russian authorities, in control of Poland at that time, from learning the location and arresting the teachers and students. At age 24, she followed her older sister Bronisława to study in Paris, where she earned her doctorate degree and conducted her substantial scientific work. Marie Curie shared her 1903 Nobel Prize in Physics with her husband (Pierre Curie) and a fellow physicist (Henri Becquerel, a French physicist, Nobel laureate, who discovered evidence of radioactivity). She won the 1911 Nobel Prize in Chemistry [259].

da Vinci, Leonardo di ser Piero

Once one soars in the wild blue yonder, one forever walks the earth with the tearful eyes turned to the sky, yearning to be there, among the clouds, on top of them, wrapped in their compassion, floating over the fields of green, gold, and brown, with the faint rainbow on the horizon.

An Italian polymath, da Vinci was a Renaissance man in 1476, with areas of interest in mathematics, science, engineering, invention, music, sculpting, literature, anatomy, geology, astronomy, botany, history, writing, and cartography. He is called *the father of paleontology, ichtnology, and architecture*, and is considered one of the greatest painters of all time, well-known for his works *Mona Lisa* and *The Last Supper*. Parachute, helicopter, and tank are among his concept inventions. da Vinci conceptualized flying machines, an armored fighting vehicle and solar power. Very few of his designs were constructed or were feasible during his lifetime due to the lack of contemporary technological capabilities [260].

Einstein, Albert

Every kind of peaceful cooperation among people is primarily based on mutual trust, respect, and appreciation for diversity of talents and characters and comes well ahead of the imposed moral beliefs and law-enforcing institutions such as courts of justice and police.

A German-born theoretical physicist, Einstein is famous for his impactful contribution in modern and quantum physics as well as the philosophy of science. He is well-known for developing the theory of relativity and his formula relating energy and mass ($E = mc^2$). His theory of relativity was based on his belief that Newtonian mechanics were not necessarily applicable when classical laws of mechanics and electromagnetics are both in place. Einstein was the recipient of the 1921 Nobel Prize in Physics for the discovery of the law of the photoelectric effect, which became the foundation for quantum theory. After graduation, he spent a long time searching for a teaching position; however, he was not successful, so he worked in a patent office where he was able to critique ideas and develop ones of his own. He was awarded his Ph.D. from the University of Zurich and later was appointed as lecturer at the University of Bern. While Einstein was visiting the United States in 1933, Nazis came to power in Germany. Knowing that he would be eventually barred from holding academic positions in Germany due to his ethnicity, he traveled to Belgium, England, and finally to the United States, where he took a position at the Institute for Advanced Study in Princeton, New Jersey, where he eventually settled despite numerous offers from European academic institutions. At the beginning of World War II, he composed a letter to the President Franklin Roosevelt, informing him of “extremely powerful bombs of a new type” and recommended researching the subject; the Manhattan Project was the result of this recommendation. It is reported that although he supported the Allies, he did not approve of the idea of nuclear fission as a weapon. To emphasize his views, together with Bertrand Russell, a British philosopher, he wrote the Russell-Einstein manifesto, in which they explained the hazards of a nuclear weapon. He developed interest in music since his early childhood and learned to play violin. He published over 300 scientific papers and 150 non-scientific papers. He was nicknamed as the Genius for his deep understanding and intellectual contributions [261].

Ford, Henry

Aircraft takes off against the wind and not with it; it does it with its heart—its engine—urging it on ahead.

An American industrialist and businessman, Ford is well-known for founding Ford Motor Company and developing the assembly line technique, also known as mass production. He changed the automobile from an expensive curiosity into a practical means for running everyday errands and as a result improving the prospect of life in the twentieth century. Ford's philosophy was to lower the costs in a smart fashion, resulting in introducing more efficient ways to get the job done. He entered the aviation industry during World War I, building Liberty engines. In 1925, he acquired Stout Metal Airplane Company. His best aircraft—first flown in June 1926—was the Ford 4AT Trimotor, also known as the *Tin Goose* due to its corrugated metal construction, made of a new alloy (Alclad) that improved its mechanical properties by combining the corrosion resistance of aluminum and the strength of duralumin. The plane is very similar to Fokker's V.VII-3m [262].

Harrington, H. James

To understand one's performance, one must measure it and watch the milestones as one advances through the phases; so that one may model his progress and adjust the variables, which can be controlled, resulting in one's success. No improvement is possible without understanding, measurement, and control.

An American author, lecturer, consultant, international performance improvement engineer, and businessman, Harrington has developed many concepts such as poor-quality cost, total improvement management, and business process improvement. He has authored over 35 books and software packages in the fields of organizational performance improvement and project management. Harrington also served as the president and chairman of the board for The American Society for Quality and International Academy. The cost of poor quality (COPQ) concept, also known as Poor Quality Costs (PQC), was introduced in his book *Poor Quality Costs* in 1987. COPQ refers to the costs that would disappear if systems, processes, and products were perfect. The gap between the full potential that may be achieved and the current status of product satisfaction is the missed entitlement (opportunity). The concept of quality cost was initiated in the

1960s when IBM decided to investigate its own quality costs and apply the quality concepts to its processes. The term *poor quality cost* was adopted to counteract Feigenbaum's term *quality costs* that focuses on the cost associated with producing better-quality products. This is to emphasize that investing in defect prevention and detection does indeed offset the associated costs both in the short and long terms [263].

Lilienthal, Otto

The thin air is significant enough to support the entirety of an aircraft; that, along with trusting that the laws of physics consistently apply throughout the universe, inspires confidence and faith.

A German aviator, nicknamed the Flying Man, Lilienthal was the first person to successfully fly with gliders and record his flight experiences in an organized fashion. His major contribution to aviation was the development of heavier-than-air flight. The Wright brothers were inspired by his work. He made his flights from an artificial hill, constructed near Berlin to test his glider. In his United States patent filed in 1894, he instructs the pilots to grip the bar for carrying and flying their hang gliders. The A-frame structure suggested by this inventor and Percy Pilcher (a British aviator and inventor) is the foundation of modern hang gliders and ultralight aircraft. Collaborating with his brother (Gustav), Lilienthal successfully designed and made more than 2,000 glider flights from 1891 until his final flight in 1896, when he died as a result of injuries sustained from a dive which he could not control. He achieved a total flying time of 5 hrs, covering a distance as long as 250 m (820 ft). He was able to take advantage of updrafts of up to 10 m/s to remain stationary so that his photographs could be taken from the ground. He investigated the flight of birds in detail (e.g., storks), collected data, and correlated polar diagrams to present the relationship between the sink rate and horizontal airspeed of gliders to model the aerodynamics of glider wings [264].

Nietzsche, Friedrich Wilhelm

Before learning to fly, one is to learn to crawl, stand, wobble, walk, run, jump, climb, dance, and then fly; one cannot fly into flying.

A German philosopher, composer, poet, philologist, and language scholar, Nietzsche is well-known for his influential work in Western philosophy and modern history. He was merely 24 when he became the

youngest to hold the chair of classical philology at the University of Basel in 1869. His works are interpreted and misinterpreted due to their provocative nature, invoking passionate reactions, and are considered the foundation for the European cultural rebirth [265].

Quran, The Holy Book

Earth is created like a carpet under one's feet; sky is created like a canopy over one's head; rain is sent down from the sky and trees bear fruits as a result [266].

The clouds are driven around; then are folded together; then are agglomerated to the mountain of clouds; precipitation is generated from their midst; rain, snow, and hail follow one another [267].

The Sun and the Moon move with precision along their path [268].

The term *Quran* means recitation and is the central religious book in Islam, believed to be God's revelation to his prophet Muhammad (PBOH) over a period of 23 years, commencing December 609 CE, with monotheism at its heart. It is said to be one of the finest works of Arabic literature. It consists of four main sections (*hezb*), 30 subsections (*joze*), and 114 chapters (*surah*). Each *surah* consists of several verses, and there are over 6,000 verses in total. Detailed information on historical accounts is occasionally presented with emphasis on the moral significance of events. It does share familiar ethical narratives with the Biblical scriptures. The *tafsir* (the interpretation of the Quran is occasionally done) in combination with *Hadith* (the traditions that have been passed on over generations, which influence the interpretation outcome depending on the school of thought). For example, it is not stated in the Book that women are to cover their faces; however, modesty, virtue, and family values as well as compassion and respect for humankind are emphasized. The Book is significant in terms of scientific concepts it represents in terms of the sky and precipitation seen in the previously provided examples. The Book has been the source of inspiration to many artists in the field of calligraphy, recitation, architecture, literature, medicine, and science. It emphasizes logic, evidence, and argument and prohibits blind following. Moses is mentioned in the Book more than any other individual, Jesus is mentioned more than Muhammad, and Mary is mentioned more than in the New Testament [269].

comte de Saint-Exupéry, Antoine Marie Jean-Baptiste Roger

To be a human is to be responsible for the actions and accountable for the results, to believe that the combination of the two is the stepping stone, contributing to building a better world.

A French writer, poet, aristocrat, journalist, and pioneering aviator, the Earl of Saint-Exupéry became a laureate of several of France's highest literary awards and the United States National Book Award. His famous novella, *The Little Prince* (*Le Petit Prince*), has been translated into 300 languages. He also wrote lyrical aviation-inspired writings such as *Wind, Sand and Stars* (*Terre des Hommes*) and *Night Flight* (*Vol de Nuit*). His 1939 philosophical memoir was adopted for the name of an international humanitarian group, which also inspired the central theme of the most successful world's fair of the twentieth century, *Expo 67* (*Man and His World—Terre des Hommes*) held in Montreal, Quebec, Canada. Saint-Exupéry was a successful commercial pilot before WWII, and worked airmail routes in Europe, Africa, and South America. He joined the French Air Force (*Armée de l'Air*), flying reconnaissance missions that ended with France's armistice with Germany in 1940 after which he traveled to the United States, where he wrote three of his most important works. He later joined the Free French Air Force in North Africa despite being far past the maximum age for such pilots and in declining health. He disappeared over the Mediterranean on a reconnaissance mission in July 1944, and is believed to have died at that time [270].

Saimdang, Shin

Only the impregnated white clouds can fly over the mountains with their shades of grey, green, and blue in the distant horizon.

A Korean artist, writer, and calligraphist in 1504–1551, mother of the Korean Confucian scholar Yi I, Shin is often held up as a model of Confucian ideals; her respectful nickname was Eojin Eomeoni, meaning Wise Mother. Other versions of the poem's translation read: (a) Only the white clouds float over the mountains; and (b) White clouds rush down the darkening blue mountains [271].

Socrates

Only if one rises above the Earth and crosses all the atmosphere layers, all the way to the exosphere, can one fully fathom the beauty of the world they live in and belong.

A Greek philosopher, Socrates is well-known as one of the founders of western philosophy, the first moral philosopher, promoting work ethics through his school's teachings. It is believed that he did not actively write during his period; however, his work was recorded through his students such as Plato and Xenophon who promoted his ideas after his passing away. Plato's dialogues are among the most comprehensive of all works associated with Socrates. Republic represents the conversations between Socrates and Plato on politics surrounding the state of affairs in Athens, concluding that ideals belong in a world where only the wise man can understand. The "wise man" here is referred to the philosopher capable of governing others. Plato might have affected our perception and interpretation of Socrates political and democratic views due to his own understanding of the state of affairs during his mental note-taking sessions [272].

Tengesdal, Merryl

The flat Earth at a low altitude and its curvature as one flies to higher altitudes are experienced by so few special blessed ones, who have flown at 70,000 feet altitude and beyond, who can express the real beauty, peace, quiet of hearing one's heartbeat, breath, hum of the engine, and do not fail to properly appreciate it.

An African American, Colonel Tengesdal is the first woman to fly the United States U-2 plane, used for specialized high-altitude reconnaissance missions. She is one of only five women and three African-Americans ever to be accepted into the U-2 program. Colonel Tengesdal has accumulated over 3,400 flight hours, and has ratings on T-34S, TH-57 B/C, SH-60B, T-6A, T-38A/B/C, and U-2S. She has a science degree in electrical engineering. After officer candidate school commissioning, she began a career as a naval aviator by flying the SH-60B Seahawk helicopter at Naval Station Mayport, Florida. After a three-year sea tour in helicopters, she became a T-34C and T-6A instructor pilot, one of four Navy T-6A Instructors to train Navy and Air Force students at Joint Student Undergraduate Pilot Training (JSUPT) at Moody Air Force Base, General Aviation (GA) [273].

Wright, Orville and Wilbur

Imagine the combination of peace and excitement with the stress taking toll on the nerves in a sensational experience called flight. Are there any sports equal to what aviators relish, being carried through the air, over the

white clouds, fields, mountains, and lakes on great metal wings, with a radio as the only source of communication, and physics as their bosom friends?

The Wright brothers are American aviators, engineers, inventors, and aviation pioneers well-known for inventing, building, and flying the world's first successful airplane. They made the first controlled and sustained flight of a powered aircraft on December 17, 1903, at a sight located four miles south of Kitty Hawk, North Carolina. They developed their aircraft into the first fixed-wing type in 1904. They were the inventors of three-axis control surfaces including rudders and ailerons that made controlled fixed-wing powered flight possible [274].

Yeager, Charles Elwood

While there may exist those very few without any fears, most pilots with countless flight hours will agree that it is the fear of dying that makes them learn everything possible about their aircraft, to maintain a sense of curiosity about anything connected with it and its surroundings, its emergency equipment and procedures; this fear makes them treat their machines with respect and always stay alert in the cockpit.

An American Air Force officer, flying ace, and record-setting test pilot, Brigadier General Yeager became the first pilot who officially exceeded the speed of sound flying an experimental Bell X-1 at Mach 1.07 at an altitude of 45,000 ft (13,700 m) in a level flight in 1947. Also called Chuck, he was a private in the United States Air Force during World War II. He served as an aircraft mechanic, after which he was promoted to the rank of flight officer after completion of the pilot training program and became a P-51 fighter pilot. Yeager then advanced in his career as a test pilot of experimental (e.g., rocket powered) aircraft [275].

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and by the Persian Gulf on the south, with a population over 79,100,000 (as of 2015). The country was a monarchy until 1979 when it was renamed the Islamic Republic after a popular uprising. Iran's language is Persian (Farsi) and the letters are written in the Persian alphabet that is rooted in Phoenician alphabet, the oldest verified alphabet. It is an *abjad* style of writing, meaning that each letter is a consonant and the reader is to add the vowels. This 22-letter Phoenician alphabet was expanded to 32 letters into the current Farsi alphabet. Although some Farsi letters are similar in writing to Arabic letters, the languages are different, as Arabic is a Semitic language versus Persian (Farsi), which is an Indo-European language. Persian has had a lexical influence on other languages such as Armenian, Turkic, and Indo-Aryan languages, for it has represented a prestigious cultural language in regions such as Western, Central, and Southern Asia. Although it has influenced Arabic language to some extent, new words were borrowed from Arabic by the Persians after the Arab Conquest of Iran. Due to its ancient historical significance, Persian was able to resist the Arabic language dominance in reading and writing. Farsi (Persian) was strengthened by fields such as literature, art, poetry, music, science, and mathematics through the works of Persian scholars (e.g., *Shahnameh* by Ferdowsi, *Rubaiyat* by Omar Khayyam, *Haft Peykar*—the inspiration for the *Turandot* opera by Puccini—by Nizami Ganjavi, *Gulistan* by Sadi—the decorator of the United Nations building door head) in addition to the polymaths and physicians (e.g., Avicenna), mathematicians and astronomers (e.g., Khwarizmi), and other polymaths and scientists who were the renaissance people of their time. https://en.wikipedia.org/wiki/Persian_language

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198 *The Mountain between Us* is a motion picture made in 2017 and directed by Hany Abu-Asad. It may be used as a lesson for how to better prepare for flying over mountainous areas. The movie depicts an unexpected landing turned lucky for a doctor and a journalist of different backgrounds, each bringing their own strength to survive their unforgettable journey.

199 *The Spirit of St. Louis* is a motion picture made in 1957 and directed by Billy Wilder. It may be used to learn about flight mechanics, flying in general, and navigation using maps over long hauls. The movie depicts the unforgettable 1927 journey from New York to Paris of Charles Augustus Lindbergh, the American aviator and explorer.

200 https://en.wikipedia.org/wiki/Universal_Time#Universal_Time_and_standard_time

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202 Charles Augustus Lindbergh crossed the Atlantic on an aircraft christened *Spirit of St. Louis*, a semi-monocoque, fabric-covered, single-seat, single-engine

high-wing monoplane (the Ryan NYP CAB registration: N-X-211). The AC was designed jointly by C. A. Lindbergh and Ryan's chief engineer Donald A. Hall. Lindbergh took off from San Diego on May 10, 1927, stopped at St. Louis, arriving at Roosevelt Field on New York's Long Island from which he made the famous flight to Paris. His autobiography entitled *WE* was the first of 15 books he eventually wrote or to which he made significant contributions; it was about the spiritual partnership that he had developed between himself and his airplane during the dark hours of his flight. https://en.wikipedia.org/wiki/Charles_Lindbergh

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